Optimal Income Taxation in Unionized Labor Markets*

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Abstract

This paper extends the Diamond (1980) model with labor unions to study optimal income taxation and to analyze whether unions can be desirable for income redistribution. Unions bargain with firms over wages in each sector and firms unilaterally determine employment. Optimal unemployment benefits and optimal income taxes are lower in unionized labor markets. Unions raise the efficiency costs of income redistribution, because unemployment benefits and income taxes raise wage demands and thereby generate involuntary unemployment. We show that unions are socially desirable only if they represent (low-income) workers whose participation is subsidized on a net basis. By creating implicit taxes on work, unions alleviate the labor-market distortions caused by income taxation. We empirically verify whether i) participation tax rates are lower if unions are more powerful, and ii) unions are desirable by compiling our own data set with union densities and participation tax rates for 18 sectors in 23 advanced countries. In line with our theoretical predictions, we find that participation tax rates are lower if unions are stronger. Moreover, the desirability condition for unions is never met empirically. Numerical simulations for the Netherlands confirm that unions are typically not desirable and optimal participation taxes are lower if unions are stronger.

Keywords: optimal taxation, unions, wage bargaining, labor participation
JEL classification: H21, H23, J51, J58

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1 Introduction

Unions play an important role in modern labor markets. Figure 1 plots union membership and coverage rates among three groups of OECD-countries over the period 1960-2011. While union membership has shown a steady downward trend since the early 1980s, the fraction of labor contracts covered by collective labor agreements has decreased by much less and remains high, especially in continental European and Nordic countries. Despite their importance, surprisingly little is known about the impact of unions on the optimal design of redistributive policies. Therefore, this paper aims to study optimal income redistribution in unionized labor markets. It asks two main questions: *'How should the government optimize income redistribution if labor markets are unionized?'* And: *'Can labor unions be socially desirable if the government wants to redistribute income?'* Although some papers have analyzed optimal taxation in unionized labor markets, no paper has, to the best of our knowledge, studied the desirability of unions for income redistribution.

To answer these questions, we extend the extensive-margin models of Diamond (1980), Saez (2002), and Choné and Laroque (2011) with unions. Workers are heterogeneous with respect to their costs of participation and the sector (or occupation) in which they can work. Workers choose whether to participate or not, and supply labor on the extensive margin if they succeed in finding a job. In the baseline version of our model, we abstract from an intensive labor-supply margin.[1] The extensive margin is often considered empirically more relevant compared to the intensive margin, especially at the lower part of the income distribution.[2] Workers within a sector are represented by a union, which maximizes the expected utility of its members. Firm-owners employ a stock of capital and different labor types to produce a final consumption good. Our baseline is the canonical Right-to-Manage (RtM) model of Nickell and Andrews (1983). The wage in each sector is determined through bargaining between firm-owners and unions. Firm-owners, in turn, unilaterally determine how many workers to hire.[3] Finally, there is a redistributive government that sets income taxes, unemployment benefits, and profit taxes to maximize social welfare. Our main findings are the following.

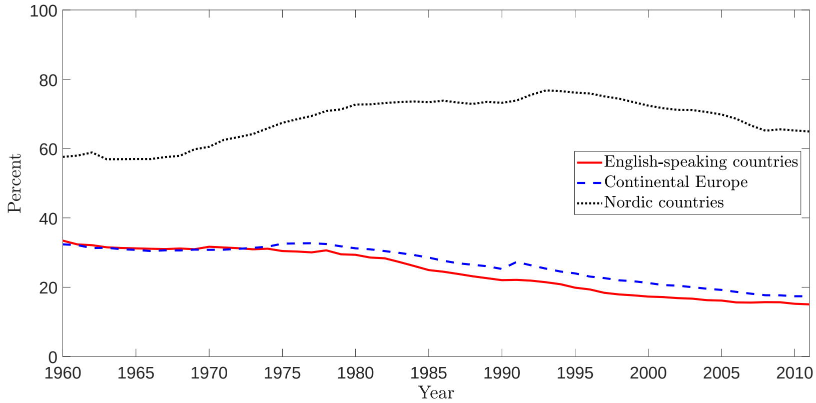
First, we answer the question how income taxes should be set in unionized labor markets. We show that optimal participation taxes (i.e., the sum of income taxes and unemployment benefits) are lower if unions are more powerful.[4] Intuitively, high income taxes and unemployment benefits worsen the inside option of workers relative to their outside option. Hence, higher participation taxes induce unions to bid up wages above market-clearing levels. This results in involuntary unemployment, which generates a welfare loss. Involuntary unemployment creates an implicit tax, which exacerbates the explicit tax on labor participation. Consequently,

[1] In an extension we analyze the case where individuals can choose their occupation, which Saez (2002) refers to as the 'intensive margin' in a model with discrete labor choices.

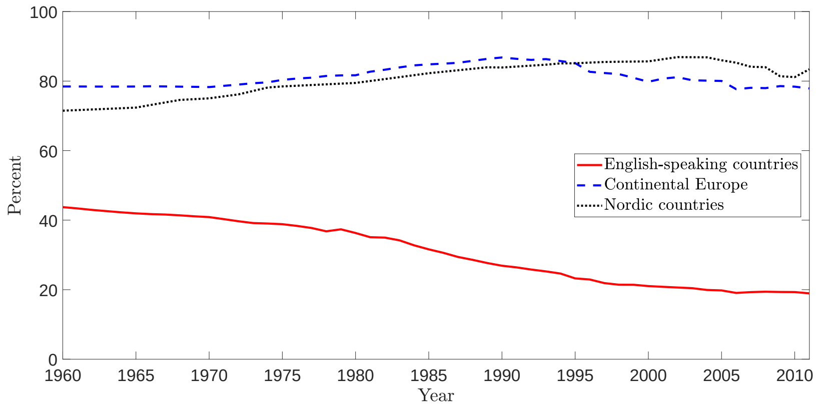
[2] See, for instance, Heckman (1993), Eissa and Liebman (1996), and Meyer (2002).

[3] The RtM-model nests both the monopoly-union (MU) model of Dunlop (1944) and the competitive model as special cases. We also analyze the efficient bargaining (EB) model of McDonald and Solow (1981) in an extension. Together with the RtM-model, these are the canonical union models, see Layard et al. (1991), Booth (1995), and Boeri and Van Ours (2008).

[4] Because participation no longer equals employment if there is involuntary unemployment, Jacquet et al. (2014) and Kroft et al. (2020) prefer the term *employment tax* over the term *participation tax*. In line with most of the literature, we use the term 'participation tax', keeping this caveat in mind.



(a) Union membership



(b) Union coverage

Figure 1: Union membership (a) and union coverage (b). Data are obtained from the ICTWSS Database version 5.1 (ICTWSS, 2016). Membership is measured as the fraction of wage earners in employment who are member of a union, and coverage as the fraction of employees covered by collective labor agreements. Missing observations are linearly interpolated. The countries included are: Australia, Canada, the United Kingdom, the United States ('English-speaking countries'), Austria, Belgium, France, Germany, the Netherlands, Switzerland ('Continental Europe'), Denmark, Finland, Norway and Sweden ('Nordic countries'). Averages are computed using population weights, which are obtained from the OECD database (OECD, 2020).

optimal participation taxes are lowered. It may be optimal to subsidize participation even for workers whose social welfare weight is below average, which never occurs if labor markets are competitive, cf. Diamond (1980), Saez (2002), and Choné and Laroque (2011). Participation subsidies are therefore more likely to be desirable if unions are more powerful.

Second, we answer the question whether unions are desirable for income redistribution. We show that, if taxes are optimally set, and labor rationing is efficient, then unions are desirable if and only if they represent workers with an above-average social welfare weight.[5,6] Intuitively, participation is subsidized on a net basis for these workers, see also Diamond (1980), Saez

[5]Efficient rationing in our model means that the burden of unemployment is borne by the workers with the highest participation costs.

[6]The social welfare weight is defined as the monetary welfare gain of transferring a euro to a worker with a particular income. In the optimal tax system, the average social welfare weight over all workers equals one, since the government ensures that the marginal social value of resources is the same in the public and private sector.

(2002), and Choné and Laroque (2011). Consequently, labor participation is distorted upwards. Unions alleviate the upward distortion in employment by bidding up wages. Hence, involuntary unemployment acts as an implicit tax, which partially off-sets the explicit subsidy on labor participation.[7] Consequently, participation subsidies and labor unions are complementary instruments to raise the net incomes of the low-skilled. The reverse is also true: unions are never desirable if they represent workers with a below-average social welfare weight, since labor participation is then taxed on a net basis.[8] In that case, implicit taxes from involuntary unemployment exacerbate explicit taxes on labor participation.

We compile our own data set of 294 observations in 23 advanced countries and 18 sectors to empirically verify i) whether stronger unions are associated with lower participation tax rates (the sum of income taxes and unemployment benefits as a fraction of the wage), and ii) whether the desirability condition for unions is met. We deploy union densities at the sectoral from the so-called "Jelle Visser Database" as our measure for union power. Moreover, we calculate participation tax rates at the sectoral level using the online tax-benefit calculator of the Organisation for Economic Co-operation and Development (OECD), and by using sectoral wage and employment data from the OECD and the International Labor Organization (ILO). Our analysis demonstrates that unions are indeed associated with lower participation tax rates, as we theoretically predicted. In our sample, average participation tax rates are predicted to be on average about 4%-points lower if union densities are set to zero. Moreover, we find that participation tax rates are always positive, which implies the desirability conditions for unions is not met empirically for any country in our sample.

To further explore the quantitative importance of unions for optimal tax policy, and to study whether an increase in union power is socially desirable, we simulate a structural version of our model for the Netherlands, where in 2015 approximately 79.4% of all employees were covered by collective labor agreements (OECD, 2020). For plausible values of labor-demand and participation elasticities, optimal participation tax rates are on average 7.4 percentage points lower in unionized labor markets than in perfectly competitive labor markets. The reduction in participation tax rates is brought about by lower income taxes and lower unemployment benefits. Furthermore, in most of our simulations unions are not socially desirable, corroborating our empirical findings. However, this finding is sensitive to the redistributive preferences of the government. It can be overturned if the government attaches a sufficiently large social welfare weight to low-income workers.

We also investigate the robustness of our findings by relaxing a number of important assumptions. Specifically, we study extensions where: i) unions respond to marginal tax rates, ii) labor rationing is not fully efficient, iii) individuals can endogenously choose the sector in which they work, iv) a national union bargains over all sectoral wages with the aim to compress the wage distribution, and v) unions and firms bargain over wages and employment, as in the efficient bargaining model of McDonald and Solow (1981). We show that expressions for op-

[7]This finding echoes the results of Lee and Saez (2012) and Gerritsen and Jacobs (2020), who show that, if labor rationing is efficient, a binding minimum wage raises social welfare if the social welfare weight of the workers for whom the minimum wage binds is above the average.

[8]The net tax on participation is the sum of the participation tax and the implicit tax on labor. As indicated above, it is possible to have an explicit participation subsidy even with a below-average social welfare weight. This is the case if the implicit tax is larger than the explicit subsidy on labor.

timal participation taxes remain the same if sectoral choice is endogenous or a national union bargains over all wages, since we express our tax rules in terms of sufficient statistics. If unions moderate their wage demands in response to higher marginal tax rates, optimal marginal tax rates are reduced if labor participation is subsidized on a net basis or if a higher wage generates redistributional gains. Moreover, optimal taxes are modified to account for implicit taxes under inefficient rationing and implicit labor subsidies under efficient bargaining. We show that our condition for the desirability of unions carries over to the cases where sectoral choice is endogenous, a national union bargains over all wages, and there is efficient bargaining. In contrast, if unions respond to marginal tax rates, the desirability condition for unions is slightly weaker as it depends on both social welfare weights and participation taxes. In addition, the desirability condition becomes tighter if labor rationing is not fully efficient, since the union exacerbates inefficiencies in labor rationing.

The remainder of this paper is organized as follows. Section 2 discusses the related literature. Section 3 outlines the basic structure of the model, characterizes general equilibrium, and discusses the comparative statics. Section 4 analyzes how participation taxes, unemployment benefits, and profit taxes should optimally be set. Section 5 analyzes the desirability of labor unions. Section 6 summarizes the main findings of several robustness checks that are analyzed in the online Appendix. Section 7 empirically studies whether participation tax rates are lower if unions are stronger and whether the desirability condition for unions holds in the data. Section 8 presents our simulations. Section 9 concludes. Finally, the Appendix to this paper contains the proofs and provides additional details on the simulations. An online Appendix contains a number of extensions and proofs of the claims we make in Section 6 and describes the details on the compilation of the dataset that is used Section 7.

2 Related literature

Our paper relates to several branches in the literature. First, there is an extensive literature which analyzes the comparative statics of taxes on wages and employment in union models, see, e.g., Hersoug (1984), Lockwood and Manning (1993), Bovenberg and van der Ploeg (1994), Koskela and Vilmunen (1996), Fuest and Huber (1997), Sørensen (1999), Fuest and Huber (2000), Lockwood et al. (2000), Bovenberg (2006), Aronsson and Sjögren (2004), Sinko (2004), van der Ploeg (2006), and Aronsson and Wikström (2011). In these papers, high unemployment benefits and high income taxes (i.e., high *average* tax rates) improve the position of the unemployed relative to the employed, which raises wage demands and lowers employment. Moreover, high marginal tax rates (for given average tax rates) moderate wage demands and boost employment, since wage increases are taxed at higher rates. If, however, individuals can also adjust their working hours, the impact of higher marginal tax rates on overall employment (i.e., total hours worked) becomes ambiguous (Sørensen, 1999, Fuest and Huber, 2000, Aronsson and Sjögren, 2004, and Koskela and Schöb, 2012). We contribute to this literature by studying optimal taxation rather than deriving comparative statics.

Second, there is a literature on optimal taxation in unionized labor markets to which we contribute. Palokangas (1987), Fuest and Huber (1997), and Koskela and Schöb (2002) analyze models with exogenous labor supply. They show that the first-best optimum can be achieved, provided that the government can tax profits and it can prevent unions from setting above market-clearing wages via income or payroll taxes. First-best cannot be achieved in our model, because labor supply is endogenous and the government does not observe participation costs. Aronsson and Sjögren (2003), Aronsson and Sjögren (2004), and Kessing and Konrad (2006) study labor supply on the intensive margin, which also prevents a first-best outcome. These studies find that the impact of unions on optimal taxes is ambiguous, because higher marginal tax rates moderate wage demands, and thus reduce unemployment, but they also increase labor-supply distortions on the intensive margin.[9] Instead, in our model labor supply responds only on the extensive margin.[10] Consequently, optimal participation taxes are lower because higher taxes induce unions to bid up wages, which generates involuntary unemployment.

Third, our paper is related to Diamond (1980), Saez (2002), and Choné and Laroque (2011), who analyze optimal redistributive income taxation with extensive labor-supply responses. Christiansen (2015) extends these analyses by allowing for imperfect substitutability between different labor types, so that wages are endogenous. These studies show that participation subsidies are optimal for low-income workers with an above-average social welfare weight. We extend these analyses to settings where wages are determined endogenously through bargaining between unions and firm-owners. Our model nests Diamond (1980), Saez (2002), and Choné and Laroque (2011) if labor types are perfect substitutes and it nests Christiansen (2015) if there are no unions. We find that optimal income taxes are less progressive and benefits are lower if unions create involuntary unemployment. In addition, we show that participation subsidies may be optimal even for workers with a below-average social welfare weight.

Fourth, our study is related to Christiansen and Rees (2018), who study optimal taxation in a model with occupational choice and a single union, which is concerned with wage compression. In contrast to our paper, they abstract from involuntary unemployment and focus instead on the misallocation generated by wage compression. They show that unions have an ambiguous effect on optimal taxes, because wage compression alters both the distortions and the distributional benefits of income taxes. In contrast to Christiansen and Rees (2018), we find in an extension of our model that optimal tax rules – expressed in sufficient statistics – do not change if unions are concerned with wage compression.

3 Model

We consider an economy which includes workers, unions, firm-owners, and a government. The basic structure of the model follows Diamond (1980), except that in the baseline we consider a finite number of labor types which are imperfect substitutes in production.[11] Within each sector (or occupation), workers are represented by a single labor union that negotiates wages with (representatives of) firm-owners. The latter exogenously supply capital and produce a final consumption good using the labor input of workers in different sectors. The government aims

[9] For instance, Aronsson and Sjögren (2004) show that the optimal labor income tax might be either progressive or regressive depending on whether working hours are determined by the union or by workers themselves.

[10] We study an extension with an occupational decision in the online Appendix. Moreover, we also study an extension where unions respond to marginal tax rates in the online Appendix.

[11] In the extension where unions respond to marginal tax rates, we allow for a continuum of labor types.

to maximize social welfare by redistributing income between unemployed workers, employed workers, and firm-owners. We assume that each union takes tax policy as given and does not internalize the impact of its decisions on the government budget.[12]

3.1 Workers

Workers differ in two dimensions: their participation costs and the sector in which they can work. There is a discrete number of I sectors. A worker type $i \in \mathcal{I} \equiv \{1, \cdots, I\}$ can work only in sector i, where she earns wage w_i and pays taxes T_i. We denote by N_i the mass of individuals who can work in sector i. If an individual works, she incurs a monetary participation cost φ, which is private information and has domain $[\underline{\varphi}, \overline{\varphi}]$, with $\underline{\varphi} < \overline{\varphi} \leq \infty$. The cumulative distribution function of participation costs of workers is allowed to vary across sectors and is denoted by $G_i(\varphi)$. We assume that workers cannot switch between sectors in the baseline version of the model and analyze the case with an occupational choice in an extension.

Each worker is endowed with one indivisible unit of time and decides whether she wants to work or not. All workers derive utility from consumption net of participation costs that are modeled as utility costs of working.[13] Their utility function $u(\cdot)$ is increasing and weakly concave. The *net* consumption of an employed worker in sector i with participation costs φ equals labor income w_i minus income taxes T_i and participation costs φ: $c_{i,\varphi} = w_i - T_i - \varphi$. Unemployed workers consume c_u, which equals an unemployment benefit of $-T_u$, hence $c_u = -T_u$. An individual in sector i with participation costs φ is willing to work if and only if

$$u(c_{i,\varphi}) = u(w_i - T_i - \varphi) \geq u(-T_u) = u(c_u). \tag{1}$$

For each sector i, equation (1) defines a cut-off φ_i^* at which individuals are indifferent between working and not working: $\varphi_i^* \equiv w_i - T_i + T_u$. Higher wages w_i, lower income taxes T_i, and lower unemployment benefits $-T_u$ all raise the cut-off φ_i^*, and, thus, raise labor participation in sector i. Workers are said to be *involuntarily* unemployed if condition (1) is satisfied, but they are not employed.

3.2 Firms

There is a unit mass of firm-owners, who inelastically supply K units of capital, and employ all types of labor to produce a final consumption good.[14] The production technology is described

[12]Consequently, the government is the Stackelberg leader relative to firms and unions. This assumption seems most natural given that in our model workers are represented by unions at the *sectoral* level (as is the case, for instance, in the Netherlands). The distortions from unions would typically be smaller if they (partly) internalize the impact of their decisions on the government budget constraint, see, e.g., Calmfors and Driffill (1988) and Summers et al. (1993).

[13]This is a slight abuse of terminology, since we assume that individuals who are *involuntarily* unemployed do *not* incur these costs. Nevertheless, we prefer to use the term 'participation costs' rather than 'costs of working', to stay close to the literature on optimal taxation with extensive-margin labor-supply responses. For analytical convenience, we model participation costs as a pecuniary cost rather than a utility cost, see also Choné and Laroque (2011). Utility is then a function of consumption net of participation costs. Whether participation costs are modeled as pecuniary or a utility costs has no bearing on our results.

[14]Alternatively, we could assume that there are sector-specific firms producing a single, final consumption good. As long as the government is able to observe (and tax) profits of all firms, none of our results change. The same is true if firm-ownership would be equally shared among workers or, in case of unequal ownership, if profits can

by a production function:

$$F(K, L_1, \cdots, L_I), \quad F_K(\cdot), F_i(\cdot) > 0, \quad F_{KK}(\cdot), F_{ii}(\cdot), -F_{Ki}(\cdot) \leq 0. \tag{2}$$

Here, the subscripts refer to the partial derivatives with respect to capital and labor in sector i. We assume that capital and labor have positive, non-increasing marginal returns. Moreover, capital and labor in sector i are co-operant production factors ($F_{Ki} \geq 0$). We do not make specific assumptions regarding the complementarity of different labor types. In a number of special cases, we invoke the assumption that labor markets are independent.

Assumption 1. (Independent labor markets) *The marginal product of labor in sector i is unaffected by the amount of labor employed in sector $j \neq i$, i.e., $F_{ij}(\cdot) = 0$ for all $i \neq j$.*

Under Assumption 1, there are no spillover effects between different sectors in the labor market.[15]

Profits Π equal output minus wage costs:

$$\Pi = F(K, L_1, \cdots, L_I) - \sum_i w_i L_i. \tag{3}$$

Firm-owners maximize profits, while taking sectoral wages w_i as given. The first-order condition for profit maximization in each sector i is given by:

$$w_i = F_i(K, L_1, \cdots, L_I). \tag{4}$$

Firms demand labor until its marginal product is equal to the wage. The labor-demand elasticity ε_i in sector i is defined as $\varepsilon_i \equiv -F_i(\cdot)/(L_i F_{ii}(\cdot)) > 0$.[16]

Firm-owners consume their profits net of taxes. Their utility is given by $u(c_f) = u(\Pi - T_f)$, where T_f denotes the profit tax. The profit tax is non-distortionary, as it affects none of the firms' decisions.

3.3 Unions and labor-market equilibrium

All workers in sector i are organized in a union, which aims to maximize the expected utility of its members.[17] We characterize labor-market equilibrium in sector i using a version of the Right-to-Manage (RtM) model due to Nickell and Andrews (1983). In this model, the wage w_i is determined through bargaining between the union in sector i and (representatives of) firm-owners. Individual firm-owners in each sector take the negotiated wage w_i as given and have the 'right to manage' how much labor to employ. The RtM-model nests both the competitive equilibrium (CE) as well as the monopoly-union (MU) model of Dunlop (1944) as special cases.

be taxed and rebated to employed and unemployed workers in a lump-sum way.

[15]Such spillover effects may also occur with an occupational-choice margin. We return to this point in more detail below.

[16]If Assumption 1 holds, then the demand for labor in sector i depends only on the wage in sector i (i.e., $L_i = L_i(w_i)$, where $L_i'(\cdot) = 1/F_{ii}(\cdot)$) and the labor-demand elasticity ε_i depends only on L_i.

[17]The qualitative predictions of the model are robust to changing the union objective as long as the union cares about *both* wages and employment, and as long as the negotiated wage extends to the non-union members. For example, we could allow for different degrees of union membership across workers with different participation costs.

Because union members differ in their participation costs, we have to make an assumption on labor rationing: which workers become unemployed if the wage is set above the market-clearing level? In most of what follows, we assume that labor rationing is efficient (cf. Lee and Saez, 2012, Gerritsen, 2017, and Gerritsen and Jacobs, 2020).

Assumption 2. (Efficient Rationing) *The incidence of involuntary unemployment is borne by the workers with the highest participation costs.*

If labor markets are competitive, there is no involuntary unemployment and Assumption 2 is trivially satisfied. However, if there is involuntary unemployment, there is no reason to believe that only individuals with the highest participation costs bear the burden of unemployment, see also Gerritsen (2017). The assumption of efficient rationing clearly biases our results in favor of unions and will be relaxed in Section 6.

Let $E_i \equiv L_i/N_i$ denote the employment rate for workers in sector i. Under Assumption 2, workers with participation costs $\varphi \in [\underline{\varphi}, \hat{\varphi}_i]$, where $\hat{\varphi}_i \equiv G_i^{-1}(E_i)$, are employed, whereas those with participation costs $\varphi \in (\hat{\varphi}_i, \overline{\varphi}]$ are not employed. Workers with participation costs $\varphi \in (\hat{\varphi}_i, \varphi_i^*]$ are involuntarily unemployed, since they prefer to work but cannot find employment. Workers with participation costs $\varphi \in (\varphi_i^*, \bar{\varphi}]$ do not participate ('voluntary unemployment'). Because participation is voluntary, the fraction of workers willing to participate is weakly larger than the rate of employment: $E_i = G_i(\hat{\varphi}_i) \leq G_i(\varphi_i^*)$. If union i maximizes the expected utility of its members, and labor rationing is efficient, the union's objective function can be written as:

$$\Lambda_i = \int_{\underline{\varphi}}^{\hat{\varphi}_i} u(c_{i,\varphi}) \mathrm{d}G_i(\varphi) + \int_{\hat{\varphi}_i}^{\overline{\varphi}} u(c_u) \mathrm{d}G_i(\varphi) = E_i \overline{u(c_i)} + (1 - E_i) u(c_u), \tag{5}$$

where $\overline{u(c_i)} \equiv \int_{\underline{\varphi}}^{\hat{\varphi}_i} u(c_{i,\varphi}) \mathrm{d}G_i(\varphi)/E_i$ denotes the average utility of employed workers in sector i.

To characterize equilibrium, we employ a version of the RtM-model that allows for any degree of union power. This is graphically illustrated in Figure 2. The competitive equilibrium (CE) lies at the intersection of the labor-supply curve and the labor-demand curve. The monopoly-union (MU) outcome, in turn, lies at the point where the union's indifference curve is tangent to the labor-demand curve. Any point on the bold part of the labor-demand curve corresponds to an equilibrium in the RtM-model. The higher (lower) is union power, the closer is the outcome to the monopoly-union (competitive) outcome. Therefore, the monopoly-union outcome and the competitive outcome represent the two polar cases in our analysis.

We refer to the monopoly-union (MU) model if the union in sector i has full bargaining power. In this case, the union chooses the combination of the wage w_i and the rate of employment E_i, which maximizes its objective (5) subject to the labor-demand equation (4). This leads to the following (implicit) wage-demand equation for each sector i:

$$1 = \varepsilon_i \frac{u(\hat{c}_i) - u(c_u)}{\overline{u'(c_i)} w_i}, \tag{6}$$

where $u(\hat{c}_i)$ denotes the utility of the marginally employed worker (i.e., the worker with participation costs $\hat{\varphi}_i$), and $\overline{u'(c_i)}$ is the average marginal utility of employed workers in sector i. If the union has full bargaining power, it demands a wage w_i in sector i such that the marginal

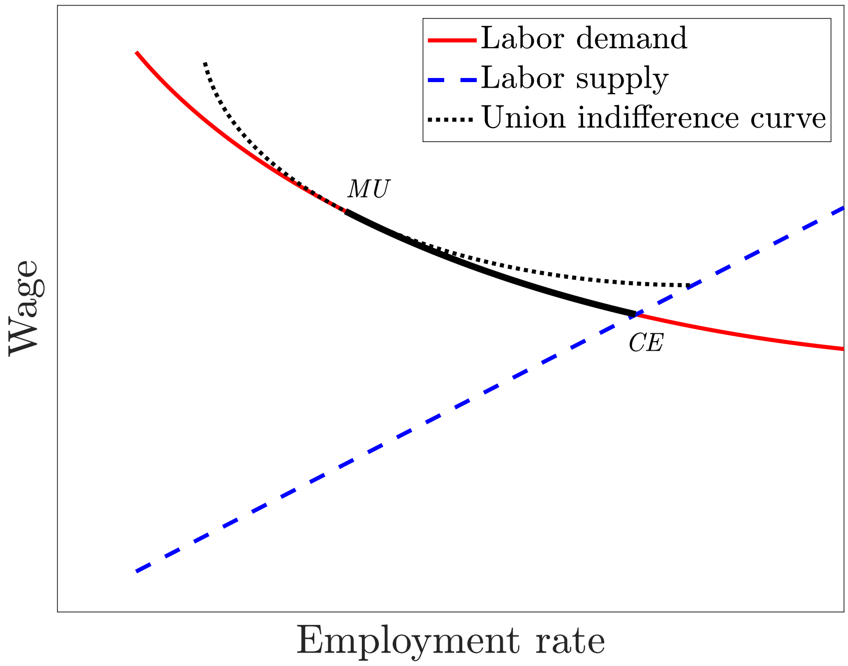


Figure 2: Labor-market equilibria in the Right-to-Manage model

benefit of raising the wage for the employed with one euro (on the left-hand side) equals the marginal cost of higher unemployment (on the right-hand side). The marginal cost of setting the wage above the market-clearing level equals the elasticity of labor demand multiplied with the marginal worker's monetized utility gain of finding employment as a fraction of the wage: $\frac{u(\hat{c}_i)-u(c_u)}{u'(c_i)w_i}$. Importantly, because rationing is efficient, the costs of setting a higher wage depend only on the utility loss of the marginally employed workers, since they lose their jobs first following an increase in the wage. Furthermore, equation (6) implies that an increase in either the income tax T_i or the unemployment benefit $-T_u$ raises wage demands. Intuitively, higher income taxes T_i or unemployment benefits $-T_u$ make the outside option of not working more attractive relative to the inside option of working.

The polar opposite case is the competitive outcome, where unions have no bargaining power at all. In this case, the wage is driven to the point where the marginally employed worker is indifferent between participating and not participating (i.e., $u(\hat{c}_i) = u(c_u)$) and labor demand equals labor supply for each sector i:

$$E_i = G_i(\varphi_i^*). \tag{7}$$

Since there is no involuntary unemployment, we have $\hat{\varphi}_i = \varphi_i^* = w_i - T_i + T_u$. A reduction in either the income tax T_i or the unemployment benefit $-T_u$ leads to higher employment and, through the labor-demand equation (4), to a lower wage. The reduction in the wage and the increase in employment comes about through an increase in labor participation, rather than through a reduction in the union's wage demand.

A common approach to characterize the labor-market equilibrium for an intermediate degree of union power is to solve the Nash bargaining problem between the union and the firm. Here, we choose a different approach. Rather than using bargaining weights, we introduce a *union power parameter* $\rho_i \in [0,1]$, which directly determines which equilibrium is reached in the wage negotiations. In particular, we modify the wage-demand equation (6) and characterize labor-market equilibrium for each sector i as:

$$\rho_i = \varepsilon_i \frac{u(\hat{c}_i) - u(c_u)}{\overline{u'(c_i)}w_i}. \tag{8}$$

The union power parameter ρ_i determines which point on the labor-demand curve between MU and CE is reached in the wage negotiations. If $\rho_i = 1$, the outcome corresponds to the equilibrium in the MU-model. If $\rho_i = 0$, the outcome corresponds to the CE. Consequently, $\rho_i \in (0,1)$ corresponds to any intermediate degree of union bargaining power in the RtM-model. The higher (lower) is ρ_i, the higher (lower) is the negotiated wage.

In what follows, union power ρ_i is treated as policy-invariant. In Appendix A, we derive that there is a direct relationship between our measure of union power ρ_i and the union's Nash-bargaining parameter in the RtM-model. Hence, we can use either parameter to rationalize any point on the labor-demand curve in equilibrium. However, treating the Nash-bargaining parameter as fixed leads to technical complications that we circumvent. For this reason, we prefer to characterize equilibrium and derive the optimal tax results using our measure of union power.

3.4 Government

The government is assumed to maximize a social welfare function $\mathcal{W}$:

$$\mathcal{W} \equiv \sum_i \psi_i N_i (E_i \overline{u(c_i)} + (1 - E_i)u(c_u)) + \psi_f u(c_f), \tag{9}$$

where ψ_f is the Pareto weight that the government attaches to firm-owners and ψ_i is the Pareto weight that the government attaches to individuals who work in sector i. We assume throughout that Pareto weights are lower for workers in sectors where wages are higher. By attaching the same Pareto weight to all workers within the same sector, this government objective respects the union's objective by imposing the same preferences for income redistribution within a sector.[18]

The informational assumptions in our model are as follows. The government observes the employment status of all workers, all sectoral wages, and firm profits. However, individual participation costs φ are private information, as in Diamond (1980), Saez (2002), and Choné and Laroque (2011).[19] This assumption is the most natural one to make, as in reality the government lacks the information to redistribute income between workers who have the same income but different participation costs. The non-observability of participation costs also implies that the government is unable to distinguish workers who are voluntarily unemployed and those who are involuntarily unemployed. In particular, only workers with participation costs $\varphi \in (\hat{\varphi}_i, \varphi_i^*]$

[18] Conflicting government and union objectives would introduce unnecessary complications, from which we like to abstain.

[19] This assumption is the analogue of the non-observability of earning ability in the Mirrlees (1971) model.

are involuntarily unemployed, while workers with participation costs $\varphi \in (\varphi_i^*, \bar{\varphi}]$ are voluntary unemployed. To distinguish both types of workers thus requires information on the participation costs φ of each worker. Therefore, if participation costs are realistically not observable, then tax policy cannot be conditioned on φ. Hence, the assumption that participation costs are not observable implies the government needs to resort to distortionary taxes and transfers to redistribute income and optimal tax policy can at best implement a second-best allocation.[20]

In line with our informational assumptions, the government can set income taxes T_i, as well as a profit tax T_f to finance an unemployment benefit $-T_u$ and an exogenous revenue requirement R. The government's budget constraint is given by:

$$\sum_i N_i(E_i T_i + (1 - E_i)T_u) + T_f = R. \tag{10}$$

3.5 Equilibrium and behavioral responses

General equilibrium with unions is defined as follows.

Definition 1. *An equilibrium with unions consists of wages w_i and employment E_i in each sector i such that, for given union power ρ_i and taxes T_i, T_u, and T_f:*

1. *For all sectors i, firms maximize profits:*

$$w_i = F_i(\cdot). \tag{11}$$

2. *For all sectors i, wages and employment satisfy the wage-demand equation of unions:*

$$\rho_i \int_{\underline{\varphi}}^{G_i^{-1}(E_i)} u'(w_i - T_i - \varphi)\mathrm{d}G_i(\varphi)F_{ii}(\cdot)N_i + \big(u(w_i - T_i - G_i^{-1}(E_i)) - u(-T_u)\big) = 0, \tag{12}$$

3. *The government runs a balanced budget as given by equation (10).*

Equations (11) and (12) determine equilibrium wages and employment in each sector i as a function of union power, unemployment benefits, and income taxes in all sectors. Without additional structure on the production function, it is generally not possible to derive the comparative statics of a change in union power or income taxes on equilibrium wages and employment rates. However, if labor markets are independent (i.e., if Assumption 1 holds), equilibrium in sector i does not depend on union power or income taxes in other sectors.[21] In that case, we can write $E_i = E_i(T_i, T_u, \rho_i)$ and $w_i = w_i(T_i, T_u, \rho_i)$ for all sectors i. Appendix B shows that an increase in union power ρ_i, income taxes T_i, or the unemployment benefit $-T_u$ raises the equilibrium wage w_i and lowers the equilibrium employment rate E_i in sector i.

Before turning to the optimal tax problem, we make the following assumption in most of what follows.

[20] A first-best allocation can be implemented only if participation costs φ would be fully verifiable and tax policy can be conditioned on participation costs φ. See Appendix C for details.

[21] With independent labor markets, one can also show that the equilibrium is unique if the union objective is concave in E_i after substituting $w_i = F_i(\cdot)$ and $\hat{\varphi}_i = G^{-1}(E_i)$. If that is the case, the first-order condition (6) of the monopoly union's maximization problem is both necessary and sufficient.

Assumption 3. (No income effects at the union level) *The equilibrium wage and employment in sector i respond symmetrically to an increase in the income tax T_i or an increase in the unemployment benefit $-T_u$: $\frac{\partial w_i}{\partial T_i} = -\frac{\partial w_i}{\partial T_u}$ and $\frac{\partial E_i}{\partial T_i} = -\frac{\partial E_i}{\partial T_u}$.*

Under Assumption 3, giving both the employed and the unemployed an additional euro does not affect equilibrium wages and employment rates.[22] This assumption is made solely for analytical convenience, as none of our results critically depend on it, see Appendix D.2 and E.1. If Assumptions 1 and 3 hold, the equilibrium wage and employment in sector i depend only on union power and the participation tax $T_i - T_u$ in sector i. The behavioral responses are given in the following Lemma.

Lemma 1. *If Assumptions 1 (independent labor markets), 2 (efficient rationing), and 3 (no income effects at the union level) are satisfied, then the comparative statics of an increase in the participation tax $T_i - T_u$ on equilibrium wages and employment rates in each sector i are given by:*

$$\frac{\mathrm{d}E_i}{\mathrm{d}(T_i - T_u)} = \frac{\rho_i E_i \overline{u_i''} N_i F_{ii} + \hat{u}_i'}{\rho_i E_i \overline{u_i''}(F_{ii}N_i)^2 + \rho_i E_i \overline{u_i'} F_{iii} N_i^2 + \hat{u}_i'((1+\rho_i)F_{ii}N_i - 1/G_i')} < 0, \tag{13}$$

$$\frac{\mathrm{d}w_i}{\mathrm{d}(T_i - T_u)} = \frac{(\rho_i E_i \overline{u_i''} N_i F_{ii} + \hat{u}_i')F_{ii}N_i}{\rho_i E_i \overline{u_i''}(F_{ii}N_i)^2 + \rho_i E_i \overline{u_i'} F_{iii} N_i^2 + \hat{u}_i'((1+\rho_i)F_{ii}N_i - 1/G_i')} > 0, \tag{14}$$

where we ignored function arguments to save on notation, and $G_i' \equiv G_i'(E_i)$.

Proof. See Appendix B. □

According to Lemma 1, an increase in the participation tax (resulting from either an increase in the income tax or the unemployment benefit) raises the union's wage demand, which reduces labor demand, and thus lowers employment.

4 Optimal taxation

The government optimally chooses participation taxes $T_i - T_u$, the unemployment benefit $-T_u$, and profit taxes T_f to maximize social welfare (9), subject to the government budget constraint (10), while taking into account the behavioral responses to tax policy. We characterize optimal tax policy in terms of elasticities and social welfare weights.[23] Social welfare weights of employed workers in sector i and the firm-owners are denoted by $b_i \equiv \psi_i \overline{u'(c_i)}/\lambda$ and $b_f \equiv \psi_f u'(c_f)/\lambda$, where λ is the multiplier on the government budget constraint. The social welfare weight of the unemployed is given by the weighted average of the social welfare weights of the unemployed $\psi_i u'(c_u)/\lambda$ in each sector i:

$$b_u \equiv \frac{\sum_i N_i(1-E_i)\psi_i u'(c_u)/\lambda}{\sum_i N_i(1-E_i)}. \tag{15}$$

The social welfare weight measures the monetized increase in social welfare resulting from a one unit increase in income. The following Proposition characterizes optimal tax policy.

[22] This is an assumption on the individual utility function $u(\cdot)$ that is always satisfied if $u(\cdot)$ is linear. Appendix B shows that income effects at the union level are also absent if $u(\cdot)$ is of the CARA-type. We are not aware of other utility functions for which this assumption holds.

[23] We also implicitly characterize the optimal tax system in terms of the model's primitives in Appendix E.1.

Proposition 1. *Suppose Assumptions 2 (efficient rationing) and 3 (no income effects at the union level) hold, then the optimal unemployment benefit $-T_u$, profit taxes T_f, and participation taxes $T_i - T_u$ are determined by:*

$$\omega_u b_u + \sum_i \omega_i b_i = 1, \tag{16}$$

$$b_f = 1, \tag{17}$$

$$\sum_j \omega_j \left(\frac{t_j + \tau_j}{1 - t_j}\right) \eta_{ji} = \omega_i(1 - b_i) + \sum_j \omega_j (b_j - b_f)\kappa_{ji}, \quad \forall i, \tag{18}$$

where

$$\omega_i \equiv \frac{N_i E_i}{\sum_j N_j}, \quad \omega_u \equiv \frac{\sum_i N_i(1 - E_i)}{\sum_j N_j}, \quad t_j \equiv \frac{T_j - T_u}{w_j}, \quad \tau_j \equiv \frac{\psi_j(\hat{u}_j - u_u)}{w_j \lambda} = \frac{\rho_j b_j}{\varepsilon_j}, \tag{19}$$

$$\eta_{ji} \equiv -\left(\frac{\partial E_j}{\partial (T_i - T_u)} \frac{w_i - (T_i - T_u)}{E_j}\right) \frac{w_j(1 - t_j)}{w_i(1 - t_i)}, \tag{20}$$

$$\kappa_{ji} \equiv \left(\frac{\partial w_j}{\partial (T_i - T_u)} \frac{w_i - (T_i - T_u)}{w_j}\right) \frac{w_j}{w_i(1 - t_i)}. \tag{21}$$

ω_i and ω_u are the shares of employed workers in sector i and the unemployed, t_j is the participation tax rate in sector j, τ_j is the union wedge in sector j, η_{ji} and κ_{ji} are the elasticities of employment and wages in sector j with respect to the participation tax $T_i - T_u$ weighted with relative net wages.

Proof. See Appendix D.1. □

Equation (16) states that a weighted average of the social welfare weights of the employed and unemployed workers equals one.[24] This is a well-known result in optimal tax theory. Intuitively, the government uniformly raises transfers to all individuals until the marginal utility benefits of a higher transfer (left-hand side) are equal to the unit marginal costs (right-hand side).[25] Unless the utility function $u(\cdot)$ is linear, and the government attaches equal Pareto weights to workers in all sectors, i.e., $\psi_i = \psi_f = 1$, there will be at least one sector where $b_i < 1$. Depending on the redistributive preferences of the government, there may also be employed workers whose social welfare weight is above one, see also Diamond (1980), Saez (2002), and Choné and Laroque (2011). In the remainder, we refer to workers for whom $b_i > 1$ as low-income, or low-skilled workers. If the utility function is concave, then typically the unemployed have the highest social welfare weight. Given that the social welfare weights are on average equal to one, this implies that $b_u > 1$.

Condition (17) for optimal profit taxes states that the government taxes firm-owners until their social welfare weight equals one. Since the profit tax is non-distortionary, the government raises profit taxes until it is indifferent between raising firm-owners' consumption with one unit and receiving a unit of public funds.

Equation (18) gives the first-order condition with respect to the participation tax $T_i - T_u$.

[24] If there are income effects at the union level, i.e., if Assumption 3 does not hold, this equation is slightly modified, see Appendix D.2 for details.

[25] This confirms Jacobs (2018), who shows that the marginal cost of public funds equals one in the policy optimum even under distortionary taxation.

The left-hand side gives the marginal costs in the form of larger labor-market distortions, whereas the right-hand side gives the marginal distributional benefits (or losses) of higher participation taxes in sector i. At the optimum, the distortionary costs of raising the participation tax in sector i are equated to the distributional gains over all sectors.

The overall distortion of the participation tax in sector i is given by the sum over all sectors of the total tax wedge in sector j multiplied by the weighted (cross) elasticity of employment in sector j with respect to the participation tax in sector i. The total tax on labor participation in sector j equals $t_j + \tau_j$ and consists of the explicit tax on participation t_j and the union wedge $\tau_j \equiv \psi_j(\hat{u}_j - u_u)/(w_j\lambda) = \rho_j b_j/\varepsilon_j$. A reduction in employment reduces social welfare by government revenue from the participation tax $T_j - T_u$, and it lowers social welfare through the union wedge τ_j, which is the monetized loss in social welfare as a fraction of the wage if the marginal worker in sector i loses employment. Unions generate welfare losses by bidding up wages above the market-clearing level. As a result, the marginal worker (i.e., the employed worker with the highest participation costs) is no longer indifferent between working and not working. Therefore, τ_j acts as an *implicit* tax on labor participation. The union wedge τ_j is proportional to union power ρ_j and inversely related to the labor-demand elasticity ε_j. Hence, $\tau_j = 0$ if either labor markets are competitive so that the union has no bargaining power ($\rho_j = 0$), or if labor demand is infinitely elastic ($\varepsilon_j \to \infty$). In the latter case, unions refrain from demanding a wage above the market-clearing level, since doing so would result in a complete breakdown of employment.

The main insight from Proposition 1 is that, for constant social welfare weights and behavioral responses, optimal participation taxes are lower if unions are stronger (i.e., if union wedges τ_j are larger). Intuitively, the tax system is not only geared toward income redistribution, but also aims to reduce involuntary unemployment generated by unions bidding up wages above the market-clearing level. Lower participation taxes induce unions to moderate their wage demands, and this alleviates the welfare costs of involuntary unemployment.

A higher participation tax in sector i raises wages demanded by unions in sector i. *Ceteris paribus,* this leads to a decrease in employment in sector i. Moreover, the change in the participation tax in one sector has implications for both employment and wages in all other sectors. If labor types are complementary (i.e., $F_{ij}(\cdot) > 0$ for $i \neq j$), then the decrease in employment in sector i lowers marginal productivity, and thus labor demand, in all other sectors $j \neq i$. Consequently, both employment and wages in all other sectors are reduced. The reduction in employment is larger if the (weighted) cross elasticity η_{ji} of employment in sector j with respect to the participation tax in sector i is larger. If the sum of the explicit and implicit tax on participation is positive (negative), i.e., $t_j + \tau_j > 0$ (< 0), then a higher participation tax in sector i exacerbates (alleviates) labor-market distortions in sector j. The total wedge on labor participation $t_j + \tau_j$ is weighted by the employment elasticity in sector j with respect to the participation tax in sector i (η_{ji}). Therefore, if η_{ji} is large, optimal participation taxes are lower. This is in line with the findings from Diamond (1980) and Saez (2002).

The right-hand side of equation (18) gives the sum of the marginal distributional benefits over all sectors of a higher participation tax in sector i. An increase in the participation tax directly redistributes income from workers in sector i to the government. The associated welfare effect is proportional to $1 - b_i$, which captures the rise in government revenue minus the monetized utility loss of workers if they need to pay more taxes. Furthermore, the increase in the participation tax in sector i redistributes income from firm-owners (whose social welfare weight equals b_f) to workers in sector i (whose social welfare weight equals b_i) if the wage w_i increases. Intuitively, if an increase in the participation tax in sector i raises the wage in that sector, then the wage increase yields desirable distributional benefits if the social welfare weight of workers exceeds that of firm-owners in sector i, i.e., if $b_i > b_f$. In addition, there are indirect redistributional consequences in all other sectors $j \neq i$, because wages in all other sectors are affected if participation taxes in sector i are raised. The total impact on social welfare due to general-equilibrium effects on the wage structure is obtained by summing these effects over all sectors. If the social welfare weight of workers in sector j is larger than that of firm owners, i.e., $b_j > b_f$, then the reduction in the wage in sector j due to higher participation taxes in sector i is socially costly. However, if the social welfare weight of workers in sector j is smaller than that of firm-owners, i.e., $b_j < b_f$, the reduction in the wage in sector j is welfare-enhancing. This indirect welfare effect is weighted by the wage elasticity in sector j with respect to the participation tax in sector i (κ_{ji}).

Our main finding – optimal participation taxes are lower in unionized labor markets – holds *for given social welfare weights and behavioral responses*. Clearly, both social welfare weights and behavioral responses are endogenous to the tax system. As such, our result should not be interpreted as a comparative statics exercise of the optimal participation tax with respect to union power, since then the endogeneity of behavioral responses and social welfare weights should be taken into account as well. An increase in union power only leads to a reduction in optimal participation taxes if the 'direct' impact of a larger union wedge τ_i is sufficiently large to off-set any 'indirect' impacts on elasticities and social welfare weights.[26] Furthermore, in our numerical simulations we take the endogeneity of social welfare weights and behavioral responses into account and they never overturn the direct impact of higher union power on optimal participation taxes.

Three final remarks are in order. First, it might be optimal in unionized labor markets to subsidize participation even for workers with a below-average social welfare weight, i.e., for whom $b_i < 1$. This never occurs if labor markets are competitive, see Diamond (1980), Saez (2002), and Choné and Laroque (2011). To see this, suppose labor markets are independent so that all cross effects of wages and employment with respect to participation taxes are zero ($\eta_{ji} = \kappa_{ji} = 0$ for all $j \neq i$) and substitute $b_f = 1$ in equation (18):

$$\left(\frac{t_i + \tau_i}{1 - t_i}\right) \eta_{ii} = (1 - b_i)(1 - \kappa_{ii}). \tag{22}$$

Under weak regularity conditions, a higher participation tax leads to a less than one-for-one increase in the wage: $\kappa_{ii} < 1$, see Lemma 1. The sign of the total wedge on employment, i.e., the sum of the participation tax and the union wedge, in sector i thus equals the sign of $1 - b_i$. Like in Diamond (1980), Saez (2002), and Choné and Laroque (2011), we find that

[26] An example with a closed-form solution for the optimal participation tax that depends in an ambiguous way on union power is available from the authors upon request.

it is optimal to subsidize participation, i.e., setting $t_i < 0$, for low-income workers with an above-average social welfare weight, i.e., if $b_i > 1$. However, and in contrast to these papers, in unionized labor markets subsidizing participation can also be optimal for workers with a below-average social welfare weight ($b_i < 1$). This occurs if the welfare cost of involuntary unemployment is high, so that the implicit tax τ_i is large. Intuitively, explicit subsidies on participation can be desirable to offset the distortions from implicit taxes on participation even if $b_i < 1$. The reason is that participation subsidies are not only used for income redistribution, but also to off-set downward distortions in employment generated by labor unions. A high union wedge could therefore rationalize participation subsidies even for workers with a below-average social welfare weight. In a general framework, Kroft et al. (2020) also show that the optimal participation tax can be negative for workers whose social welfare weight is below-average if wages and unemployment are endogenous to tax policy. Through the lens of their model, unions can be seen as a micro-foundation for these wage and employment responses.

Second, the formula for the optimal participation tax simplifies considerably if labor markets are competitive – irrespective of whether wages are exogenous or endogenous. It is shown in Appendix D.3 that with competitive labor markets, the optimal tax formula (18) nests the one derived in Saez (2002) and simplifies to

$$\frac{t_i}{1-t_i} = \frac{1-b_i}{\pi_i}, \tag{23}$$

where $\pi_i \equiv \frac{G_i'(\varphi_i^*)\varphi_i^*}{G_i(\varphi_i^*)}$ is the participation elasticity, which measures the percentage increase in the fraction of participants in sector i following a one-percent increase in the net payoff from working $\varphi_i^* = w_i - (T_i - T_u)$. If labor demand is infinitely elastic (i.e., if labor types are perfect substitutes in production), equations (18) and (23) coincide. In this case, unions always refrain from demanding above market-clearing wages. The result from Saez (2002) also holds if labor types are imperfect substitutes in production and there are no unions. The same result is derived as well in Christiansen (2015). If labor markets are perfectly competitive, labor-demand considerations are irrelevant for the characterization of optimal participation tax rates.[27]

Third, earlier studies on (optimal) taxation in unionized labor markets have explicitly considered restrictions on profit taxation, either to prevent a first-best outcome or to analyze rent appropriation by unions.[28] If profit taxation is restricted, e.g., due to political-economy reasons or profit-shifting opportunities, the social welfare weight of firm-owners is below the average over all (employed and unemployed) workers: $b_f < 1$. If labor markets are independent, this calls for a higher participation tax *ceteris paribus*, see equation (18) and set $\kappa_{ji} = \eta_{ji} = 0$ if $i \neq j$. A higher participation tax puts upward pressure on the wage, cf. Lemma 1.[29] This redistributes income from firm-owners to workers. The latter is more desirable (or less costly) if profit taxation is more severely restricted. The finding that income taxes are adjusted to indirectly redistribute income from firm-owners to workers has been established as well in Fuest

[27]See also Diamond and Mirrlees (1971a,b), who show that optimal taxes are the same in partial as in general equilibrium *provided* markets are competitive. Saez (2004) refers to this finding as the 'tax-formula result'.

[28]See, among others, Fuest and Huber (1997), Koskela and Schöb (2002), and Aronsson and Sjögren (2004).

[29]Lemma 1 assumes there are no income effects at the union level. However, a higher participation tax also raises the equilibrium wage if there are income effects at the union level, see Appendix B.

and Huber (1997) and Aronsson and Sjögren (2004).

5 Desirability of unions

The previous Section analyzed the optimal tax-benefit system in unionized labor markets. In this Section we ask the question: can it be socially desirable to allow workers to organize themselves in a union? And, if so, under which conditions? The following Proposition answers both questions.

Proposition 2. *If Assumption 2 (efficient rationing) is satisfied, and taxes are set optimally, then increasing union power ρ_i in sector i raises social welfare if and only if the social welfare weight of the workers in sector i is above-average, i.e., it exceeds one: $b_i > 1$.*

Proof. See Appendix E.1. □

According to Proposition 2, unions are desirable if they represent low-income workers for whom $b_i > 1$. To understand why, suppose that the tax-benefit system is optimized and union power in sector i is marginally increased: $d\rho_i > 0$. The increase in union power leads to a higher wage and a lower employment rate in sector i, see Appendix B. Moreover, it also reduces employment and wages in other sectors j if labor types are complements in production. All the effects on employment and wages, in turn, can be perfectly offset by combining the increase in union power ρ_i with a lower income tax T_i. If the tax system is optimized, a marginal change in income taxes does not change social welfare. If the joint policy reform of raising union power and changing the income tax offsets the change in the wage and employment rate in sector i, labor-market outcomes in all other sectors j will be unaffected as well. The reduction in the income tax T_i transfers income from the government (with social welfare weight 1) to workers in sector i (whose social welfare weight is b_i). An increase in union power ρ_i is therefore welfare-enhancing if and only if $b_i > 1$.

The fundamental reason why unions can raise social welfare if the tax-benefit system is optimized is that it might be optimal for the government to subsidize participation when participation costs are not observable. This, in turn, leads to upward distortions in employment. To see this, suppose that there are no unions, i.e., $\rho_i = 0$ for all i. According to equation (23), if $b_i > 1$, then participation is optimally subsidized (i.e., $T_i < T_u$), see also Diamond (1980) and Saez (2002). Consequently, labor participation is distorted upwards: too many low-skilled workers decide to participate. Unions alleviate this distortion by offsetting the explicit subsidy on participation with an implicit tax τ_i on participation. As such, unions can meaningfully complement the tax-benefit system.

The result from Proposition 2 is related to Hungerbühler and Lehmann (2009), who study optimal non-linear taxation in a matching framework. They find that increasing the worker's bargaining power leads to higher social welfare if the latter is below the level prescribed by the Hosios (1990) condition. Intuitively, raising the worker's bargaining power puts upward pressure on wages, which partly alleviates the downward distortion on wages brought about by positive marginal tax rates. By contrast, in our framework, unions can be useful to alleviate upward distortions in employment generated by participation subsidies.

Proposition 2 is also related to the findings of Lee and Saez (2012) and Gerritsen and Jacobs (2020), who show that a similar role can be played by minimum wages. Unlike the tax system, both unions and a binding minimum wage can raise the income of low-skilled workers and simultaneously reduce employment, which is desirable if participation is distorted upwards. An important difference between unions and a minimum wage is that unions, unlike a minimum wage, respond to changes in the tax system. Moreover, a minimum wage only generates unemployment at low income levels.

Unions are never desirable if the government has Rawlsian social preferences and only cares about individuals who are least well-off. In our framework, these are the (voluntarily or involuntarily) unemployed, because participation is voluntary. As a result, the social welfare weight of all employed individuals is zero: $b_i = 0$ for all i. Proposition 2 then immediately implies that an increase in union power always lowers social welfare if the government is Rawlsian. Intuitively, by generating additional involuntary unemployment, unions make it more costly to redistribute towards the unemployed. A Rawlsian government therefore always prefers competitive over unionized labor markets.[30]

Proposition 2 holds irrespective of whether there are income effects at the union level or whether labor markets are independent or not, see Appendix E.1. Perhaps surprisingly, the result also generalizes to a setting where profits cannot be fully taxed, in which case $b_f < 1$. Hence, a restriction on profit taxes does not provide an additional reason why an increase in union power could be welfare-enhancing. The reason is that income taxes can already be used to raise wages and thereby indirectly redistribute from firm-owners to workers. As such, for the result in Proposition 2 to hold, it is important that income taxes are optimized.[31] The optimal tax-benefit system takes the indirect redistribution from firm-owners to workers into account. This explains why *ceteris paribus* income taxes are higher when profit taxation is restricted (i.e., when b_f is low), see Proposition 1. Unions are not helpful to achieve more income redistribution from firm-owners to workers over and above what can already be achieved via the tax-benefit system. Therefore, provided income taxes are optimized, the only role of labor unions is to offset upward distortions in employment generated by participation subsidies.

We can also use our model to determine the optimal union power ρ_i in each sector i. Unlike tax policy, union power is not typically considered an instrument over which policymakers have direct control.[32] Nevertheless, if such policy instruments are available, then they should ensure that union power satisfies the conditions in the next Corollary.

Corollary 1. *If Assumption 2 (efficient rationing) is satisfied, and taxes and transfers are set optimally, then the optimal degree of union power $\rho_i^* \in [0, 1]$ ensures that the social welfare weight of workers in sector i becomes equal to one: $b_i = 1$. If that is not feasible, $\rho_i^* = 1$ if*

[30] Optimal participation taxes for the Rawlsian government follow by plugging $b_i = 0$ for all i in the optimal tax formula in equation (18).

[31] Political-economy reasons can explain why the income tax is sub-optimal, either by imposing additional constraints on the tax system or by generating a misalignment between the social welfare function and the political objective function. If that is the case, unions can be welfare-enhancing by either alleviating the constraints on the tax system or reducing the misalignment.

[32] It is not obvious how the government can set union power. In this context, Hungerbühler and Lehmann (2009, p.475) remark that: "Whether and how the government can affect the bargaining power is still an open question". They suggest that changing the way how unions are financed and regulated can affect their bargaining power.

$b_i > 1$ *and* $\rho_i^* = 0$ *if* $b_i < 1$.

According to Corollary 1, for workers with an above-average social welfare weight (i.e., $b_i \geq 1$), the power of the union representing them should optimally be increased until $b_i = 1$. However, if this is not feasible (which can happen if workers have low wages w_i or if the utility function is linear), the next best thing to do is to make the labor union a monopoly union, i.e., to set $\rho_i^* = 1$.[33] For workers with a below-average social welfare weight ($b_i < 1$), the government would like to lower the power of the union representing them. However, the government cannot decrease union power below the competitive level.

A disadvantage of Proposition 2 is that it is written in terms of social welfare weights, which are generally endogenous as they depend on the entire allocation.[34] Moreover, assessing whether the condition holds requires invoking political judgments regarding the desirability of income redistribution, i.e., on the exact value of b_i. However, it is possible to judge the desirability of unions while refraining from making such political judgments. The main idea is that the increase in union power ρ_i can be combined with a set of tax adjustments such that net incomes of all workers in the economy remain unaffected, hence the distribution of utilities is kept constant in the tax reform.[35] As a result, the desirability condition for unions from Proposition 2 can be expressed solely in terms of behavioral responses, fiscal externalities, and union wedges, as the next Proposition demonstrates.

Proposition 3. *If Assumption 2 (efficient rationing) is satisfied, and taxes are set optimally, then a net-income neutral increase in union power ρ_i raises social welfare if and only if*

$$\sum_j N_j(t_j + \tau_j)w_j \mathrm{d}E_j^i > 0, \tag{24}$$

where $\mathrm{d}E_j^i$ is the change in employment in sector j induced by a joint increase in union power ρ_i in sector i and a tax reform $\{\mathrm{d}T_k^i\}_k$ that keeps all net incomes in all sectors the same. The changes in employment in all sectors j are given by

$$\mathrm{d}E_j^i = \frac{\partial E_j}{\partial \rho_i}\mathrm{d}\rho_i + \sum_k \frac{\partial E_j}{\partial T_k^i}\mathrm{d}T_k^i. \tag{25}$$

The tax reform $\{\mathrm{d}T_k^i\}_k$ can be found by solving, for all j,

$$\frac{\partial w_j}{\partial \rho_i}\mathrm{d}\rho_i + \sum_k \frac{\partial w_j}{\partial T_k^i}\mathrm{d}T_k^i - \mathrm{d}T_j^i = 0. \tag{26}$$

Proof. See Appendix E.3. □

Proposition 3 can again be understood by starting from a small increase in union power ρ_i in sector i. Such an increase raises the wage in sector i, and lowers wages in other sectors $j \neq i$,

[33] In this case, the constraint $\rho_i^* \leq 1$ becomes binding. See Appendix E.2 for details.

[34] The only instance where social welfare weights are exogenous is if the utility function is linear. However, it is always possible to make the social welfare weights exogenous at will by considering a monotone transformation of $u(\cdot)$ that makes the individual utility function linear and to (locally) describe the government's preference for income redistribution using Pareto weights ψ_i.

[35] Such tax reforms have been analyzed as well by Gerritsen and Jacobs (2020) in the context of minimum wages.

if labor types are complements in production. The net-income neutral tax reform offsets the impact on net wages by combining the increase in ρ_i with a tax reform $\{\mathrm{d}T_k^i\}_k$ that keeps all net incomes constant. This tax reform can be found by solving equation (26), which is obtained by setting $\mathrm{d}(w_j - T_j) = 0$ for each sector j. Provided the social welfare weight of firm-owners equals one (i.e., provided the profit tax is optimized), the only welfare-relevant effect of the joint increase in union power and the tax reform goes via changes in employment rates, $\mathrm{d}E_j^i$, as given by equation (25). The associated welfare impact consists of the fiscal externality $t_j w_j = T_j - T_u$ and the union wedge $\tau_j w_j$.

The total impact of the combined increase in ρ_i and the tax reform $\{\mathrm{d}T_k^i\}_k$ on employment in different sectors is generally ambiguous. The increase in union power raises the wage and lowers employment in sector i. Keeping the net wage $w_i - T_i$ in sector i fixed thus requires increasing the income tax T_i, which further lowers employment in sector i. In other sectors, both employment and wages go down following the increase in ρ_i if labor types are complements in production. Hence, keeping net wages $w_j - T_j$ in other sectors fixed requires decreasing T_j, which raises employment in other sectors. Equation (24) states that an increase in union power ρ_i in sector i is desirable if and only if the sum of the fiscal externality and the union wedge over all sectors is positive.

It is shown in Appendix E.3 that the impact of a rise in union power in sector i on employment in sector $j \neq i$ is zero in sectors where wages are determined competitively or if labor markets are independent. Furthermore, the effect is negligible if the production function can be approximated well by a second-order Taylor expansion. If $\mathrm{d}E_j^i = 0$ for $j \neq i$ and $\mathrm{d}E_i^i < 0$, then according to Proposition 3 an increase in union power ρ_i raises social welfare if and only if employment in sector i is upward distorted on a net basis, i.e., if the sum of the explicit and implicit tax are negative: $t_i + \tau_i < 0$. Because the union wedge is non-negative, this condition requires that participation is subsidized, i.e., $T_i < T_u$. In reality, as we will demonstrate below, participation is taxed for all workers in OECD countries. Hence, if the tax system in these countries is optimized, and spillover effects between different sectors are small, an increase in union power unambiguously lowers social welfare. We get back to this point in more detail in Sections 7 and 8.

6 Summary of extensions

In the online Appendix accompanying this paper, we investigate the robustness of our results by relaxing some of the key assumptions in our model. i) We study how our main results are affected if unions respond to marginal tax rates. ii) We analyze the consequences of inefficient rationing. iii) We study endogenous occupational choice, or the 'intensive margin' as in Saez (2002). iv) We analyze a single, national union that bargains with firm-owners over the entire *distribution* of wages. v) We analyze sectoral unions that bargain with firms over wages *and* employment, as in the efficient bargaining model of McDonald and Solow (1981). This Section summarizes the main results from these extensions. More details and the proofs of all claims made here can be found in the online Appendix.

6.1 Union responses to marginal tax rates

So far, we have assumed that the government sets the tax liability T_i in each sector directly, which unions subsequently take as given. However, if the government sets a tax *schedule* $T(w_i)$, rather than a tax liability T_i in each sector, unions will anticipate that a higher wage affects the tax liability. Hence, the marginal tax rate will also determine wage demands of the union. The extension in this subsection derives how our main results are affected if the government optimizes a tax schedule and unions respond to marginal tax rates. See also Section 1 in the Online Appendix.

To study this extension, it is more convenient to work with a continuum, rather than a discrete set, of sectors (or occupations), which gives rise to a continuous income distribution. Like before, sectors are indexed by $i \in \mathcal{I} = [0,1]$ and ordered in such a way that wages $w(i)$ are increasing in i.[36] To maintain tractability, we invoke Assumption 1, which guarantees that there are no spillover effects between different sectors.[37] Within each sector, workers are represented by a union that maximizes the expected utility of its members, as in the baseline. Unions bargain with firm-owners over wages and firms unilaterally determine employment. We again parameterize union power in each sector with a parameter $\rho(i)$ so that we allow for any equilibrium in the Right-to-Manage model. The modified wage-demand equation, which is the counterpart of equation (8), then reads as:

$$\rho(i)(1 - T'(w(i))) = \varepsilon(i) \frac{u(\hat{c}(i)) - u(c_u)}{u'(c(i))w(i)}. \tag{27}$$

There is one key difference relative to the baseline. Labor-market outcomes are affected by changes in the marginal tax rate $T'(w(i))$: the left-hand side of equation (27) is multiplied by the net-of-tax rate. Intuitively, unions only care about demanding higher wages if this leads to higher after-tax earnings. Consequently, a higher marginal tax rate reduces wage demands, which induces firms to hire more workers. The negative (positive) impact of the marginal tax rate on the equilibrium wage (employment rate) is referred to in the literature as the wage-moderating effect of a higher marginal tax rate.[38,39] In Appendix 1 of the online Appendix, we characterize optimal profit taxes, unemployment benefits, and the optimal tax schedule on labor income $T(\cdot)$ using the tax-perturbation approach.[40] The first two results from Proposition 1 generalize immediately. However, optimal income taxes now need to take into account two additional, welfare-relevant effects.

First, a higher marginal tax rate at w' raises the employment rate at this income level

[36] Because this extension employs a continuum of sectors, i shows up as a function argument instead of a subscript.

[37] See Sachs et al. (2020) for a derivation of the optimal non-linear tax schedule in a competitive framework with a continuum of wages and spillover (general-equilibrium) effects.

[38] The negative (positive) impact of the marginal tax rate on the equilibrium wage (employment rate) is derived in the context of unions by Hersoug (1984), but also holds when there are matching frictions (Pissarides, 1985), or when firms pay efficiency wages (Pisauro, 1991). See Lehmann et al. (2016) for empirical evidence, and Kroft et al. (2020) and Hummel (2021) for the implications for optimal taxation.

[39] Sometimes, this effect is referred to as the wage-moderating effect of 'tax progressivity'. Indeed, if marginal tax rates increase, while average tax rates remain fixed, a higher marginal tax rate also raises the progressivity of the tax system, since a tax system is progressive only if the average tax rate increases in income.

[40] The tax-perturbation approach is also employed by, Saez (2001), Golosov et al. (2014), Gerritsen (2016), and Jacquet and Lehmann (2021), among many others.

due to the wage-moderating effect of a higher marginal tax rate. This alleviates labor-market distortions from the explicit tax $t(w')$ on labor participation, and the implicit tax $\tau(w')$ from unions bidding up wages above the market-clearing level. Intuitively, if unions moderate wage demands in response to a higher marginal tax rate, employment increases, and this is welfare-improving if employment is distorted downwards, i.e., if $t(w') + \tau(w') > 0$.

Second, as the marginal tax rate moderates wages at income level w', income is redistributed among workers, firm-owners, and the government. In particular, if wages are lowered, firm-owners receive higher profits, workers see their after-tax income reduced, and the government experiences a reduction in tax revenue (provided that $T'(w') > 0$). The welfare effect of this additional redistribution is ambiguous and depends on whether $b(w') \gtrless 1$. A redistribution of one unit of income from the worker to the firm owner yields a welfare effect of $1 - b(w')$, since firm-owners have a social welfare weight of 1 (in the optimum). The subsequent reduction in tax payments of this worker with $T'(w')$ units yields a welfare effect of $T'(w')(b(w') - 1)$. As both effects are proportional to $1 - b(w')$, there is a redistributional gain (loss) due to wage moderation at w' if $b(w') < 1$ ($b(w') > 1$).

Wage-moderation effects of marginal tax rates thus trigger two welfare-relevant effects: they alleviate (exacerbate) labor-market distortions if labor participation is taxed (subsidized) on a net basis, and they generate redistributional gains (losses) if $b(w') < 1$ ($b(w') > 1$). These welfare effects are related. Loosely speaking, the government typically only provides transfers to employed workers that exceed the unemployment benefit, i.e., sets $t(w) < 0$, if these workers have an above-average social welfare weight, i.e., if $b(w) > 1$. Therefore, we conjecture that, compared to the baseline, wage-moderation effects tend to reduce (raise) optimal marginal tax rates if employment is distorted upwards (downwards) – *ceteris paribus*. However, we are not sure whether the ceteris paribus condition holds, since the optimal marginal tax schedule is dependent on all social welfare weights, the entire income distribution, and participation distortions at all income levels. Only a more elaborate quantitative analysis can give a more definitive answer to the question how wage-moderation effects affect optimal participation taxes, which is beyond the scope of the current paper.[41]

Turning to the desirability of unions, we find that an increase in union power raises social welfare if the union represents workers with an above-average social welfare weight and/or represents workers whose labor participation is subsidized on a net basis. Hence, our desirability condition carries over in slightly modified form. Intuitively, an increase in union power at income level w boosts wage demands and reduces employment at w. This results in a welfare gain i) if participation is distorted upwards on a net basis (our first effect discussed above), and/or ii) if the wage increase is associated with a positive redistributional gain (our second effect discussed above). A positive redistributional gain requires that $b(w) > 1$. Therefore, we view our adjusted desirability condition as only slightly weaker, since an above-average social welfare weight generally also implies that labor participation is distorted upwards (see, e.g., Diamond, 1980).

Moreover, we can derive a sufficiency condition for the desirability of unions: an increase in

[41]See Kroft et al. (2020) and Hummel (2021) for an analysis of the quantitative implications of the wage-moderating effect for optimal taxes.

union power at income level w raises social welfare if participation is distorted upwards on a net basis ($t(w) + \tau(w) < 0$) *and* the social welfare weight of the workers represented by the union is above-average ($b(w) > 1$). Conversely, a sufficient condition for unions not to be desirable is that workers pay positive participation taxes ($t(w) > 0$) and have a below-average social welfare weight ($b(w) < 1$).[42] Given that we empirically find that participation taxes are never negative (see the next section), the desirability condition also implies that a necessary condition for unions to be desirable is that the social welfare weight of the workers that are represented by the union is above average, i.e., $b(w) > 1$. Hence, Proposition 2 largely carries over to the current setting.

In the baseline, without union responses to marginal tax rates, social welfare weights and net participation taxes at a particular income level are tightly linked.[43] The reason is that both participation distortions and distributional effects are proportional to $1 - b(w)$. Therefore, only knowledge of social welfare weights is required to judge whether an increase in union power raises social welfare, cf. Proposition 2. However, if unions respond to marginal tax rates, such a tight link between social welfare weights and net taxes on participation no longer exists. This is because participation taxes at each income level are determined by the entire optimal non-linear tax schedule, which depends on all social welfare weights, the income distribution, and participation distortions at all income levels. Therefore, judging whether an increase in union power raises social welfare generally requires knowledge of both participation taxes and social welfare weights.

6.2 Inefficient rationing

We have deliberately biased our findings in favor of unions by assuming that labor rationing is efficient: the burden of involuntary unemployment is borne by the workers with the highest participation costs. However, there are neither theoretical nor empirical reasons to expect that labor rationing is always efficient, see Gerritsen (2017) and Gerritsen and Jacobs (2020). In this extension, we relax the assumption of efficient rationing. For analytical convenience, this extension assumes that labor markets are independent and there are no income effects at the union level. See also Section 2 in the online Appendix.

We follow Gerritsen (2017) and Gerritsen and Jacobs (2020) by defining a rationing schedule that specifies the probability that workers find employment in sector i for a given sectoral employment rate E_i and a given participation threshold φ_i^* in sector i. The probability of finding a job in sector i increases in employment E_i and decreases if labor participation rises, i.e., if φ_i^* is higher. Consequently, it is possible that a worker with lower participation costs (a higher surplus from work) is unemployed, while a worker with higher participation costs (lower surplus from work) has a job.

We show that Proposition 1 for optimal taxes generalizes to a setting with inefficient labor rationing with two modifications. First, with a general rationing scheme, the union wedge τ_i no

[42]This sufficiency condition only requires that participation taxes are positive, since implicit taxes from unions are always weakly positive (i.e., $\tau(w) \geq 0$). Hence, a positive participation tax is sufficient to guarantee downward distortions on participation.

[43]From equation (22), net participation taxes are negative if and only if the social welfare weight is above average.

longer measures the monetized utility loss of a *marginal* worker losing her job, but the expected utility loss of *all rationed workers*, i.e., the workers who lose their job if the wage is marginally increased. Second, in addition to the union wedge, there is a distortion associated with the inefficiency of the rationing scheme. The more inefficient is the rationing scheme, the *higher* should be the optimal participation tax – *ceteris paribus* – compared to the case with efficient rationing. The intuition is similar to Gerritsen (2017): if wages are above the market-clearing level and rationing is inefficient, some workers will be unemployed that have a higher surplus from work than some of the workers who are employed. By setting a higher participation tax, the workers with the lowest surplus from work opt out of the labor market. This, in turn, increases the employment prospects of the workers with a larger surplus from work. Consequently, the government replaces involuntary unemployment by voluntary unemployment, which reduces the inefficiency of labor-market rationing.

In addition, the desirability condition for unions in Proposition 2 is modified to account for inefficient rationing. In particular, an increase in union power is less likely to be desirable than in the case with efficient rationing. The implicit tax on labor caused by unions not only alleviates possible upward distortions in labor supply, it also generates more inefficiencies in labor rationing. Hence, the desirability condition for unions becomes tighter. Unions can be desirable only if the social welfare weight b_i in sector i is sufficiently above the average of one so as to compensate for the larger inefficiencies in labor rationing.

6.3 Endogenous sectoral choice

We abstracted from an intensive margin of labor supply for the following reasons. First, including an intensive margin requires us to take a stance on whether working hours are determined by the worker, the union, or some combination. Second, we also need to know the incidence of involuntary unemployment: does it fall on the intensive margin, the extensive margin, or both? We are neither aware of good theoretical models nor empirical evidence on the joint determination of hours worked and the incidence of involuntary unemployment on the intensive and extensive margin. Therefore, in this extension (studied in Section 3 of the online Appendix), we follow Saez (2002) and model the 'intensive margin' by letting workers optimally choose the sector in which they want to work. As before, we assume that there are no income effects at the union level.

To model endogenous sectoral choice, we assume that all workers draw a vector of participation costs $\boldsymbol{\varphi} \equiv (\varphi_0, \varphi_1, \cdots, \varphi_I)$ indicating how costly it is to work in each sector i. Based on their participation costs, individuals optimally choose in which sector (or: occupation) to look for a job. We assume that the probability $p_i \in [0, 1]$ that an individual finds employment in sector i can be written as a reduced-form function of the participation taxes in all sectors $p_i(\boldsymbol{\varphi}, T_1 - T_u, \cdots, T_I - T_u)$. If the individual is unsuccessful in finding a job in his/her preferred sector, she cannot move to another sector but instead becomes unemployed. We extend our notion of efficient rationing to this environment by assuming that, if there is involuntary unemployment, individuals who are indifferent between choosing sector i and another sector (possibly non-employment) do not find a job if wages in sector i are set above the market-clearing level.[44]

[44]Our notion of efficient rationing is similar to Lee and Saez (2012), but we extend it to multiple sectors.

We demonstrate that Proposition 1 generalizes to a setting where workers can switch between occupations with two modifications. First, the union wedge τ_i no longer captures the utility loss of the marginal worker, but instead captures the average utility loss of all workers who lose their job if employment in sector j is marginally reduced – like in the case with inefficient rationing, see above. Second, the employment and wage responses η_{ji} and κ_{ji} remain sufficient statistics, but they not only capture 'demand interactions' through complementarities in production (as in the baseline model), but also 'supply interactions' through occupational choice. Moreover, the desirability condition for unions in Proposition 2 generalizes completely to an environment with occupational choice. The reason is that if labor rationing is efficient, individuals who are marginally indifferent between two sectors will not switch between sectors if there is involuntary unemployment. Therefore, the welfare effects of a combined increase in union power and a tax reform that leaves the wage unaffected are the same as before.

6.4 National unions

In our baseline model, bargaining takes place at the sectoral level. Each sectoral union faces a trade-off between employment and wages, but does not care about the overall *distribution* of wages. There is, however, ample empirical evidence that a higher degree of unionization is associated with lower wage inequality.[45] How do our results for optimal taxes and the desirability of unions change if unions care about the entire distribution of wages?

To answer this question, Section 4 of the online Appendix analyzes a model where a single union bargains with firm-owners over *all* wages in all sectors, while firms (unilaterally) determine employment, as in the RtM-model. To maintain tractability, we assume efficient rationing and we assume away income effects at the union level. The union maximizes the sum of all workers' expected utilities. Since the utility function $u(\cdot)$ is concave, the union has an incentive to compress the wage distribution. We explicitly solve the Nash-bargaining problem between unions and firms to characterize labor-market equilibrium. To maintain comparability with our previous findings, we assume that firm-owners are risk neutral. It should be noted that a national union does not necessarily find it in its best interest to bargain wages in *all* sectors above the market-clearing level. This is because an increase in the wage for high-skilled workers depresses the wages for low-skilled workers. A national union may therefore refrain from demanding an above market-clearing wage for high-skilled workers.

We demonstrate that Proposition 1 carries over fully to a setting with a national union bargaining over the entire wage distribution. The reason is that the optimal tax rules in Proposition 1 are expressed in terms of sufficient statistics for the employment and wage responses η_{ji} and κ_{ji}. Hence, a different bargaining structure gives rise to different elasticities, but the optimal tax formulas remain the same. We derive the counterpart of Proposition 2 for the desirability of a national union bargaining over all wages. In particular, we show that increasing power of a national union raises social welfare if and only if weighted average social welfare weight of workers in sectors with involuntary unemployment exceeds the average social welfare weight of all (employed and unemployed) workers.

[45] See, for instance, Freeman (1980, 1993), Lemieux (1993, 1998), Machin (1997), Card (2001), DiNardo and Lemieux (1997), Card et al. (2004), Visser and Checchi (2011), and Western and Rosenfeld (2011).

6.5 Efficient bargaining

The baseline assumed that bargaining takes place in a Right-to-Manage setting. This bargaining structure generally leads to outcomes that are not Pareto efficient (McDonald and Solow, 1981). This inefficiency can be overcome if unions and firm-owners bargain over both wages *and* employment.[46] Therefore, we explore whether our results generalize to a setting with efficient bargaining (EB), as in McDonald and Solow (1981). For simplicity, we assume efficient rationing, independent labor markets, and no income effects at the union level. See also Section 5 in the online Appendix.

The key feature of the EB-model is that any potential labor-market equilibrium (w_i, E_i) in sector i lies on the *contract curve*, which is the line where the union's indifference curve and the firm's iso-profit curve are tangent:

$$\frac{u(w_i - T_i - \hat{\varphi}_i) - u(-T_u)}{E_i \overline{u'(w_i - T_i - \varphi)}} = \frac{w_i - F_i(\cdot)}{E_i}. \tag{28}$$

Intuitively, if the equilibrium wage and employment level are on the contract curve, then it is impossible to raise either union i's utility while keeping firm profits constant, or vice versa. Which labor contract (w_i, E_i) is negotiated depends on the power of union i relative to that of the firm. We model union i's power as its ability to bargain for a wage that exceeds the marginal product of labor with a rent-sharing rule.[47] In stark contrast to the RtM-model, an increase in union power will not only result in a higher wage, but also in *higher* employment. Intuitively, unions can use their power to bargain both for a higher wage and a higher employment rate. Moreover, and also in contrast to the RtM-model, there is now a labor-demand distortion: the wage exceeds the marginal product of labor. As a result, there will be an implicit subsidy on labor demand.

We show that Proposition 1 generalizes to a setting with efficient bargaining with one important modification. The larger is the implicit subsidy on labor demand, the higher is the optimal participation tax – *ceteris paribus*. Therefore, the impact of unions on optimal participation taxes has become ambiguous with efficient bargaining, in contrast to our findings with the RtM-model. On the one hand, employment is too low, because unions generate involuntary unemployment (as captured by the union wedge τ_i), which calls for lower participation taxes. On the other hand, employment is too high, because unions generate implicit subsidies on labor demand in the EB-model, which calls for higher participation taxes. Furthermore, we demonstrate that the desirability condition of Proposition 2 remains the same in the EB-model. Therefore, the question whether unions are desirable or not does not depend on the bargaining structure. Intuitively, also in the EB-setting, unions will generate more *involuntary* unemploy-

[46] We consider the EB-model less appealing for two reasons. First, the assumption that firms and unions can write contracts on both wages *and* employment is problematic with national or sectoral unions, since individual firm-owners then need to commit to employment levels that are not profit-maximizing (Boeri and Van Ours, 2008). Oswald (1993) argues that firms unilaterally set employment, even if bargaining takes place at the firm level. Second, employment is higher in the EB-model compared to the competitive outcome, since part of firm profits are converted into jobs. This property of the EB-model is difficult to defend empirically.

[47] If unions have zero bargaining power, the outcome in the EB-model coincides with the competitive equilibrium. If, on the other hand, union i has full bargaining power, it can offer a contract which leaves no surplus to firm-owners.

ment if they are more powerful. Hence, an increase in union power is desirable only if labor participation (and not employment) is distorted upwards, just like in the RtM-model.

7 Empirical analysis

According to Proposition 1, more powerful unions should be associated with lower participation tax rates. Moreover, Proposition 3 gives a necessary condition for the desirability of unions in a sector: if taxes are optimized, unions are desirable only if participation is subsidized on a net basis.[48] Furthermore, according to Proposition 2 unions are only desirable for workers with the lowest incomes, who feature the highest (above-average) social welfare weights. In this Section, we empirically verify whether more powerful unions are associated with lower participation tax rates, whether participation is subsidized, and whether unions are stronger among the lower income groups. We so by compiling our own data set with 294 country-sector observations on union densities, wages, and participation tax rates from 23 OECD countries and 18 sectors.

7.1 Data

This Section summarizes the construction of our data set of union densities, wages, and participation tax rates at the sectoral level.[49] All details can be found in online Appendix 6. We use union densities at the sectoral level from the OECD Bargaining and Trade Union Data. To calculate participation tax rates, we use the online tax-benefit calculator from the OECD. Making these calculations requires information on the earnings of workers at the sectoral level. We obtain the earnings data from the STAN database from the OECD and the Statistics on Wages Database of the ILO.

7.1.1 Union densities

Our analysis uses sectoral union density as a measure for union power. Union density measures the percentage of (employed) workers who are member of a labor union. Our assumption is that if union densities are larger, then unions are more powerful. To the best of our knowledge, this is the only available union variable that is consistently measured across countries and across sectors.[50]

Data on union density come from the "Jelle Visser database", which is officially referred to as the Institutional Characteristics of Trade Unions, Wage Setting, State Intervention and Social Pacts (Visser, 2019).[51] This panel data set spans 55 countries over the time-period 1960-2018 and contains union densities at the sectoral level. However, many observations are missing,

[48] Participation is typically only subsidized on a net basis if social welfare weights are above-average, that is, for the low-income workers, see equation (22).

[49] Unfortunately, micro data on individual union membership are scarce. Therefore, our primary unit of observation is the sector level. An important advantage of using sectoral data is that it allows us to include many countries.

[50] Alternatively, one may use union coverage by sector as measure for union power, see also Figure 1. Such a measure would also take into account that in some countries, collective labor agreements are extended to the entire sector. However, to our knowledge, no data on union coverage are available at the sectoral level.

[51] This database forms the basis of the OECD Bargaining and Trade Union Data. We employ version 6.1 of the database (2019), which is the latest version that contains data on union membership at the sectoral level.

since union densities are not measured every year, not for every country, and not for every sector. To obtain a more complete data set, we pool the observations on union membership for each country-sector over a 10-year time window.[52] Doing so gives us a coverage of union densities at the sectoral level of approximately 75%. The reference year of each country is the latest year for which data on sectoral union densities are available.

Our final sample contains 23 countries. Table 1 in Appendix 6 lists the countries that are included.

7.1.2 Wages

Data on wages of workers at the sectoral level are obtained mainly from the STAN (Structural Analysis) industry database from the OECD (OECD, 2022d). The STAN database covers sectoral data for OECD countries at the International Standard Industrial Classification of All Economic Activities (ISIC4) 2-digit level from 1970-2021. The wage refers to gross wages and salaries for employees, excluding employer contributions, for example for social insurance and pensions. Moreover, by focusing on the wage bill minus employer contributions, this wage measure corresponds most closely to the gross earnings variable in the OECD tax benefit calculator. Of the 23 countries that we include in our final sample (see Table 1 in Appendix 6), the OECD STAN database does not contain sectoral wage data for Switzerland, Japan, South Korea, New Zealand, and Turkey. For these countries, we rely on the Statistics on Wages Database of the ILO (2022d). This database contains mean monthly gross earnings of employees measured in local currency units at the ISIC4 1-digit level, which are multiplied with 12 to obtain yearly figures. Furthermore, the STAN wage data cover fewer sectors than the ILO data for Australia. Therefore, we also use the ILO wage data for Australia to obtain more observations.

In all our calculations, wages are measured per full-time equivalent worker per year. The STAN data provide the total wage bill at the sectoral level. In addition, data on full-time equivalent employment are available for seven countries (Austria, Spain, France, Italy, Netherlands, Norway, and the United States). For these countries, we calculate the wage per full-time equivalent worker in each sector. For the 16 remaining countries, only data on total employment are available. We translate wages per worker to full-time equivalents by means of a country-sector specific part-time factor, which is defined as the ratio of average weekly hours worked relative to the statutory length of the working week in that country. We rely on data from the OECD and the ILO to compute the sectoral part-time factor, see online Appendix 6 for more details.

To merge the sectoral union densities from the ICTWSS-database and the sectoral wage data from the STAN and ILO databases, a concordance between the sectoral classifications of each database is employed. Table 2 in Appendix 6 shows the sectoral mapping between all datasets. The data set with union densities and wages ultimately consists of observations during the period 2014-2018. Given that the coverage of sectoral union densities and wage data is incomplete, we obtain a cross section of countries with 294 observations spread out over 23 countries and 18 sectors.[53]

[52]This procedure rests on the assumption that union membership rates are only slow-changing over time, which is empirically the case.

[53]The sectors are: Agriculture, Industry*, Services*, Mining, Manufacturing, Utilities, Construction, Trade, Transport and communication, Hotels and restaurants, Finance, Real estate and business services, Commercial

7.1.3 Participation tax rates

We employ the OECD tax-benefit web calculator to compute participation tax rates for all 294 country-sector observations in our data (OECD, 2022c). To do so, we first calculate the sum of taxes paid minus transfers received at the household level if the primary earner is full-time employed at the sectoral wage. Subsequently, we calculate the sum of taxes paid minus transfers received at the household level when the primary earner is unemployed and entitled to social-assistance benefits (in the baseline) or unemployment benefits (in the robustness check).[54] In line with our theoretical definition, the participation tax rate is defined as the difference between taxes paid minus transfers received when the primary earner is employed and unemployed, expressed as a fraction of gross earnings of the primary earner. The total net tax burden in work is the sum of the income tax and social-security contributions minus family benefits, and in-work tax credits.[55] The total tax burden for households where the primary earner is out of work is based on the same tax items except that we account for social-assistance benefits (in the baseline) or unemployment benefits (in the robustness check).

We use the default settings of the tax-benefit calculator and focus on a two-earner couple with two dependent children. The earnings of the primary earner are taken to the sector-specific yearly full-time equivalent wage. Regarding the secondary earner, we assume positive assortative mating such that there is a perfect correlation between earnings of primary and secondary earners. We calculate the secondary earner's income by multiplying the primary earner's income with a country-specific ratio that is computed based on average monthly earnings and total employment by gender, which are obtained from ILO (2022a,b,c). See online Appendix 6 for details.[56]

7.2 Descriptive statistics

Before diving into our empirical exploration, this Section provides some descriptive statistics of our data set. Table 4 in Appendix F gives the means, standard deviations, minima and maxima broken down by country and sector. On average, the union density is 27% in our sample. The participation tax rate is on average 37%. Moreover, Table 4 reveals that all countries set positive participation tax rates. Furthermore, participation tax rates can sometimes be as high as 100% in sectors where earnings are very low. Figure 9 in Appendix F also provides scatter plots of participation tax rates against union densities for all countries.

Figure 3 gives the (unweighted) average union density and average participation tax rate by country. The countries with low union densities are, for example, the United States (11%),

services*, Social services, Public administration, Education, Health care, and Other services, where an asterisk refers to an aggregated sector.

[54] Because our theoretical model is static, it is not obvious if the empirical counterpart of income in non-employment includes only social assistance or also unemployment benefits (which only have a limited duration). Therefore, we decided to calculate the participation tax rate at each country-sector observation twice.

[55] We set the housing benefits (e.g., rent assistance) to zero, since we do not want to distinguish between renters and home-owners.

[56] Specifically, the fraction is calculated as the product of average monthly earnings of females multiplied by total female employment divided by the product of average monthly earnings of males multiplied by total male employment. In our data set, this fraction is always between 0 and 1 (see Table 3 in Appendix 6) and captures differences in labor participation, unemployment rates, working hours and hourly wages (e.g., due to labor-market discrimination) between females and males.

France (11%), Hungary (11%), and New Zealand (14%).[57] At the high end of union densities, we find the Scandinavian countries: Finland (63%), Sweden (63%), and Denmark (68%).

There is substantial cross-country heterogeneity in average participation tax rates. They are highest in Denmark (64%), Japan (50%), and Germany (49%), followed by Australia (47%), Austria (46%), and Canada (45%). On average, participation tax rates are lowest in Turkey (21%), Spain and Sweden (25%), and Slovakia (26%).

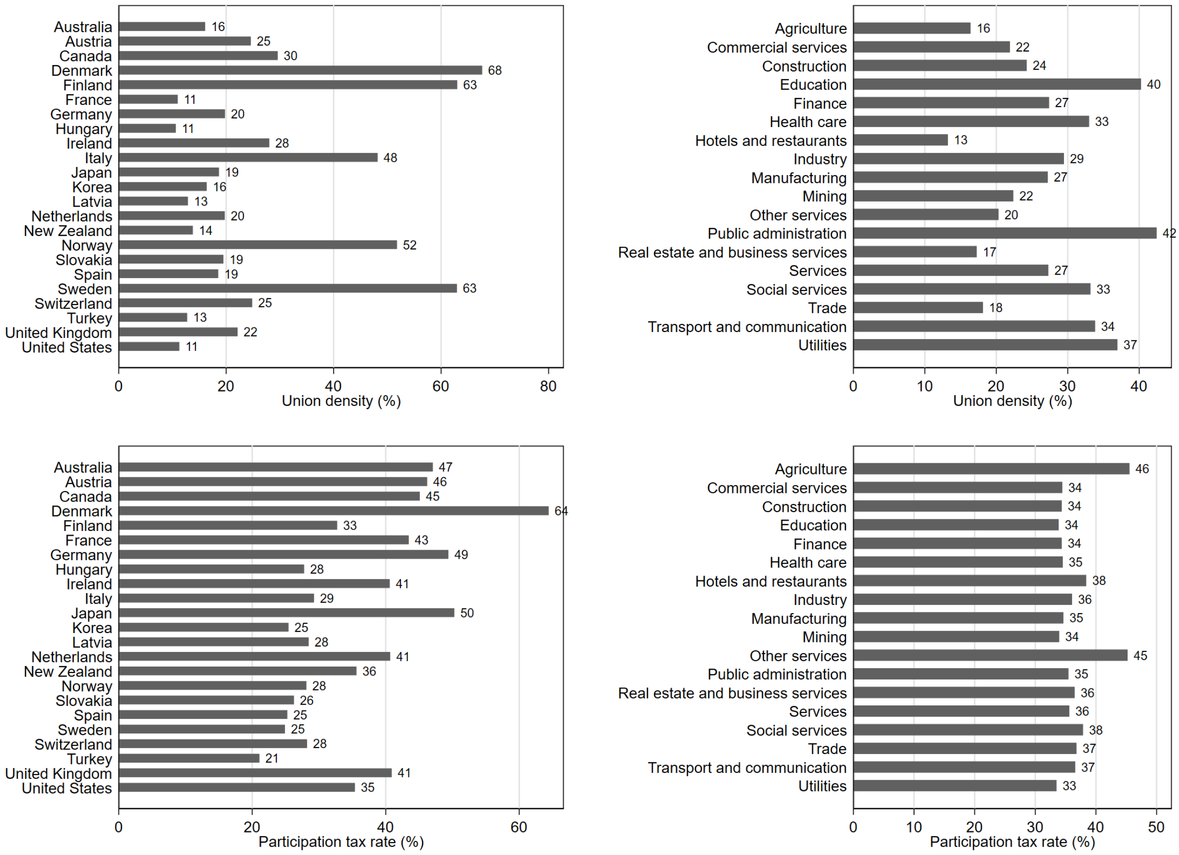


Figure 3: Average union densities and participation tax rates across countries and sectors

Similarly, Figure 3 breaks down our data across sectors. Clearly, there is quite some variation in union densities across sectors. Not surprisingly, Hotels and restaurants (13%) and Agriculture (16%) are the sectors that have on average very low union densities. At the same time, Public Administration (42%), Education (40%), and Utilities (37%) are the sectors that are most strongly unionized.

There is much less variation in participation tax rates across sectors: most sectors feature a participation tax rate of around 35-40%. The sectors with – on average – the lowest wages, Agriculture and Other Services, feature substantially higher participation tax rates, since unemployed workers in these sectors receive large income support relative to their earnings.

[57] Despite low union densities in France, union coverage is very large, around 98%, because collective labor agreements are extended to entire sectors in the economy, see also OECD (2022b).

7.3 Analysis

We start by exploring whether union densities and participation tax rates are negatively associated, in line with the prediction from Proposition 1. Figure 4 gives a scatter plot of participation tax rates against union densities in our data set. At first sight, there does not seem to be much of an association between participation tax rates and union densities. Indeed, the coefficient of a simple regression of participation tax rates on union densities (0.003, s.e. 0.035) is not significantly different from zero. However, this correlation may be driven by substantial cross-country heterogeneity, as we documented above.

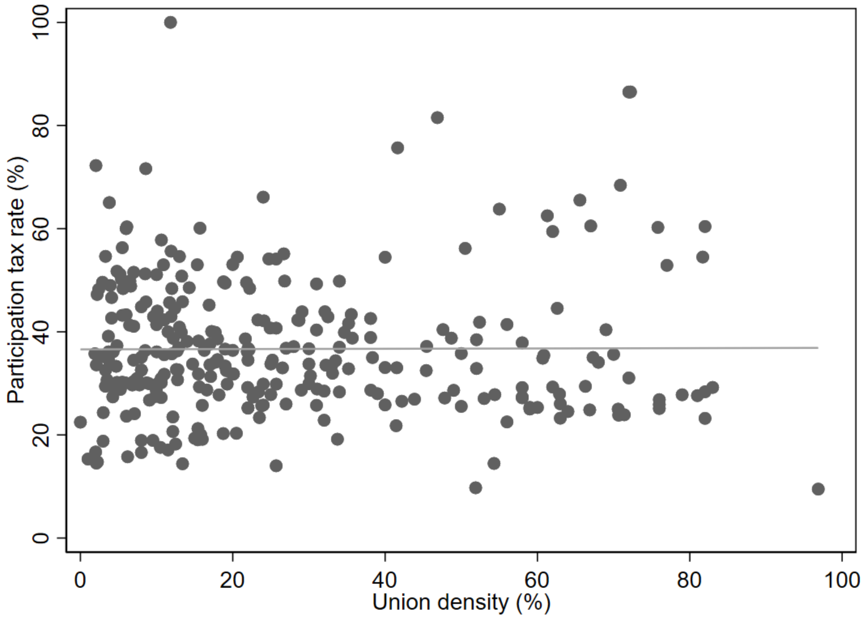


Figure 4: Participation tax rates and union densities

Table 1: Fixed-effects regressions of participation tax rates on union densities

Variable	Coefficient	Standard error	t-value
Union density	-0.142	0.039	-3.62
Constant	37.0	2.14	17.3
R^2	0.60	R^2 adj.	0.57

Country-fixed effects included, United States is the reference country

To control for the unobserved, sector-invariant heterogeneity between countries (e.g., in preferences for income redistribution), we also run a country-fixed effects regression of participation tax rates on union densities. Table 1 gives the regression results. Now, the coefficient on the union density is -0.14 and is statistically significant at the 1-percent level. Our estimate implies that a one-percentage point increase in union density is associated with a 0.14 percentage-point decrease in the participation tax rate. This association is in line with the prediction from Proposition 1: higher union densities are associated with lower participation tax rates. Evaluated at the mean union density of 27%, participation tax rates would be on average about 4 percentage points lower if there were no unions, which is a reduction in participation tax rates of 11% on average.[58]

Next, we verify empirically if the desirability condition is met. According to Proposition 3, unions can be desirable only if employment is distorted upwards as a result of participation subsidies. Clearly, the desirability condition is never met in our data, as can be verified upon inspection of Table 4 and the scatter plot in Figure 4. Participation tax rates are positive in all sectors and all countries under consideration. This empirical observation implies that, through the lens of our model, unions are never socially desirable for income redistribution in any country or sector in our data set.[59]

According to Proposition 2, unions would be desirable only for the lowest income groups, since they have the highest social welfare weights for standard social welfare functions, see also Proposition 2.[60] With our data, we can verify whether union power – as measured by union density – is indeed largest for the workers with lower incomes and lowest for high-income workers. In Figure 5, we plot union densities against wages, where sectoral wages are taken relative to their national average to account for the fact that wages are measured in national currencies. A clear positive correlation is visible between union densities and wages. Moreover, this correlation survives in a country-fixed effects regression of union densities on relative wages, see Table 2. A one percentage-point increase in the wage relative to the national average is associated with a 0.12 percentage points higher union density. Hence, it appears that unions are actually strongest in sectors where wages are relatively high.[61] This finding corroborates our earlier result that unions are not desirable for income redistribution. Unions are on average most powerful among the higher income groups, while they should be most desirable for the lower income groups.

Table 2: Fixed-effects regressions of union density on relative wages

Variable	Coefficient	Standard error	t-value
Relative wage	11.5	2.45	4.70
Constant	-0.74	4.03	-0.18
R^2	0.68	R^2 adj.	0.65

Country-fixed effects included, United States is the reference country

7.4 Robustness

As a robustness check, we also compute participation tax rates using unemployment benefits rather than social-assistance benefits, see online Appendix 6.5. Average participation tax rates are 68% based on unemployment benefits, compared to an average of 37% in the baseline. Participation tax rates based on unemployment benefits are, on average, much higher, because

[58] This result should be interpreted with caution, because our results cannot be given a causal interpretation, since we do not exploit exogenous variation union density.

[59] As a Corollary, our analysis also implies that the desirability condition for minimum wages – as derived by Gerritsen and Jacobs (2020) – is rejected empirically. The logic is similar: minimum wages are not desirable because there are no upward distortions in employment resulting from participation subsidies.

[60] We assume declining Pareto weights with income, which can be rationalized by many theories of redistributive justice or inequality aversion in a social welfare function. Declining private marginal utility of income can also generate declining social welfare weights.

[61] Again, this finding should be interpreted with caution because wages might also be high partly as a result of strong unions.

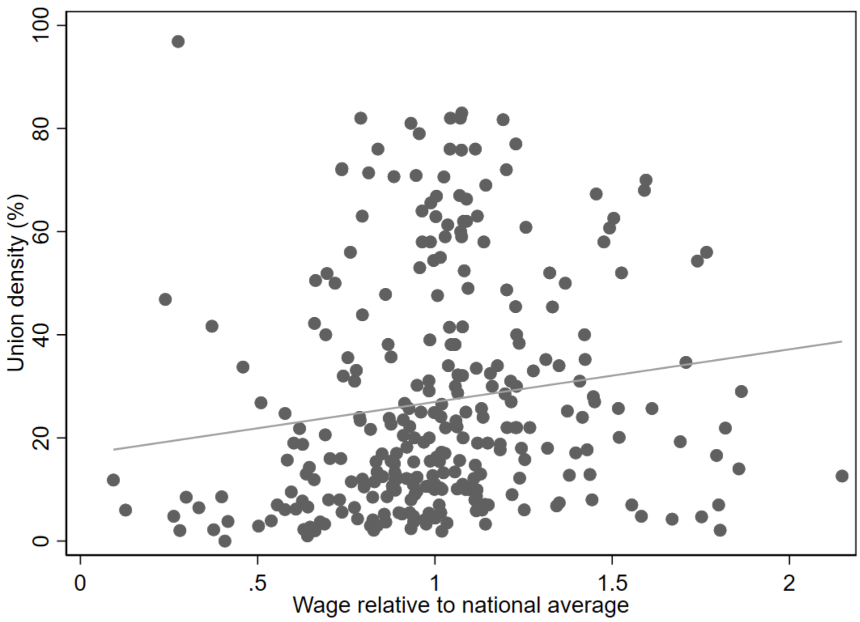


Figure 5: Union densities and wages

in many countries social-assistance benefits are means tested on partner income, while unemployment benefits are only linked to past earnings. Redoing the analysis with this alternative measure for the participation tax rate strengthens our main findings. Indeed, the country-fixed effects regression of participation tax rates on union densities returns an even smaller coefficient of -0.17 (significant at 1%-level), suggesting that participation tax rates are lower if union densities are higher, see Table 6 in online Appendix 6.5. Moreover, nowhere are participation tax rates negative, like in the baseline. Hence, the desirability condition for unions is still not satisfied.

A potential concern is that for some individuals or household types at the bottom of the income distribution, participation taxes could be lower than in our data, in which case the desirability condition could be met. Indeed, most wage levels in our sample are substantially above guaranteed minimum incomes. Some countries, for example the United States, may target in-work tax credits especially at the working poor, which would lower their participation tax rates, and thus would potentially make unions (more) desirable. To address this concern, we calculate participation taxes for a household where one individual is full-time employed at the minimum wage, and the second individual is not employed. Data on minimum wages are obtained from OECD (2022a) for a selection of 16 out of our 23 countries. For the remaining countries, we set the income of the household to 25% of average earnings.[62] We maintain the other assumptions of the baseline; the couple has two children, is not entitled to housing benefits, and receives social assistance when out of work.[63] Figure 6 shows the cross-section of participation tax rates for these workers. For most countries, participation tax rates at minimum-income levels are substantially higher (66%) compared to the average participation

[62]The Table does not include Italy as the OECD tax benefit calculator does not return meaningful participation taxes at this low income level.

[63]As explained above, with unemployment benefits participation tax rates would be substantially higher, in which case the desirability condition would be even more difficult to meet.

tax rate (37%), see Table 4 of Section 6 in the online Appendix. Participation tax rates for Austria, Norway, Spain, and Sweden are (close to) 100%. The international outlier is the United States, where the participation tax rate at minimum income levels is only 7%. Hence, the desirability condition for unions is still never met as participation taxes remain positive, even at very low earnings levels.

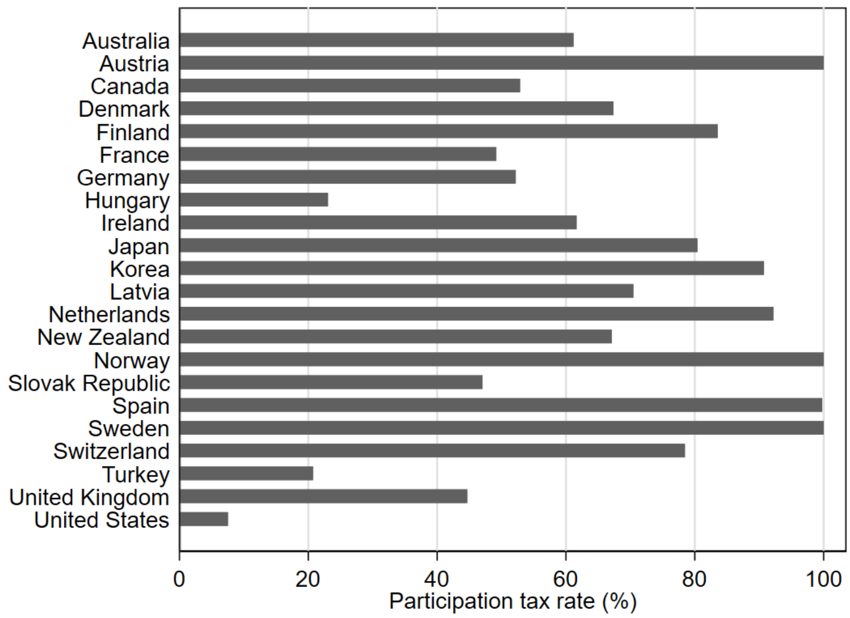


Figure 6: Participation tax rates for minimum-wage earners

8 Simulations

In this final Section, we analyze how the presence of unions affects the optimal tax-benefit system and study the desirability of unions in a structural version of the model that is calibrated to the Netherlands The main difference with the previous section is that we now explicitly specify a social welfare function and numerically solve for optimal taxes. We can thus explore whether unions can meaningfully complement an optimal tax-benefit system – for a well-defined social welfare function – instead of assessing the desirability of unions under the *current* tax-benefit system. The reason for choosing to calibrate our model to the Netherlands is that the RtM-model we use throughout this paper shares important features with the actual bargaining process between unions and employers in the Netherlands.[64]

8.1 Calibration

To calculate the optimal tax-benefit system and to study the desirability of unions, we calibrate a structural version of our baseline model where income effects at the union level are absent and labor rationing is efficient (cf. Assumptions 2 and 3). We allow for spillover effects between

[64]Unions and representatives of firms bargain over wages (mainly) at the sectoral level. Employment is subsequently determined unilaterally by firms. Furthermore, in 2015, the year of our calibration, 79.4% of all employees are covered by collective labor agreements (OECD, 2020).

different sectors as labor types are complements in aggregate production, but abstract from the extensions presented in Section 6. After discussing the data, we present the functional forms for the social welfare function, the utility function, the production function, the distribution of participation costs, and explain how the parameters of our model are calibrated.

8.1.1 Data

Most of our data come from Statistics Netherlands, which provides information on employment and average wages for $I = 65$ industries based on the two-digit NACE industry classification (Statistics Netherlands, 2020c). Consequently, we have more income levels than in the STAN data of our empirical analysis.[65] To correct for differences in hours worked and part-time jobs, we express sectoral employment L_i in full-time equivalents. Aggregate employment is slightly above 5.8 million full-time equivalents. The average sectoral wage w_i is the yearly wage for an employee who works full time.[66] It varies between €27,600 (catering services) and €89,500 (mineral extraction), with an average of €44,777. By having a relatively large number of sectors, we are able to approximate the income distribution reasonably well, while maintaining the sectoral structure of the model. We combine sectoral data on wages and employment with a number of labor market aggregates, in particular the labor income share of 75.2% (Statistics Netherlands, 2020b), the labor force participation rate of 70.2% and the involuntary unemployment rate of 6.9% (Statistics Netherlands, 2020a).

To calibrate the primitives of our structural model, we also need information on income taxes and unemployment benefits in the current tax-transfer system. Instead of using the OECD online tax-benefit calculator to compute participation taxes for a specific household type, we calculate income taxes T_i by multiplying annual labor earnings w_i by the average tax rate that applies at that income level. The average tax rates are obtained from Quist (2015), who uses detailed, micro-level data from the CPB Netherlands Bureau of Economic Policy Analysis to compute the average tax liability for individuals throughout the income distribution, based on all taxes, tax credits, and tax rebates that are applicable for each individual.[67] For a detailed discussion of the data and which taxes are included, see Quist (2015). The average yearly social assistance benefit $-T_u$ paid to the non-employed is set at €12,223. This figure is based on the weighted average benefit of €961 for singles (14% of recepients) and €1,372 for couples (86% of receipients) (Rijksoverheid, 2016).

8.1.2 Social welfare function

We assume a utilitarian social welfare function, by setting the Pareto weight of workers in each sector i and firm-owners to one: $\psi_i = \psi_f = 1$. Moreover, without much loss of generality we can

[65] Specifically, the STAN data has 17 income levels for the Netherlands, see Table 4. An important advantage of the STAN data is that we could include many more countries.

[66] As in our empirical analysis from Section 7, the annual gross wage includes all taxes and social-security contributions levied at the individual, which are typically withheld by firms, but it does not include the social-security contributions and employment subsidies levied at firms.

[67] By computing averages over all demographic groups at each income level, this approach differs from the OECD online tax-benefit calculator, where a tax liability is computed for a specific household type based on particular demographic characteristics. This explains why the numbers for the participation taxes are not directly comparable and why in Section 7 we also conduct robustness exercises by computing participation tax rates based on unemployment benefits and for minimum-wage earners alone.

simplify the analysis considerably by letting profits flow directly to the government's budget. Neither capital nor firm-owners play an important role in our analysis. What ultimately matters in our calibration is the difference between the government revenue requirement and the profit tax, i.e., $R - T_f$, and not the composition over R and T_f.[68] This short-cut implies that we do not need to obtain empirical measures for the level of the profit tax as it would simply translate into a different value for the revenue requirement.

8.1.3 Utility function

We assume a utility function with a constant coefficient of absolute risk-aversion $\theta > 0$ (CARA):

$$u(c - \varphi) = -\exp(-\theta(c - \varphi))/\theta. \tag{29}$$

Since the labor union maximizes the expected utility of its members, θ also captures the willingness of unions to tolerate more unemployment when demanding higher wages. The CARA utility function ensures that income effects at the union level are absent, cf. Assumption 3. Hence, an increase in the benefit level has the same effect on the wage demanded by the union as an increase in the tax level with the same amount.

The parameter θ measures the concavity in the utility function and thereby determines the social preference for income redistribution. The larger is θ, the stronger is the government's inequality aversion. We set $\theta = 0.139$ in the baseline to make sure the average participation tax rate in the optimal tax system is roughly equal to (income-weighted) average participation tax rate of 58% in the calibrated economy.[69] In Section 7 of the online Appendix, we explore the sensitivity of our results with respect to θ.

8.1.4 Production function

To allow for interdependent labor markets with general-equilibrium effects on the wage structure, we assume the following CES production function, which is defined over aggregate capital K and labor L_i in each sector i:

$$Y = F(K, L_1, \cdots, L_I) = AK^{1-\alpha}\left(\sum_i a_i L_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\alpha\sigma}{\sigma-1}}, \tag{30}$$

where $\sigma > 0$ is the constant elasticity of substitution between different labor types, and $\alpha \in (0, 1)$ is the aggregate labor share. The latter is set at the empirically observed value of $\alpha = 0.757$, which is obtained from Statistics Netherlands (2020b). We harmlessly normalize $AK^{1-\alpha} = 1$.

[68] The government is indifferent between taxing firm profits or setting a lower revenue requirement if firm-owners have a linear utility function. Moreover, in the optimum, the government is indifferent between a marginally higher profit tax and a marginally lower revenue requirement, since the social welfare weight of firm-owners is one.

[69] This value is considerably higher than the average participation tax rate computed using the OECD online tax-benefit calculator for two main reasons (see Table 4). First, Quist (2015) uses averages at each income level containing all demographic groups, whereas in our calculations the OECD tax-benefit calculator is based on two-earner couples that have fewer entitlements to income-support programs due to means-testing on household income. Second, Quist (2015) includes all income-support programs, including rent assistance, which we have set to zero in the OECD tax-benefit calculator.

A different value for this composite parameter would only change the coefficients a_i, which are used to match data on wages in each sector i.

We calibrate σ to match the employment-weighted average labor-demand elasticity. The labor-demand elasticity in each sector i is given by (see Appendix G.1 for the derivation):

$$\varepsilon_i = \frac{\sigma}{1 + \phi_i(\sigma(1-\alpha) - 1)}, \tag{31}$$

where $\phi_i \equiv w_i L_i / \sum_j w_j L_j$ is the labor share of sector i in aggregate labor income. We draw on Lichter et al. (2015) who conduct an extensive meta-analysis of labor-demand elasticities. They find an average wage elasticity of labor demand of around 0.55. However, this average contains numerous short-run estimates and we think of our model as describing the economy's long-run equilibrium. Therefore, we use their long-run estimates to account for changes in, e.g., technology and substitution across labor types. Of all studies that explicitly estimate a long-run elasticity of labor demand, the average equals 0.70. We calibrate $\sigma = 0.672$ to match an employment-weighted average labor-demand elasticity of $\bar{\varepsilon} = 0.70$. Since the labor-demand elasticity governs the trade-off between employment and wages at the union level, we conduct several robustness checks with respect to the labor-demand elasticity in Section 7 of the online Appendix.

The productivity shifters a_i can be calculated from the labor-demand equation by using data on employment L_i and wages w_i in each sector i – given the values of α and σ:

$$w_i = F_i(\cdot) = \alpha a_i Y^{\frac{1-(1-\alpha)\sigma}{\alpha\sigma}} L_i^{-\frac{1}{\sigma}}, \tag{32}$$

where aggregate output follows from $Y = \sum_i w_i L_i / \alpha$.

8.1.5 Distribution of participation costs

We impose the following functional form for the distribution of participation costs, which is assumed to be common across all sectors i:

$$G(\varphi) = \frac{\gamma\varphi^{\zeta}}{1 + \gamma\varphi^{\zeta}}, \tag{33}$$

where $\gamma, \zeta > 0$. The reason for choosing this functional form is twofold. First, because participation costs are defined on the interval $\varphi \in [0, \infty)$, full employment is never optimal. This prevents boundary solutions in each sector that could, for instance, occur if $G(\varphi)$ is iso-elastic (so that the participation elasticity is constant) and one considers large tax reforms, such as those from the current to the optimal tax-benefit system. Second, equation (33) generates participation elasticities that are declining in income, in line with empirical evidence, see Hansen (2021) for references. To see this, note that the participation elasticity can be written as

$$\pi_i \equiv \frac{G'(\varphi_i^*)\varphi_i^*}{G(\varphi_i^*)} = \frac{\zeta}{1 + \gamma(\varphi_i^*)^{\zeta}}, \tag{34}$$

where $\varphi_i^* = w_i - T_i + T_u$ is the net gain from working. The latter is larger for individuals who earn a higher net wage $w_i - T_i$.

The parameter ζ is calibrated to match an average participation elasticity of $\bar{\pi} = 0.25$. This value is in line with common empirical estimates, but somewhat higher than estimates for the participation elasticity for the Netherlands. In particular, Mastrogiacomo et al. (2013) documents estimates ranging from 0.10 to 0.16. The reason for choosing a higher value is twofold. First, estimates of the participation elasticity with respect to the unemployment benefit are typically larger. Gercama et al. (2020) estimate a value for this elasticity of around 0.30 for the Netherlands. Second, other extensive margins (e.g., schooling and retirement) may also result in a higher participation elasticity. Because of its importance for the optimal tax-benefit system (especially in the absence of unions), we investigate the robustness of our results with respect to the participation elasticity in Section 7 of the online Appendix.

The average participation elasticity is given by

$$\bar{\pi} = \sum_i \left(\frac{N_i}{\sum_j N_j} \right) \pi_i = \zeta \sum_i \left(\frac{N_i}{\sum_j N_j} \right) \left[1 - \frac{\gamma(\varphi_i^*)^\zeta}{1+\gamma(\varphi_i^*)^\zeta} \right] = \zeta \left[1 - \frac{\sum_i N_i G(\varphi_i^*)}{\sum_j N_j} \right], \tag{35}$$

where the last term in brackets equals one minus the aggregate participation rate, as obtained from Statistics Netherlands (2020a). For an average participation elasticity of 0.25, this gives a value of $\zeta = 0.25/(1 - 0.702) = 0.839$.

The parameter γ determines how many individuals decide to participate in the labor market. We calibrate this parameter to match the aggregate participation rate. Because we only have data on employment $L_i = N_i E_i$, and not on labor force sizes N_i or sectoral employment rates E_i, the parameter γ needs to be calibrated jointly with the degree of union power, as the latter also affects the employment rate.

8.1.6 Union power

Given that there are no direct empirical counterparts of union power ρ_i, neither in the aggregate, nor at the sectoral level, we assume that union power is the same across all sectors: $\rho_i = \rho$ for all i and that, in line with our theoretical analysis, all unemployment observed in the data is caused by unions. The higher the degree of union power ρ, the further away the equilibrium is from the labor-supply curve, and the higher is the unemployment rate, see Figure 2.

We calibrate the value for ρ, joint with γ, such that the unemployment and participation rates in our model match the data. Doing so requires, first, solving the union wage-demand equation (8) for employment E_i for each sector i:

$$\rho \left(\frac{\int_0^{G^{-1}(E_i)} u'(w_i - T_i - \varphi) \mathrm{d}G(\varphi)}{E_i} \right) \frac{w_i}{\varepsilon_i} = u(w_i - T_i - G^{-1}(E_i)) - u(-T_u). \tag{36}$$

Parameters $\rho = 0.215$ and $\gamma = 0.229$ are then chosen in such a way that the involuntary unemployment rate equals 6.9% and the aggregate participation rate equals 70.2% based on data from Statistics Netherlands (2020a). The size of the labor force in each sector i then follows residually from $N_i = L_i/E_i$. We will conduct robustness checks for different values of union power ρ in Section 7 of the online Appendix.

8.1.7 Revenue requirement

The final parameter that needs to be calibrated is the revenue requirement R. Given our assumption that profits flow to the government budget, R follows directly from the budget constraint

$$R = \sum_i N_i(E_i T_i + (1 - E_i)T_u) + (1 - \alpha)\sum_i w_i L_i/\alpha. \tag{37}$$

The revenue requirement equals approximately 36.8% of GDP. Although this number appears high, it includes all capital income, as captured by the last term of equation (37). Correcting for the capital share of $1 - \alpha = 0.243$, the revenue requirement equals 12.5% of GDP, which is close to non-redistribution government spending in the Netherlands of approximately 10% of GDP (Jacobs et al., 2017).

All simulation inputs are summarized in Table 3. Figure 10 in Appendix G.3 plots participation rates, employment rates, and unemployment rates by earnings level in the baseline economy. Sectoral participation rates range from 59.7% (at the lowest wage) to 83.4% (at the highest wage). The sectoral employment rates are between 47.5% and 83.1%, implying that sectoral unemployment rates range from 20.4% (at the lowest wage) to 0.4% (at the highest wage). Figure 11 in Appendix G.3 plots the participation elasticity and the labor-demand elasticity by income level. The participation elasticity declines from 0.34 at the lowest income level to 0.14 at the highest income level. There is little variation in the labor-demand elasticities, which range from 0.67 to 0.72. As can be seen from equation (31), the variation in labor-demand elasticities across sectors is driven solely by the labor shares ϕ_i, which turn out to only have a limited impact.

Table 3: Baseline calibration

Parameter	Value	Calibration target
CARA	$\theta = 0.139$	Avg. participation tax rate 58.3%
Labor income share	$\alpha = 0.757$	Labor income share 75.7%
Elasticity of substitution	$\sigma = 0.672$	Labor-demand elasticity $\bar{\varepsilon} = 0.697$
Union power	$\rho = 0.215$	Unemployment rate 6.9%
Participation curvature	$\zeta = 0.839$	Participation elasticity $\bar{\pi} = 0.25$
Participation shifter	$\gamma = 0.229$	Participation rate 70.2%
Revenue requirement	$R/Y = 0.368$	Government budget constraint

8.2 Optimal taxes and the desirability of unions

The numerical methods for solving the optimal tax system are described in Appendix G. Figure 7 shows the optimal participation tax rates $t_i = (T_i - T_u)/w_i$ in the calibrated economy with unions. The figure also plots the optimal participation tax rates if labor markets are competitive, which are obtained by setting $\rho = 0$, and the participation tax rates in the current tax system.[70] To facilitate comparison, all participation tax rates are plotted against *current* income.

[70] We prefer a 'pure' comparative statics exercise by *only* changing the degree of union power from its value in the calibrated economy to zero (competitive labor markets), while not recalibrating the parameter γ to match the aggregate participation rate. If we would do this, labor force sizes $N_i = L_i/E_i$ would change as well, which complicates the comparison of optimal tax systems with and without unions. Nevertheless, if we recalibrate γ, we obtain very similar conclusions as in the main text.

Comparing the first two lines from Figure 7 shows our most important finding: optimal participation tax rates are substantially lower in unionized labor markets than in competitive labor markets. The average participation tax rate with unions equals approximately 58.3%, as it is calibrated to be the same as in the current tax system. By contrast, if labor markets are competitive (i.e., if $\rho = 0$), the average optimal participation tax rate equals approximately 65.8%. Unions lower the optimal participation tax rates on average by approximately 7.4 percentage points. This reduction is brought about both by a reduction in income taxes and a reduction in the non-employment benefit. On average, income taxes are approximately €1,310 lower in unionized than in competitive labor markets. The optimal non-employment benefit with unions equals approximately €12,560, close to its current value of around €12,223. However, if labor markets are competitive, the optimal non-employment benefit is higher and equals approximately €14,534. The reason why participation tax rates are optimally lower with unions is the presence of the union wedge τ_i. The government optimally lowers participation taxes to moderate union wage demands and to reduce involuntary unemployment.

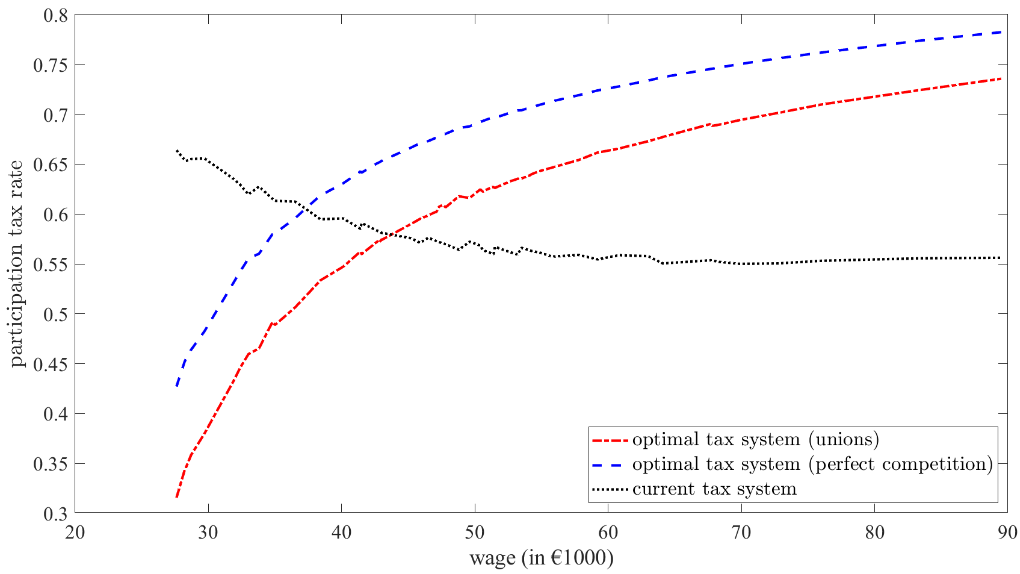


Figure 7: Optimal participation tax rates (baseline)

There is a substantial discrepancy between the current tax system and the optimal tax system, as can be seen from Figure 7. Income taxes for low-income individuals exceed the taxes that would be set by a utilitarian government. This finding confirms earlier research on optimal taxes for the Netherlands in Zoutman et al. (2013). Using the inverse optimal tax approach, Jacobs et al. (2017) demonstrate that the social welfare weights implied by the current tax system in the Netherlands are much larger for the middle-income groups than for the low- and high-income groups, presumably for political-economy reasons. Hence, the current government does not optimize a social welfare function with smoothly declining social welfare weights as in our model.[71]

[71] It is perhaps surprising that participation tax rates at the current tax system are *declining* in income. The reason is quite mechanical. Participation taxes consist of both income taxes and social assistance benefits. In our model, the latter do not vary with earnings. Consequently, if they are expressed as a fraction of the wage, they are lower for high-income earners.

In Section 7, we document that stronger unions are associated with lower participation tax rates. In particular, a reduction in union density from approximately 20% (the average union density in the Netherlands) to zero is associated with a 2.8 percentage-point reduction in the participation tax rate, cf. Table 1. This number is not directly comparable to the 7.4 percentage-point reduction brought about by unions that is documented in Figure 7 for at least three reasons. First, while we can use the structural version of our model to study the causal impact of union power on optimal participation taxes, our estimates from Section 7 do not exploit any exogenous variation in union power and cannot be given a causal interpretation. Second, Figure 7 studies the impact of unions on the *optimal* tax-benefit system, whereas our empirical analysis studies the association between union density and participation tax rates in the *current* tax-benefit system. As discussed above, the current and optimal tax-benefit system differ substantially from each other. However, if we were to calculate optimal participation taxes with and without unions under the assumption that the current tax-transfer system is optimized, we are still confident that optimal participation taxes would be lower with unions than without, as is also demonstrated in the robustness analysis where we vary the degree of inequality aversion, see Section 7 in the online Appendix. Third, our numerical simulations assume that all unemployment is caused by unions demanding wages that are above market-clearing levels. This may create an upward bias for the difference between optimal taxes in unionized and competitive labor markets. Nevertheless, despite these caveats, both findings suggest that stronger unions reduce (optimal) participation tax rates.

Turning to the desirability of unions, Figure 8 plots the social welfare weights against current labor income at the optimal tax system in unionized and in perfectly competitive labor markets. Given that the tax system is optimized, the average social welfare weight in both cases equals one, cf. Proposition 1. Moreover, the concavity in the utility function ensures that the social welfare weights are monotonically declining in income. As can be seen from the figure, the social welfare weight for the unemployed workers (whose wage equals zero) exceeds one and is higher if there are unions. The reason is that the optimal unemployment benefit is lower (i.e., €12,560 with unions versus €14,534 without unions). Furthermore, employed workers in *all* sectors have a social welfare weight that is smaller than the average of one. Hence, there are no employed individuals whose social welfare weight exceeds one.[72] Proposition 2 then immediately implies that if the tax system is optimized, an increase in union power in any sector of the Dutch economy reduces social welfare. Even starting from a competitive labor market, introducing a union for low-income workers is not socially desirable. A utilitarian government would always prefer to increase the net incomes of low-skilled workers directly by reducing taxes (or increasing subsidies) rather than indirectly by increasing the bargaining power of the union representing them. The finding that unions cannot meaningfully complement an optimal tax-benefit system is consistent with the findings from Section 7 that, as a result of positive participation taxes, the desirability condition for unions is not met.

In Section 7 of the online Appendix, we investigate the sensitivity of our results by studying the desirability of unions and optimal participation tax rates in unionized labor markets for different assumptions on the labor-demand and participation elasticity, union power, and the

[72] Recall from Section 8.1.1 that the lowest level of positive earnings in the data is €27,600.

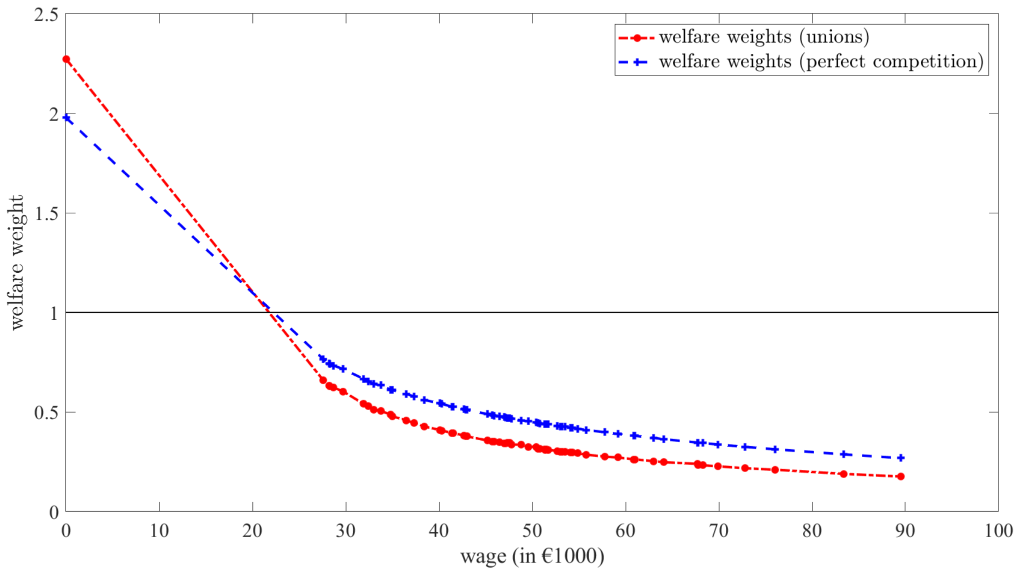


Figure 8: Social welfare weights (baseline)

degree of inequality aversion. Quantitatively, the results change for different assumptions on the main behavioral elasticities. Nevertheless, we find that, in all cases, unions reduce optimal participation tax rates, and increasing union power does not raise social welfare, with one exception. In particular, a very substantial reduction in inequality aversion brings the social welfare weights of unemployed and employed workers closer to each other and raises the social welfare weight of low-skilled workers above the average of one.[73] In that case, participation is optimally subsidized and an increase in union power representing workers at the bottom of the income distribution is welfare-improving. However, a much lower inequality aversion reduces optimal participation tax rates to 27.8% (on average), which is much lower than the optimal participation tax rate of 58.3% (on average) in the calibrated economy. Therefore, unions can only be desirable for social preferences for income redistribution that deviate substantially from redistributive preferences that would rationalize the current tax-benefit system. Hence, the results from this Section corroborate our findings from the empirical analysis: stronger unions are associated with lower participation tax rates and, given that participation is typically taxed (both in the current and in the optimal tax system), unions cannot be used to alleviate upward distortions in labor participation.

9 Conclusion

The aim of this paper has been to answer two questions concerning optimal income redistribution in unionized labor markets. Our first question was: *'How should the government optimize income redistribution if labor markets are unionized?'* Our answer is that the optimal tax-benefit system is less redistributive than in competitive labor markets. Intuitively, the tax system is

[73]There could also be non-welfarist motives why the social welfare weights of low-income workers are raised relative to the social welfare weight of the non-employed. This would be the case, for instance, if the working poor are considered more deserving or if work is a merit good. We abstract from this in our analysis.

not only used to redistribute income, but also to alleviate the distortions induced by unions. Lower income taxes and lower benefits motivate unions to moderate their wage demands, which results in less involuntary unemployment. We show that participation taxes should be lower the larger are the welfare gains from reducing involuntary unemployment. Therefore, it may be optimal to subsidize participation even for workers with a below-average social welfare weight, which cannot happen if labor markets are competitive (see, e.g., Diamond, 1980, Saez, 2002, and Choné and Laroque, 2011). We collect data on participation tax rates and union densities – a proxy for union power – from 18 sectors in 23 OECD countries. In line with our theoretical predictions, we find there is a negative association between participation tax rates and union densities. Furthermore, we simulate a structural version of our model, which is calibrated to the Netherlands. Our simulations suggest that optimal participation tax rates are substantially lower if unions are more powerful.

Our second question was: *'Can labor unions be socially desirable if the government wants to redistribute income?'* Our answer is that increasing the power of the unions representing workers with an above-average social welfare weight is welfare-enhancing, while the opposite holds true for workers with a below-average social welfare weight. Since Diamond (1980), it is well known that participation is optimally subsidized for workers with an above-average social welfare weight, i.e., they receive an income transfer that exceeds the unemployment benefit. Consequently, participation for these workers is distorted upwards, which results in *overemployment*. By bidding up wages, unions create implicit taxes on employment, which reduce the upward distortions from participation subsidies. Whether unions are desirable thus depends critically on whether low-income workers are subsidized or taxed on a net basis. We calculate participation taxes throughout the income distribution and find that they are always positive in nearly all OECD countries. Our data thus reveal that unions are nowhere desirable for income redistribution. Moreover, in our simulations, we find that increasing union power typically lowers social welfare, but this finding is sensitive to the government's preference for income redistribution. Hence, increasing union power would typically not be socially desirable, as it would only exacerbate labor-market distortions.

We have made some assumptions that warrant further research. First, we assumed throughout the paper that the government is the Stackelberg leader relative to firms and unions. However, unions may internalize some of the macro-economic and fiscal impacts of their decisions in wage negotiations, see also Calmfors and Driffill (1988). Second, we have abstracted from labor supply on the intensive (hours, or effort) margin. For future research, it would be interesting to study a setting where unions and the government interact strategically and labor supply also responds on the intensive margin.

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A Derivation of ρ_i from the Right-to-Manage model

In this Appendix, we derive the relationship between our measure of union power ρ_i and the bargaining power in the Nash product that is more commonly used to characterize equilibrium in the RtM-model (see, for instance, Boeri and Van Ours, 2008). In particular, the Nash bargaining problem is given by:

$$\begin{aligned}\max_{w_i,E_i}\ \Omega_i = {} & \delta_i \log\left(\int_{\underline{\varphi}}^{G_i^{-1}(E_i)} (u(w_i - T_i - \varphi) - u(-T_u))\mathrm{d}G_i(\varphi)\right) \\ & + (1-\delta_i)\log\left(u(F(\cdot) - \sum_i w_i N_i E_i - T_f) - u(F(\cdot)|_{E_i=0} - \sum_{j\neq i} w_j N_j E_j - T_f)\right) \\ \text{s.t. } & w_i = F_i(\cdot), \\ & G_i(w_i - T_i + T_u) - E_i \geq 0, \end{aligned} \tag{38}$$

where $\delta_i \in [0,1]$ is the weight attached to the union's payoff in the Nash product, and $F(\cdot)|_{E_i=0}$ is the firm's output if it does not reach an agreement with the union in sector i, and, hence, none of the workers in sector i find employment. The payoffs are taken in deviation from the payoff associated with the disagreement outcome. It is important to take the voluntary participation constraint in equation (38) explicitly into account, as it will bind for small values of δ_i. If δ_i

is close to zero, labor-market equilibrium is characterized by the final two conditions, which jointly determine the competitive equilibrium.

The Lagrangian reads as:

$$\begin{aligned}\mathcal{L} = \delta_i \log \left(\int_{\underline{\varphi}}^{G_i^{-1}(E_i)} (u(w_i - T_i - \varphi) - u(-T_u)) \mathrm{d}G_i(\varphi) \right) \\ + (1-\delta_i) \log \left(u(F(\cdot) - \sum_i w_i N_i E_i - T_f) - u(F(\cdot)|_{E_i=0} - \sum_{j \neq i} w_j N_j E_j - T_f) \right) \\ + \vartheta_i (w_i - F_i(\cdot)) + \mu_i (G(w_i - T_i + T_u) - E_i). \end{aligned} \tag{39}$$

The first-order conditions are given by:

$$w_i : \ \frac{\delta_i}{(\overline{u_i} - u_u)} \overline{u_i'} - \frac{(1-\delta_i)}{(u_f - u_f^{-i})} u_f' N_i E_i + \vartheta_i + \mu_i G_i' = 0, \tag{40}$$

$$E_i : \ \frac{\delta_i}{E_i(\overline{u_i} - u_u)} (\hat{u}_i - u_u) - \vartheta_i F_{ii} N_i - \mu_i = 0, \tag{41}$$

$$\vartheta_i : \ w_i - F_i = 0, \tag{42}$$

$$\mu_i : \ \mu_i (G_i - E_i) = 0, \tag{43}$$

where the bars indicate averages over all employed workers in sector i, $\hat{u}_i$ is the utility of the marginal worker in sector i and $u_f^{-i} \equiv u(F(\cdot)|_{E_i=0} - \sum_{j \neq i} w_j N_j E_j - T_f)$ is the utility of firm-owners if they fail to reach an agreement with the union in sector i. If $\delta_i = 1$, equations (40)–(41) imply that $\mu_i = 0$, and we find the equilibrium of the monopoly-union model. For small values of δ_i, the constraint $G_i = E_i$ becomes binding, and the labor-market equilibrium coincides with the competitive outcome. This can be verified by setting $\delta_i = 0$. Equations (40)–(41) then imply that $\mu_i > 0$. This is the case for all values of $\delta_i \in [0, \delta_i^*]$, where $\delta_i^* \in (0,1)$ solves:

$$\frac{\delta_i^*}{1-\delta_i^*} = \frac{E_i(\overline{u_i} - u_u)}{(u_f - u_f^{-i})} \frac{u_f' N_i}{\overline{u_i'}}. \tag{44}$$

This equation is obtained by setting $G_i = E_i$ and $\mu_i = 0$ in the system of first-order conditions in equations (40)–(43). The reason is that, at exactly this value of δ_i, the constraint $G_i = E_i$ becomes binding. For values of $\delta_i \in [\delta_i^*, 1]$, we thus have $\mu_i = 0$. Combining equations (40)–(41) then leads to:

$$1 - \left(\frac{1-\delta_i}{\delta_i} \right) \frac{E_i(\overline{u_i} - u_u)}{(u_f - u_f^{-i})} \frac{u_f' N_i}{\overline{u_i'}} = \varepsilon_i \frac{(\hat{u}_i - u_u)}{\overline{u_i'} w_i}. \tag{45}$$

If we write the left-hand side of this equation as

$$\rho_i = 1 - \left(\frac{1-\delta_i}{\delta_i} \right) \frac{E_i(\overline{u_i} - u_u)}{(u_f - u_f^{-i})} \frac{u_f' N_i}{\overline{u_i'}}, \tag{46}$$

we arrive at our equilibrium condition (8). Clearly, if $\delta_i = 1$, we have $\rho_i = 1$, so that the MU-model applies. If $\delta_i = \delta_i^*$, from equation (44) it follows that $\rho_i = 0$, and the equilibrium coincides with the competitive outcome. Hence, the relationship between our measure of union power ρ_i and the Nash-bargaining parameter δ_i is:

$$\rho_i = \begin{cases} 0 & \text{if} \quad \delta_i \in [0, \delta_i^*), \\ 1 - \frac{(1-\delta_i)}{\delta_i} \frac{E_i(\overline{u_i} - u_u)}{(u_f - u_f^{-i})} \frac{u_f' N_i}{\overline{u_i'}} & \text{if} \quad \delta_i \in [\delta_i^*, 1]. \end{cases} \tag{47}$$

For a given tax-benefit system, this equation specifies a direct relationship between δ_i and ρ_i. The mapping clearly depends on endogenous objects such as the tax-benefit system and the threshold δ_i^*. For reasons explained in the main text, we prefer to characterize equilibrium using our measure of union power ρ_i instead of the Nash-bargaining parameter δ_i.[74]

B Derivation elasticities

This appendix derives the employment and wage responses to changes in the tax instruments and union power if labor markets are independent and rationing is efficient (i.e., if Assumptions 1 and 2 hold). The labor-market equilibrium conditions are as given by equations (11) and (12). Substituting the labor-demand equation $w_i = F_i(\cdot)$ in equation (12), equilibrium employment in sector i is determined implicitly by the following condition:

$$\Gamma(E_i, T_i, T_u, \rho_i) \equiv$$
$$\rho_i \int_{\underline{\varphi}}^{G_i^{-1}(E_i)} u'(F_i(\cdot) - T_i - \varphi)\mathrm{d}G_i(\varphi) F_{ii}(\cdot) N_i + \big(u(F_i(\cdot) - T_i - G_i^{-1}(E_i)) - u(-T_u)\big) = 0. \tag{48}$$

Since labor markets are independent, $F_i(\cdot)$ and $F_{ii}(\cdot)$ depend only on employment $L_i = N_i E_i$ in sector i. Hence, this equation pins down $E_i = E_i(T_i, T_u, \rho_i)$. If the union objective (5) is concave in E_i after substituting $\hat{\varphi}_i = G_i^{-1}(E_i)$ and $w_i = F_i(\cdot)$, it follows that $\Gamma(\cdot)$ is decreasing in E_i. The comparative statics can be determined through the implicit function theorem:

$$\frac{\partial E_i}{\partial T_i} = -\frac{\Gamma_{T_i}}{\Gamma_{E_i}} = \frac{\rho_i E_i \overline{u_i''} F_{ii} N_i + \hat{u}_i'}{\rho_i E_i \overline{u_i''}(F_{ii}N_i)^2 + \rho_i E_i \overline{u_i'} F_{iii} N_i^2 + \hat{u}_i'((1+\rho_i)F_{ii}N_i - 1/G_i')} < 0, \tag{49}$$

$$\frac{\partial E_i}{\partial T_u} = -\frac{\Gamma_{T_u}}{\Gamma_{E_i}} = \frac{-u_u'}{\rho_i E_i \overline{u_i''}(F_{ii}N_i)^2 + \rho_i E_i \overline{u_i'} F_{iii} N_i^2 + \hat{u}_i'((1+\rho_i)F_{ii}N_i - 1/G_i')} > 0, \tag{50}$$

$$\frac{\partial E_i}{\partial \rho_i} = -\frac{\Gamma_{\rho_i}}{\Gamma_{E_i}} = \frac{-E_i \overline{u_i'} F_{ii} N_i}{\rho_i E_i \overline{u_i''}(F_{ii}N_i)^2 + \rho_i E_i \overline{u_i'} F_{iii} N_i^2 + \hat{u}_i'((1+\rho_i)F_{ii}N_i - 1/G_i')} < 0. \tag{51}$$

We ignored function arguments to save on notation. The impact on the equilibrium wage w_i follows directly from the labor-demand equation $w_i = F_i(\cdot)$:

$$\frac{\partial w_i}{\partial x} = \frac{\partial w_i}{\partial E_i}\frac{\partial E_i}{\partial x} = F_{ii} N_i \frac{\partial E_i}{\partial x}, \quad x = T_i, T_u, \rho_i \tag{52}$$

[74] To the best of our knowledge, the Nash-bargaining parameter δ_i does not have a clear economic interpretation or game-theoretic foundation the current setting. The reason is that the equilibrium is restricted to lie on the labor-demand curve (4), which violates the axiom of Pareto optimality (see also Section 5 in the online Appendix). In addition, as mentioned, for low values of δ_i, the voluntary participation constraint $G_i \geq E_i$ becomes binding.

$$\frac{\partial w_i}{\partial T_i} = \frac{(\rho_i E_i \overline{u_i''} N_i F_{ii} + \hat{u}_i')F_{ii}N_i}{\rho_i E_i \overline{u_i''}(N_i F_{ii})^2 + \rho_i E_i \overline{u_i'} F_{iii} N_i^2 + \hat{u}_i'((1+\rho_i)F_{ii}N_i - 1/G_i')} > 0, \tag{53}$$

$$\frac{\partial w_i}{\partial T_u} = \frac{-u_u' F_{ii} N_i}{\rho_i E_i \overline{u_i''}(N_i F_{ii})^2 + \rho_i E_i \overline{u_i'} F_{iii} N_i^2 + \hat{u}_i'((1+\rho_i)F_{ii}N_i - 1/G_i')} < 0, \tag{54}$$

$$\frac{\partial w_i}{\partial \rho_i} = \frac{-E_i \overline{u_i'}(F_{ii}N_i)^2}{\rho_i E_i \overline{u_i''}(N_i F_{ii})^2 + \rho_i E_i \overline{u_i'} F_{iii} N_i^2 + \hat{u}_i'((1+\rho_i)F_{ii}N_i - 1/G_i')} > 0. \tag{55}$$

If there are no income effects at the union level (cf. Assumption 3), a change in the unemployment benefit has the same impact as an increase in the income tax. Setting $\partial E_i/\partial T_i = -\partial E_i/\partial T_u$ and $\partial w_i/\partial T_i = -\partial w_i/\partial T_u$, it follows that income effects are absent if

$$\rho_i E_i \overline{u_i''} N_i F_{ii} + (\hat{u}_i' - u_u') = 0.$$

If utility is linear, this condition is trivially satisfied. In addition, the condition also holds if utility is of the CARA-type, i.e., $u(c) = -\exp(-\theta c)/\theta$. To see this, substitute $u'(c) = \exp(-\theta c)$ in equation (48) and multiply the expression by $\exp(-\theta T_u)$. The equation then depends on the tax instruments only through the participation tax level $T_i - T_u$.

C First-best allocation

We assume throughout the paper that the government cannot observe participation costs φ. Hence, taxes cannot be conditioned on φ. However, if taxes can be conditioned on participation costs, it is possible to decentralize the first-best allocation as a competitive equilibrium.[75] In this case, the wage in each sector is equated to the marginal productivity of labor, i.e., $w_i = F_i(\cdot)$. Moreover, individuals in sector i with participation costs $\varphi \leq G_i^{-1}(E_i)$ will all be employed. The first-best allocation is characterized by choosing taxes $T_{i,\varphi}$, T_f and employment rates E_i that maximize social welfare subject only to the government budget constraint. The Lagrangian for this maximization problem is given by:

$$\begin{aligned}\mathcal{L} = &\sum_i \psi_i N_i \left[\int_{\underline{\varphi}}^{G_i^{-1}(E_i)} u(F_i(\cdot) - T_{i,\varphi} - \varphi)\mathrm{d}G_i(\varphi) + \int_{G_i^{-1}(E_i)}^{\overline{\varphi}} u(-T_{i,\varphi})\mathrm{d}G_i(\varphi) \right] \\ &+ \psi_f u(F(\cdot) - \sum_i F_i(\cdot) N_i E_i - T_f) + \lambda \left[\sum_i N_i \int_{\underline{\varphi}}^{\overline{\varphi}} T_{i,\varphi} \mathrm{d}G_i(\varphi) + T_f - R \right].\end{aligned} \tag{56}$$

[75] Because the first-best allocation can be decentralized as a competitive equilibrium, it follows immediately that unions cannot improve on the allocation.

The first-order conditions are:

$$\begin{aligned}
T_{i,\varphi}: &\quad N_i(\lambda - \psi_i u'(F_i(\cdot) - T_{i,\varphi} - \varphi))g_i(\varphi) = 0 \quad \text{if} \quad \varphi \le G_i^{-1}(E_i), &(57)\\
&\quad N_i(\lambda - \psi_i u'(-T_{i,\varphi}))g_i(\varphi) = 0 \quad \text{if} \quad \varphi > G_i^{-1}(E_i), &(58)\\
T_f: &\quad \lambda - \psi_f u'(c_f) = 0, &(59)\\
E_i: &\quad \psi_i N_i(u(F_i(\cdot) - T_{i,\varphi} - G_i^{-1}(E_i)) - u(-T_{i,\varphi}))\\
&\quad + N_i \sum_j F_{ji}(\cdot) N_j \left[\int_{\underline{\varphi}}^{G_i^{-1}(E_i)} \psi_j u'(F_j(\cdot) - T_{j,\varphi} - \varphi)\mathrm{d}G_i(\varphi) - \psi_f u'(c_f) \right] = 0, &(60)\\
\lambda: &\quad \sum_i N_i \int_{\underline{\varphi}}^{\overline{\varphi}} T_{i,\varphi}\mathrm{d}G_i(\varphi) + T_f - R = 0. &(61)
\end{aligned}$$

$c_f = F(\cdot) - \sum_i F_i(\cdot) N_i E_i - T_f$ is the consumption of firm-owners. At the first-best allocation, all social welfare weights are equalized: $\psi_f u'(c_f) = \psi_i u'(c_{i,\varphi}) = \lambda$, where $c_{i,\varphi}$ is the consumption of an individual in sector i with participation costs φ. Because all social welfare weights are equalized, the terms in the second line of equation (60) cancel. Equation (60) then implies employment is efficient: $F_i(\cdot) = G_i^{-1}(E_i)$.

D Optimal taxation

D.1 Proof Proposition 1

The Lagrangian associated with the government's optimization problem can be written as:

$$\begin{aligned}
\mathcal{L} = &\sum_i \psi_i N_i \Bigg(\int_{\underline{\varphi}}^{G_i^{-1}(E_i)} u(w_i - (T_i - T_u) - T_u - \varphi)\mathrm{d}G_i(\varphi) + \int_{G_i^{-1}(E_i)}^{\overline{\varphi}} u(-T_u)\mathrm{d}G_i(\varphi) \Bigg) \qquad (62)\\
&+ \psi_f u(F(\cdot) - \sum_i w_i N_i E_i - T_f) + \lambda \left(\sum_i N_i (T_u + E_i(T_i - T_u)) + T_f - R \right).
\end{aligned}$$

If income effects are absent (cf. Assumption 3), equilibrium wages and employment rates depend only on participation taxes $T_i - T_u$. Using the latter as instruments (instead of income taxes T_i), the first-order conditions are:

$$\begin{aligned}
T_u: &\quad -\sum_i \psi_i N_i (E_i \overline{u'_i} + (1 - E_i)u'_u) + \lambda \sum_i N_i = 0, &(63)\\
T_f: &\quad -\psi_f u'_f + \lambda = 0, &(64)\\
T_i - T_u: &\quad -N_i E_i (\psi_i \overline{u'_i} - \lambda) + \sum_j N_j E_j \left[\psi_j \overline{u'_j} - \psi_f u'_f \right] \frac{\partial w_j}{\partial (T_i - T_u)}\\
&\quad + \sum_j N_j \left[\psi_j (\hat{u}_j - u_u) + \lambda (T_j - T_u) \right] \frac{\partial E_j}{\partial (T_i - T_u)} = 0. &(65)
\end{aligned}$$

To obtain equation (16), divide equation (63) by $\lambda \sum_j N_j$ to find

$$1 = \sum_i \underbrace{\left(\frac{N_i E_i}{\sum_j N_j}\right)}_{\equiv \omega_i} \underbrace{\left(\frac{\psi \overline{u_i'}}{\lambda}\right)}_{\equiv b_i} + \underbrace{\left(\frac{\sum_i N_i(1-E_i)}{\sum_j N_j}\right)}_{\equiv \omega_u} \underbrace{\left(\frac{\sum_i N_i(1-E_i)\psi_i u_u'}{\sum_i N_i(1-E_i)\lambda}\right)}_{\equiv b_u}. \tag{66}$$

Next, divide equation (64) by λ to find equation (17).

To derive equation (18), first define the employment and wage elasticities as:

$$\eta_{ji} \equiv -\left(\frac{\partial E_j}{\partial(T_i - T_u)} \frac{(w_i - (T_i - T_u))}{E_j}\right) \frac{w_j(1-t_j)}{w_i(1-t_i)}, \tag{67}$$

$$\kappa_{ji} \equiv \left(\frac{\partial w_j}{\partial(T_i - T_u)} \frac{(w_i - (T_i - T_u))}{w_j}\right) \frac{w_j}{w_i(1-t_i)} \tag{68}$$

Then, divide equation (65) by $\lambda \sum_i N_i$, use the definitions of the employment shares and the union wedge $\tau_j \equiv \frac{\psi_j(\hat{u}_j - u_u)}{w_j \lambda} = \frac{\rho_j b_j}{\varepsilon_j}$, and rewrite to find:

$$\sum_j \omega_j \frac{(t_j + \tau_j)}{(1-t_j)} \eta_{ji} = \omega_i(1-b_i) + \sum_j \omega_j (b_j - b_f)\kappa_{ji}. \tag{69}$$

Note that this result holds irrespective of whether profits are optimally taxed (i.e., $b_f = 1$) or not (i.e., $b_f < 1$).

D.2 Allowing for income effects

If there are income effects at the union level, changes in the unemployment benefit $-T_u$ affect equilibrium employment E_i and wages w_i not only through their impact on participation taxes $T_i - T_u$. Therefore, we write $E_i = E_i(T_1 - T_u, \cdots, T_I - T_u, T_u)$ and $w_i = w_i(T_1 - T_u, \cdots, T_I - T_u, T_u)$. In this case, only the first-order condition for the optimal unemployment benefit (i.e., the counterpart of equation (63)) has to be modified:

$$\begin{aligned} T_u : &- \sum_i \psi_i N_i (E_i \overline{u_i'} + (1-E_i) u_u') + \lambda \sum_i N_i \\ &+ \sum_i N_i E_i \left[\psi_i \overline{u_i'} - \psi_f u_f'\right] \frac{\partial w_i}{\partial T_u} + \sum_i N_i \left[\lambda(T_i - T_u) + \psi_i(\hat{u}_i - u_u)\right] \frac{\partial E_i}{\partial T_u} = 0 \end{aligned} \tag{70}$$

Divide this expression by $\lambda \sum_i N_i$ to find

$$\begin{aligned} &- \sum_i \frac{N_i E_i}{\sum_i N_i} (b_i + \frac{(1-E_i)}{E_i} \psi_i u_u'/\lambda) + 1 \\ &+ \sum_i \frac{N_i E_i}{\sum_i N_i} \left[b_i - b_f\right] \frac{\partial w_i}{\partial T_u} + \sum_i \frac{N_i E_i}{\sum_i N_i} \left[(T_i - T_u) + \psi_i(\hat{u}_i/\lambda - u_u/\lambda)\right] \frac{\partial E_i}{\partial T_u} \frac{1}{E_i} = 0 \end{aligned} \tag{71}$$

Next, substitute $\omega_i \equiv \frac{N_i E_i}{\sum_i N_i}$, $\omega_u \equiv \frac{\sum_i N_i(1-E_i)}{\sum_j N_j}$, $b_i \equiv \frac{\psi_i \overline{u_i'}}{\lambda}$, $b_u \equiv \frac{\sum_i N_i(1-E_i)\psi_i u_u'/\lambda}{\sum_i N_i(1-E_i)}$ and rewrite:

$$\begin{aligned}
&-\sum_i \omega_i b_i - \omega_u b_u + 1 \\
&+\sum_i \omega_i \Big[b_i - b_f\Big] \frac{\partial w_i}{\partial T_u} + \sum_i \omega_i \Big[(T_i - T_u) + \psi_i(\hat{u}_i/\lambda - u_u/\lambda)\Big] \frac{\partial E_i}{\partial T_u} \frac{1}{E_i} = 0
\end{aligned} \tag{72}$$

To proceed, substitute $b_f = 1$ and $\tau_i = \frac{\psi_i(\hat{u}_i - u_u)}{\lambda w_i}$:

$$\sum_i \omega_i b_i + \omega_u b_u = 1 + \sum_i \omega_i \Big[b_i - 1\Big] \frac{\partial w_i}{\partial T_u} + \sum_i \omega_i \Big[(T_i - T_u) + \tau_i w_i\Big] \frac{\partial E_i}{\partial T_u} \frac{1}{E_i} \tag{73}$$

This relationship generalizes equation (16). If there are income effects at the union level, a simultaneous increase in the unemployment benefit $-T_u$ and all income taxes T_i that leaves participation taxes unchanged does *not* leave labor-market outcomes unaffected. The welfare-relevant effects are captured by the last two terms on the right-hand side of equation (73). A change in the equilibrium wage in sector i indirectly redistributes income between workers in that sector (whose social welfare weight is b_i) and firm-owners (whose social welfare weight is one). In addition, a change in the employment rate in sector i affects social welfare through the participation tax $T_i - T_u$ and the union wedge $\tau_i w_i$. The government has to take into account these responses when deciding on the optimal benefit $-T_u$.

Equation (73) can be simplified considerably if labor markets are independent. In that case, we can use the property $\frac{\partial E_i}{\partial x_i} = \frac{\partial E_i}{\partial w_i}\frac{\partial w_i}{\partial x_i}$ for $x_i \in \{T_u, T_i - T_u\}$, where $\partial E_i/\partial w_i = 1/(N_i F_{ii}(\cdot))$ is the slope of the labor-demand curve. Equation (73) can then be written as

$$\sum_i \omega_i b_i + \omega_u b_u = 1 + \sum_i \omega_i \Big[b_i - 1\Big] \frac{\partial w_i}{\partial T_u} + \sum_i \omega_i \Big[(T_i - T_u) + \tau_i w_i\Big] \frac{\partial E_i}{\partial w_i} \frac{\partial w_i}{\partial T_u} \frac{1}{E_i} \tag{74}$$

$$\sum_i \omega_i b_i + \omega_u b_u = 1 + \sum_i \left(\omega_i \Big[b_i - 1\Big] + \omega_i \Big[(T_i - T_u) + \tau_i w_i\Big] \frac{\partial E_i}{\partial w_i} \frac{1}{E_i}\right) \frac{\partial w_i}{\partial T_u} \tag{75}$$

If labor markets are independent, the term in brackets on the right-hand side can be obtained from the first-order condition with respect to $T_i - T_u$:

$$\omega_i(1 - b_i) + \omega_i \Big[(T_i - T_u) + \tau_i w_i\Big] \frac{1}{E_i} \frac{\partial E_i}{\partial (T_i - T_u)} + \omega_i(b_i - 1) \frac{\partial w_i}{\partial (T_j - T_u)} = 0 \tag{76}$$

$$\left(\omega_i \Big[b_i - 1\Big] + \omega_i \Big[(T_i - T_u) + \tau_i w_i\Big] \frac{1}{E_i} \frac{\partial E_i}{\partial w_i}\right) \frac{\partial w_i}{\partial (T_i - T_u)} = -\omega_i(1 - b_i), \tag{77}$$

where we imposed independent labor markets and again used the property $\frac{\partial E_i}{\partial x_i} = \frac{\partial E_i}{\partial w_i}\frac{\partial w_i}{\partial x_i}$. We then arrive at the following condition:

$$\sum_i \omega_i b_i + \omega_u b_u = 1 - \sum_i \omega_i (1 - b_i) \frac{\partial w_i/\partial T_u}{\partial w_i/\partial (T_i - T_u)} \tag{78}$$

$$\sum_i \omega_i b_i + \omega_u b_u = 1 - \sum_i \omega_i (1 - b_i) \iota_i, \tag{79}$$

where $\iota_i \equiv \frac{\partial w_i}{\partial T_u} / \frac{\partial w_i}{\partial (T_i - T_u)}$. Appendix B shows that $\iota_i = 0$ if the utility function $u(\cdot)$ is linear, i.e., $u(c) = c$ or if the utility function is of the CARA-type, i.e., $u(c) \equiv -\frac{1}{\theta}\exp[-\theta c]$.

D.3 Optimal participation tax with perfect competition

To derive an expression for the optimal participation tax with competitive labor markets (i.e., $\rho_i = 0$ for all i), we reformulate the optimal tax problem. Instead of taking the impact of the tax instruments on labor-market outcomes into account through the reduced-form equations $E_i = E_i(\cdot)$ and $w_i = w_i(\cdot)$, we substitute $w_i = F_i(\cdot)$ and make the equilibrium employment rate in each sector an additional choice variable in the government's optimization problem. The labor-market equilibrium condition $G_i(F_i(\cdot) - (T_i - T_u)) = E_i$ for each i then enters the optimal tax problem explicitly as a constraint. The Lagrangian associated with the government's optimization problem is then given by:

$$\begin{aligned}\mathcal{L} = &\sum_i \psi_i N_i \Big(\int_{\underline{\varphi}}^{G_i^{-1}(E_i)} u(F_i(\cdot) - (T_i - T_u) - T_u - \varphi)\mathrm{d}G_i(\varphi) + \int_{G_i^{-1}(E_i)}^{\overline{\varphi}} u(-T_u)\mathrm{d}G_i(\varphi) \Big) \\ &+ \psi_f u(F(\cdot) - \sum_i F_i(\cdot) N_i E_i - T_f) + \lambda \left(\sum_i N_i (T_u + E_i(T_i - T_u)) + T_f - R \right) \\ &\sum_i \mu_i \Big[G(F_i(\cdot) - (T_i - T_u)) - E_i \Big]. \end{aligned} \tag{80}$$

The first-order conditions with respect to $T_i - T_u$ and E_i are:

$$T_i - T_u: \quad N_i E_i (\lambda - \psi_i \overline{u_i'}) - \mu_i G_i' = 0, \tag{81}$$

$$E_i: \quad \lambda N_i (T_i - T_u) - \mu_i + N_i \sum_j F_{ji} \Big[N_j E_j (\psi_j \overline{u_j'} - \psi_f u_f') + \mu_j G_j' \Big] = 0. \tag{82}$$

If the profit tax is optimally set, $\psi_f u_f' = \lambda$, and hence, $b_f = 1$. The first-order condition (81) then implies that the term in brackets in equation (82) that is summed over j equals zero. Next, use equation (81) to substitute for μ_i in equation (82), divide the equation by λN_i, and use the property $E_i = G_i$. Rearranging gives the result stated in the main text:

$$\frac{t_i}{1 - t_i} = \frac{1 - b_i}{\pi_i}, \quad \pi_i \equiv \frac{G_i'(\varphi_i^*)\varphi_i^*}{G_i(\varphi_i^*)}, \tag{83}$$

where $\varphi_i^* = w_i - (T_i - T_u)$ is the participation threshold.

E Desirability of unions

E.1 Proof Proposition 2

To determine how an increase in union power affects social welfare, we set up the optimal tax problem while taking the labor-market equilibrium conditions explicitly into account as constraints, rather than deriving our results in terms of sufficient statistics. The reason for doing so is that this approach allows us to directly derive the welfare effect of an increase in union power. The maximization problem for the government is:

$$
\begin{aligned}
\max_{T_u, T_f, \{T_i - T_u, w_i, E_i\}_{i=1}^{I}} \quad \mathcal{W} = & \sum_i \psi_i N_i \biggl(\int_{\underline{\varphi}}^{G_i^{-1}(E_i)} u(w_i - (T_i - T_u) - T_u - \varphi) \mathrm{d}G_i(\varphi) \\
& + \int_{G_i^{-1}(E_i)}^{\overline{\varphi}} u(-T_u) \mathrm{d}G_i(\varphi) \biggr) + \psi_f u(F(\cdot) - \sum_i w_i N_i E_i - T_f), \\
\text{s.t.} \qquad & \sum_i N_i (T_u + E_i (T_i - T_u)) + T_f = R, \\
& w_i = F_i(\cdot), \quad \forall i, \\
& \rho_i \int_{\underline{\varphi}}^{G_i^{-1}(E_i)} u'(w_i - (T_i - T_u) - T_u - \varphi) \mathrm{d}G_i(\varphi) F_{ii}(\cdot) N_i \\
& + u(w_i - (T_i - T_u) - T_u - G_i^{-1}(E_i)) - u(-T_u) = 0, \quad \forall i. \qquad (84)
\end{aligned}
$$

By using the labor-demand equations to substitute for wages $w_i = F_i(\cdot)$, the corresponding Lagrangian is given by:

$$
\begin{aligned}
\mathcal{L} = & \sum_i \psi_i N_i \left(\int_{\underline{\varphi}}^{G_i^{-1}(E_i)} u(F_i(\cdot) - (T_i - T_u) - T_u - \varphi) \mathrm{d}G_i(\varphi) + \int_{G_i^{-1}(E_i)}^{\overline{\varphi}} u(-T_u) \mathrm{d}G_i(\varphi) \right) \\
& + \psi_f u(F(\cdot) - \sum_i F_i(\cdot) N_i E_i - T_f) + \lambda \left(\sum_i N_i (T_u + E_i (T_i - T_u)) + T_f - R \right) \\
& + \sum_i \mu_i \biggl(\rho_i \int_{\underline{\varphi}}^{G_i^{-1}(E_i)} u'(F_i(\cdot) - (T_i - T_u) - T_u - \varphi) \mathrm{d}G_i(\varphi) F_{ii}(\cdot) N_i \\
& + u(F_i(\cdot) - (T_i - T_u) - T_u - G_i^{-1}(E_i)) - u(-T_u) \biggr). \qquad (85)
\end{aligned}
$$

To save on notation, in the remainder we ignore function arguments and use bars to denote averages. The first-order conditions are then given by:

$$
\begin{aligned}
& T_i - T_u: \ -N_i E_i (\psi_i \overline{u_i'} - \lambda) - \mu_i \left(\rho_i \overline{u_i''} F_{ii} N_i E_i + \hat{u}_i' \right) = 0, && (86) \\
& T_u: \ -\sum_i N_i E_i \psi_i \overline{u_i'} - \sum_i N_i (1 - E_i) \psi_i u_u' + \lambda \sum_i N_i \\
& \qquad - \sum_i \mu_i \left(\rho_i \overline{u_i''} F_{ii} N_i E_i + \hat{u}_i' - u_u' \right) = 0, && (87) \\
& T_f: \ -\psi_f u_f' + \lambda = 0 && (88) \\
& E_i: \ N_i \psi_i (\hat{u}_i - u_u) + \lambda N_i (T_i - T_u) + N_i \sum_j N_j E_j (\psi_j \overline{u_j'} - \psi_f u_f') F_{ji} + \mu_i \left(\rho_i \hat{u}_i' F_{ii} N_i - \hat{u}_i' / G_i' \right) \\
& \qquad + N_i \sum_j \mu_j \left[\left(\rho_j E_j \overline{u_j''} F_{jj} N_j + \hat{u}_j' \right) F_{ji} + \rho_j E_j \overline{u_j'} N_j F_{jji} \right] = 0, && (89) \\
& \lambda: \ \sum_i N_i (T_u + E_i (T_i - T_u)) + T_f - R = 0 && (90) \\
& \mu_i: \ \rho_i E_i \overline{u_i'} F_{ii} + (\hat{u}_i - u_u) = 0 && (91)
\end{aligned}
$$

This system of first-order conditions implicitly characterizes optimal tax policy in terms of the primitives of the model (in particular, union power, Pareto weights, the revenue requirement and properties of the utility and production function). Unfortunately, these equations are rather difficult to interpret or to simplify. This explains why, in the main text, we focus on the characterization of optimal tax policy in terms of sufficient statistics.

To examine how an increase in union power ρ_i in sector i affects social welfare, differentiate the Lagrangian (85) with respect to ρ_i, and apply the envelope theorem:

$$\frac{\partial \mathcal{W}}{\partial \rho_i} = \frac{\partial \mathcal{L}}{\partial \rho_i} = \mu_i E_i \overline{u_i'} F_{ii} N_i. \tag{92}$$

Since $E_i \overline{u_i'} F_{ii} N_i < 0$ (provided that labor demand is not perfectly elastic), the expression in equation (92) is positive if and only if $\mu_i < 0$. To determine the sign of μ_i, rearrange the first-order condition (86) with respect to the participation tax $T_i - T_u$:

$$\lambda N_i E_i \left(1 - \frac{\psi_i \overline{u_i'}}{\lambda}\right) = \mu_i \left(\rho_i \overline{u_i''} F_{ii} N_i E_i + \hat{u}_i'\right). \tag{93}$$

By concavity of the utility function $u(\cdot)$ and the production function $F(\cdot)$, $\rho_i \overline{u_i''} F_{ii} N_i E_i + \hat{u}_i' > 0$. Denoting by $b_i = \psi_i \overline{u_i'}/\lambda$, it follows that

$$\mu_i < 0 \quad \Leftrightarrow \quad b_i > 1. \tag{94}$$

Hence, an increase in ρ_i leads to an increase in social welfare if and only if $b_i > 1$. Importantly, nowhere in the proof is it necessary to assume that income effects are absent or that profit taxation is unrestricted (i.e., $b_f = 1$). Proposition 2 thus generalizes to settings with income effects and a binding restriction on profit taxation.

E.2 Optimal union power

Suppose that the government could optimally determine union power ρ_i. If we denote by $\underline{\chi}_i \geq 0$ the Kuhn-Tucker multiplier on the restriction $\rho_i \geq 0$, and by $\overline{\chi}_i \geq 0$ the multiplier on the restriction $1 - \rho_i \geq 0$, the first-order condition for optimal union power ρ_i in sector i (obtained from differentiating the Lagrangian (85) augmented with the additional inequality constraints) is given by

$$\mu_i E_i \psi_i \overline{u_i'} F_{ii} N_i + \underline{\chi}_i - \overline{\chi}_i = 0. \tag{95}$$

This expression should be considered alongside the other first-order conditions of the optimization program. In an interior optimum (i.e., where the optimal $\rho_i \in (0,1)$), the Kuhn-Tucker conditions require that $\underline{\chi}_i = \overline{\chi}_i = 0$. Equations (95) and (93) then imply that in these sectors $b_i = 1$. If the solution is at the boundary, then by the Kuhn-Tucker conditions it must be that either $\overline{\chi}_i = 0$ and $\underline{\chi}_i > 0$ or $\underline{\chi}_i = 0$ and $\overline{\chi}_i > 0$. If labor demand is not perfectly elastic, equation (95) implies that $\mu_i > 0$ in the first case (in which case $b_i < 1$) and $\mu_i < 0$ in the second case (in which case $b_i > 1$). Optimal union power thus equals $\rho_i = \min[\rho_i^*, 1]$ if $b_i \geq 1$, and $\rho_i = \max[\rho_i^*, 0]$ if $b_i \leq 1$, where ρ_i^* is the bargaining power of the union for which $b_i = 1$.

E.3 Proof Proposition 3

As in the proof of Proposition 1, in this Appendix we work with the reduced-form equations describing labor-market equilibrium:

$$E_i = E_i(\rho_1, \cdots, \rho_I, T_1, \cdots, T_I, T_u), \tag{96}$$

$$w_i = w_i(\rho_1, \cdots, \rho_I, T_1, \cdots, T_I, T_u). \tag{97}$$

These relationships can be found by solving the labor-demand and the wage-demand equations (11)–(12) for all i. Importantly, we neither impose that labor markets are independent, nor that income effects at the union level are absent.

Consider a marginal increase in union power in sector i, $\mathrm{d}\rho_i > 0$, and a tax reform $\{\mathrm{d}T_k^i\}_k$ that keeps after-tax wages $w_j - T_j$ in all sectors constant following the increase in ρ_i. This tax reform can be found by equating $\mathrm{d}w_j^i = \mathrm{d}T_j^i$, where

$$\mathrm{d}w_j^i = \frac{\partial w_j}{\partial \rho_i}\mathrm{d}\rho_i + \sum_k \frac{\partial w_j}{\partial T_k^i}\mathrm{d}T_k^i \tag{98}$$

is the change in the wage in sector j following an increase in ρ_i and the tax reform $\{\mathrm{d}T_k^i\}_k$. Setting $\mathrm{d}w_j = \mathrm{d}T_j^i$ and rearranging gives equation (26):

$$\frac{\partial w_j}{\partial \rho_i}\mathrm{d}\rho_i + \sum_k \frac{\partial w_j}{\partial T_k^i}\mathrm{d}T_k^i - \mathrm{d}T_j^i = 0. \tag{99}$$

The impact of the joint increase in union power ρ_i and the tax reform $\{\mathrm{d}T_k^i\}_k$ on employment is given by:

$$\mathrm{d}E_j^i = \frac{\partial E_j}{\partial \rho_i}\mathrm{d}\rho_i + \sum_k \frac{\partial E_j}{\partial T_k^i}\mathrm{d}T_k^i. \tag{100}$$

To analyze the impact of the tax reform and the increase in union power on social welfare, recall that the Lagrangian associated with the government's optimization problem is given by:

$$\begin{aligned}\mathcal{L} = \sum_i \psi_i N_i \Big(\int_{\underline{\varphi}}^{G_i^{-1}(E_i)} u(w_i - T_i - \varphi)\mathrm{d}G_i(\varphi) + \int_{G_i^{-1}(E_i)}^{\overline{\varphi}} u(-T_u)\mathrm{d}G_i(\varphi) \Big) \\ + \psi_f u(F(\cdot) - \sum_i w_i N_i E_i - T_f) + \lambda \left(\sum_i N_i(T_u + E_i(T_i - T_u)) + T_f - R \right),\end{aligned} \tag{101}$$

where equilibrium employment rates and wages are given by equations (96)–(97). The joint increase in union power and the tax reform affects wages, employment rates and government finances. The impact on social welfare can be found by taking the total differential of the Lagrangian with respect to changes in taxes, wages and employment rates:

$$\begin{aligned}\mathrm{d}\mathcal{W} = \sum_j \psi_j N_j \int_{\underline{\varphi}}^{G_i^{-1}(E_j)} u_j' \mathrm{d}G_i(\varphi)(\mathrm{d}w_j^i - \mathrm{d}T_j^i) \\ - \psi_f u_f' \sum_j N_j E_j \mathrm{d}w_j^i + \lambda \sum_j N_j E_j \mathrm{d}T_j^i + \sum_j \psi_j N_j \left(\hat{u}_j - u_u\right) G_i'(\hat{\varphi}_j) \frac{\partial G_i^{-1}(E_j)}{\partial E_j}\mathrm{d}E_j^i\end{aligned} \tag{102}$$

$$+\psi_f u_f' \sum_j (F_j - w_j) N_j \mathrm{d}E_j^i + \lambda \sum_j N_j (T_j - T_u) \mathrm{d}E_j^i.$$

This equation can be simplified in a number of steps. First, the tax reform is such that $\mathrm{d}w_j^i = \mathrm{d}T_j^i$, so the first line drops. Moreover, profit maximization implies that $F_j = w_j$, so that the first term in the last line drops out as well. Moreover, from the definition of $E_j = G_j(\hat{\varphi}_j)$ follows that $G_i'(\hat{\varphi}_j)\frac{\partial G_i^{-1}(E_j)}{\partial E_j} = 1$. Divide the expression by λ and substitute the social welfare weights. If the tax system is optimized we have $b_f = 1$, so the first two terms on the second line drop as well. Rewriting then yields:

$$\frac{\mathrm{d}\mathcal{W}}{\lambda} = \sum_j N_j \left(T_j - T_u + \frac{\psi_j (\hat{u}_j - u_u)}{\lambda} \right) \mathrm{d}E_j^i, \tag{103}$$

Setting the final expression larger than zero, and using the definition of t_j and τ_j, we find that the joint increase in union power and the tax reform that keeps net incomes constant raises social welfare if

$$\sum_j N_j (t_j + \tau_j) w_j \mathrm{d}E_j^i > 0. \tag{104}$$

The welfare impact of the tax reform $\{\mathrm{d}T_k^i\}_k$ equals zero if the tax system is optimized. Therefore, any welfare impact of the joint increase in ρ_i and the tax reform $\{\mathrm{d}T_k^i\}_k$ is driven only by the increase in union power. An increase in union power thus raises social welfare if and only if inequality (104) holds.

The impact of the joint increase in union power ρ_i and the tax reform $\{\mathrm{d}T_k^i\}_k$ on employment in other sectors is generally ambiguous (i.e., $\mathrm{d}E_j^i$ can be negative or positive for $j \neq i$). To analyze how employment in other sectors is affected, combine equations (11) and (12) for all j and write:

$$\rho_j \int_{\underline{\varphi}}^{G_i^{-1}(E_j)} u'(F_j(\cdot) - T_j - \varphi) \mathrm{d}G_i(\varphi) F_{jj}(\cdot) N_j + \big(u(F_j(\cdot) - T_j - G_i^{-1}(E_j)) - u(-T_u) \big) = 0. \tag{105}$$

These equations pin down equilibrium employment rates in all sectors given union power and the tax-benefit system that is in place. Hence, they can be used to determine how employment rates are affected by the joint increase in ρ_i and the tax reform that keeps after-tax wages constant. From equation (105), it can immediately be seen that if the wage in sector $j \neq i$ is determined competitively (i.e., $\rho_j = 0$), there will be no change in employment: $\mathrm{d}E_j^i = 0$. This is because the first term cancels and the reform keeps $w_j - T_j = F_j(\cdot) - T_j$ constant. In that case, employment E_j is not affected either. In sectors where wages are not determined competitively, the impact of the joint increase in union power ρ_i and the tax reform $\{\mathrm{d}T_k^i\}_k$ on employment is generally ambiguous. Because the reform leaves ρ_j and $F_j(\cdot) - T_j$ unchanged, any impact on equilibrium employment must come from general-equilibrium effects in $F_{jj}(\cdot)$. If this term only depends on E_j (i.e., if labor markets are independent), then again $\mathrm{d}E_j^i = 0$. Generally, the term $F_{jj}(\cdot)$ depends on employment in all sectors. If F_{jjk} is small for $j \neq k$ (i.e., if the production function can be approximated well by a second-order Taylor expansion), then $\mathrm{d}E_j^i \approx 0$ for $j \neq i$, and there will be approximately no changes in employment in sector $j \neq i$

following the joint increase in ρ_i and the tax reform $\{\mathrm{d}T_k^i\}_k$ that keeps net incomes fixed.

F Descriptive statistics

Table 4: Union densities and participation tax rates by country

	No.	Union density				Participation tax rate			
Country	sectors	Mean	Std. dev.	Min.	Max	Mean	Std. dev.	Min.	Max.
Sample	294	27.09	22.42	0.00	96.87	36.67	13.52	9.49	100.0
Australia	13	16.09	10.36	2.40	32.50	47.05	4.54	39.89	54.48
Austria	5	24.57	9.04	11.83	33.10	46.19	30.09	31.96	100.0
Canada	18	29.58	21.76	3.80	69.00	45.10	9.21	34.88	65.05
Denmark	14	67.62	10.91	41.65	82.00	64.39	11.84	44.55	86.52
Finland	12	62.99	10.09	46.86	83.00	32.71	15.73	22.52	81.53
France	15	10.98	5.69	4.10	24.10	43.44	4.23	36.92	51.74
Germany	7	19.76	10.07	6.46	34.00	49.38	3.91	41.29	53.00
Hungary	18	10.61	7.56	1.00	25.20	27.76	7.34	14.76	34.50
Ireland	16	28.02	17.12	6.00	60.84	40.60	8.13	33.60	60.00
Italy	9	48.17	21.16	23.50	96.87	29.24	12.59	9.49	41.63
Japan	11	18.64	12.75	5.40	48.70	50.27	9.34	34.60	66.12
Korea	12	16.37	16.55	2.10	54.30	25.42	10.63	14.01	48.38
Latvia	10	12.88	11.88	0.00	38.13	28.45	2.14	22.47	29.69
Netherlands	17	19.73	8.25	7.00	34.00	40.67	9.21	35.55	71.64
Norway	13	51.77	22.38	16.00	82.00	28.10	1.80	25.53	32.88
New Zealand	13	13.79	13.81	1.90	40.00	35.60	3.04	30.19	39.99
Slovakia	10	19.47	14.94	9.07	58.00	26.24	3.75	18.93	29.85
Spain	17	62.99	10.09	46.86	83.00	32.71	15.73	22.52	81.53
Sweden	15	62.92	13.92	32.00	82.00	24.90	3.06	19.17	34.08
Switzerland	7	24.83	15.64	7.42	47.83	28.21	1.55	26.93	30.93
Turkey	6	12.72	14.74	2.98	41.45	21.07	4.11	15.76	27.38
United Kingdom	18	22.12	13.99	3.30	47.60	40.89	8.85	28.71	55.63
United States	18	11.28	9.36	2.10	30.20	57.55	12.41	47.76	80.78

Table 5: Union densities and participation tax rates by sector

Sector	No. countries	Union density Mean	St. dev.	Min.	Max	Participation tax rate Mean	St. dev.	Min.	Max.
Sample	294	27.09	22.42	0.00	96.87	36.67	13.52	9.49	100.0
Agriculture	19	16.39	23.83	0.00	96.87	45.52	26.72	9.49	100.0
Commercial services	17	21.89	18.7	5.40	61.30	34.45	10.95	20.66	62.51
Construction	21	24.26	22.1	2.00	70.89	34.35	15.09	9.74	68.41
Education	17	40.24	22.19	9.60	82.00	33.85	7.89	20.02	49.82
Finance	17	27.38	24.02	2.10	70.00	34.34	6.79	14.01	44.56
Health care	10	32.98	22.31	8.70	81.00	34.52	6.37	25.79	45.19
Hotels and restaurants	15	13.21	16.68	1.00	56.00	38.39	15.18	15.32	60.36
Industry	15	29.47	23.69	4.45	75.81	36.0	9.74	23.49	60.24
Manufacturing	22	27.21	22.55	5.25	77.00	34.63	9.18	19.05	55.12
Mining	8	22.38	10.84	4.70	41.45	33.94	9.96	21.78	54.11
Other services	15	20.30	22.21	3.92	72.00	45.22	18.24	16.59	86.51
Public administration	19	42.43	26.20	4.25	83.00	35.46	8.79	25.12	60.42
Real estate and business services	14	17.25	19.75	1.90	62.00	36.48	10.88	19.36	59.45
Services	16	27.28	20.00	10.02	66.87	35.60	11.34	21.23	65.53
Social services	23	33.18	21.35	6.04	76.00	37.86	16.83	14.40	86.52
Trade	17	18.12	18.73	3.00	59.00	36.79	11.52	17.09	63.79
Transport and communication	18	33.82	19.41	2.10	67.00	36.57	12.82	14.53	66.12
Utilities	11	36.95	21.16	7.00	81.70	33.50	9.91	14.47	54.49

Figure 9: Union densities and participation tax rates by country

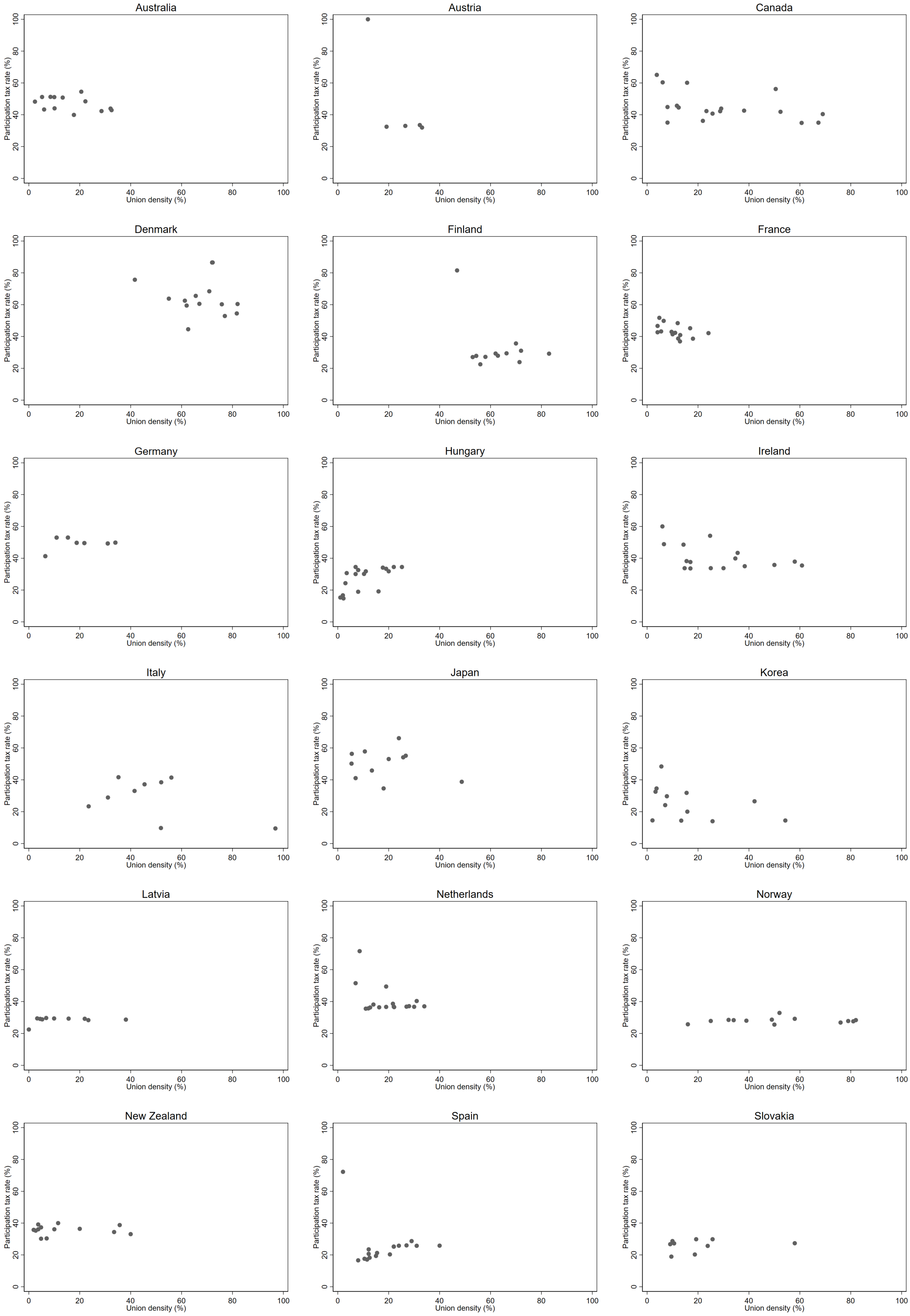
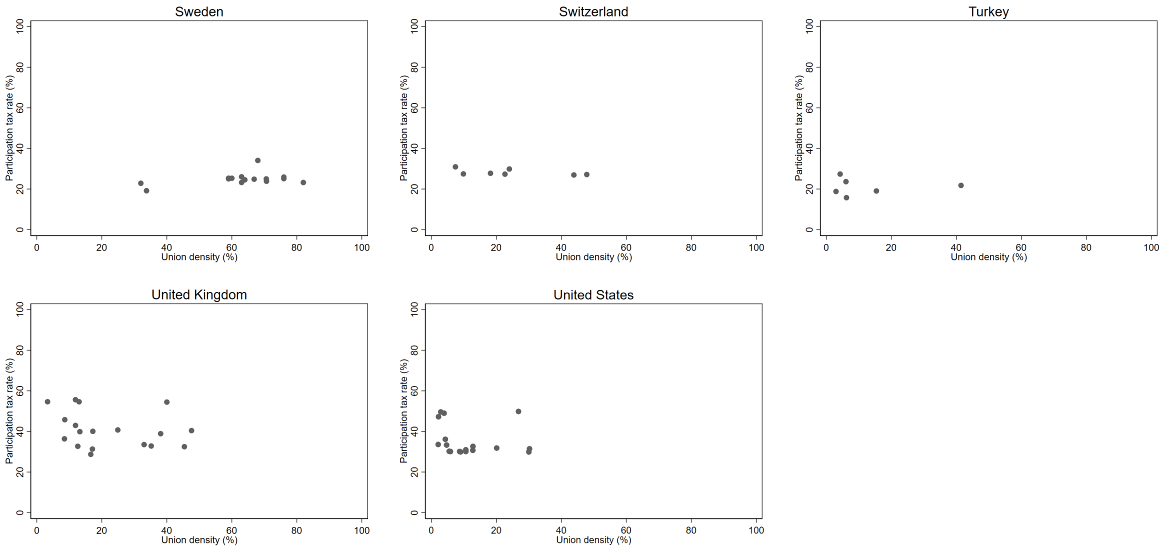


Figure 9: Union densities and participation tax rates by country – continued



G Simulations

G.1 Derivation labor-demand elasticity

Imposing the normalization $AK^{1-\alpha} = 1$, the production function is given by

$$Y = \left(\sum_i a_i L_i^{1/\delta} \right)^{\alpha\delta}, \quad \delta \equiv \frac{\sigma}{\sigma - 1}. \tag{106}$$

The derivatives are given by:

$$w_i = F_i = \alpha \left(\sum_j a_j L_j^{1/\delta} \right)^{\alpha\delta - 1} a_i L_i^{1/\delta - 1}, \tag{107}$$

$$F_{ii} = \alpha \left(\sum_j a_j L_j^{1/\delta} \right)^{\alpha\delta - 1} a_i L_i^{1/\delta - 2} \left[(1/\delta - 1) + (\alpha - 1/\delta)\phi_i \right],$$

where ϕ_i denotes the share of aggregate labor income that goes to workers in sector i:

$$\phi_i \equiv \frac{w_i L_i}{\sum_j w_j L_j} = \frac{a_i L_i^{1/\delta}}{\sum_j a_j L_j^{1/\delta}}. \tag{108}$$

Hence, using $\delta \equiv \frac{\sigma}{\sigma - 1}$ the elasticity of labor demand in sector i is thus equal to:

$$\begin{aligned} \varepsilon_i \equiv -\frac{F_i}{F_{ii} L_i} &= -\frac{\alpha \left(\sum_j a_j L_j^{1/\delta} \right)^{\alpha\delta - 1} a_i L_i^{1/\delta - 1}}{\alpha \left(\sum_j a_j L_j^{1/\delta} \right)^{\alpha\delta - 1} a_i L_i^{1/\delta - 1} \left[(1/\delta - 1) + \phi_i (\alpha - 1/\delta) \right]} \\ &= \frac{\sigma}{1 + \phi_i(\sigma(1 - \alpha) - 1)}. \end{aligned} \tag{109}$$

G.2 Numerically calculating optimal taxes

The optimal tax problem is given by

$$\max_{T_u,\{T_i,E_i\}_{i=1}^I} \mathcal{W} = \sum_i N_i \left[\int_0^{G^{-1}(E_i)} u(F_i(\cdot) - T_i - \varphi) g(\varphi) \mathrm{d}\varphi + (1 - E_i) u(-T_u) \right] \tag{110}$$

$$\text{s.t.} \quad \sum_i N_i (E_i T_i + (1 - E_i) T_u) + F(\cdot) - \sum_i F_i(\cdot) N_i E_i = R,$$

$$\rho F_{ii}(\cdot) N_i \int_0^{G^{-1}(E_i)} u'(F_i(\cdot) - T_i - \varphi) g(\varphi) \mathrm{d}\varphi + u(F_i(\cdot) - T_i - G^{-1}(E_i)) - u(-T_u) = 0, \ \forall i,$$

where we substituted the labor-demand equations $w_i = F_i(\cdot)$, imposed $G_i(\varphi) = G(\varphi)$, $\underline{\varphi} = 0$, $\rho_i = \rho$ for all i, and set the Pareto weights equal to one (utilitarian government), i.e., $\psi_i = 1$ for all i. Furthermore, we assume that all profits flow back to the government. We impose functional forms on $u(\cdot)$, $F(\cdot)$ and $G(\varphi)$, their derivatives or inverses. The primitives are the calibrated parameters of these functions (θ, α, σ, $\{a_i\}_i$, γ and ζ), union power ρ, the labor force sizes $\{N_i\}_i$ and the revenue requirement R. Our simulations exploit two possible algorithms to find optimal taxes, depending on which algorithm is faster or more stable.[76]

G.2.1 Solving unconstrained optimum

The most straightforward solution is to exploit the CARA utility function and analytically solve for the optimal participation tax level $T_i - T_u$ from the union wage-demand equation:

$$T_i - T_u = -\frac{1}{\theta} \ln \left[\frac{\theta \rho w_i}{\varepsilon_i E_i} \int_0^{G^{-1}(E_i)} \exp(-\theta(w_i - \varphi) g(\varphi) \mathrm{d}\varphi + \exp(-\theta(w_i - G^{-1}(E_i))) \right]. \tag{111}$$

Here, $w_i = F_i(\cdot)$ and $\varepsilon_i = \sigma/(1 + \phi_i(\sigma(1-\alpha) - 1))$. Hence, this is a solution for the participation tax as a function of all employment levels $\{E_i\}_i$. Next, we can use the government budget constraint to calculate:

$$T_u = \frac{1}{\sum_i N_i} \left[R + \sum_i F_i(\cdot) N_i E_i - F(\cdot) - \sum N_i E_i (T_i - T_u) \right]. \tag{112}$$

Hence, we have all taxes $\{T_i\}_i$ and T_u as a function of all employment rates $\{E_i\}_i$. After substituting these equations in the objective and constraints of problem (110), we obtain an *unconstrained* maximization problem in the employment rates $\{E_i\}_i$. Starting from the current employment rates in the calibrated economy, we numerically search for the vector of employment rates that maximizes social welfare.

G.2.2 Solving first-order conditions

Another approach is to solve for the first-order conditions associated with maximization problem (110). We specify all first-order conditions of the optimal tax problem: one for T_u, one for each T_i and E_i, the government budget constraint, and the union wage-demand equation for every sector i. This is a system of $3 \times I + 2$ equations in an equal number of unknowns: T_i, E_i,

[76] All programs are written in Matlab and are available on request from the authors.

T_u, μ_i (multiplier on union wage-demand equation), and λ (multiplier on government budget constraint). We can simplify this system as follows. The union wage-demand equation and the government budget constraint can be used to solve for T_i and T_u, as shown in the first method. Moreover, the system is linear in the multipliers μ_i, hence this multiplier can be eliminated as well. Finally, we use the first-order condition for T_u to solve for the multiplier on the resource constraint λ. We then obtain a system of I equations in I unknowns: all first-order conditions with respect to the employment rates $\{E_i\}_i$. Starting from the employment rates in the calibrated economy, we numerically solve for the vector of employment rates. We verify our solution to the first-order conditions indeed maximizes social welfare by using the candidate solution as a guess in the unconstrained maximization problem.

G.3 Additional graphs

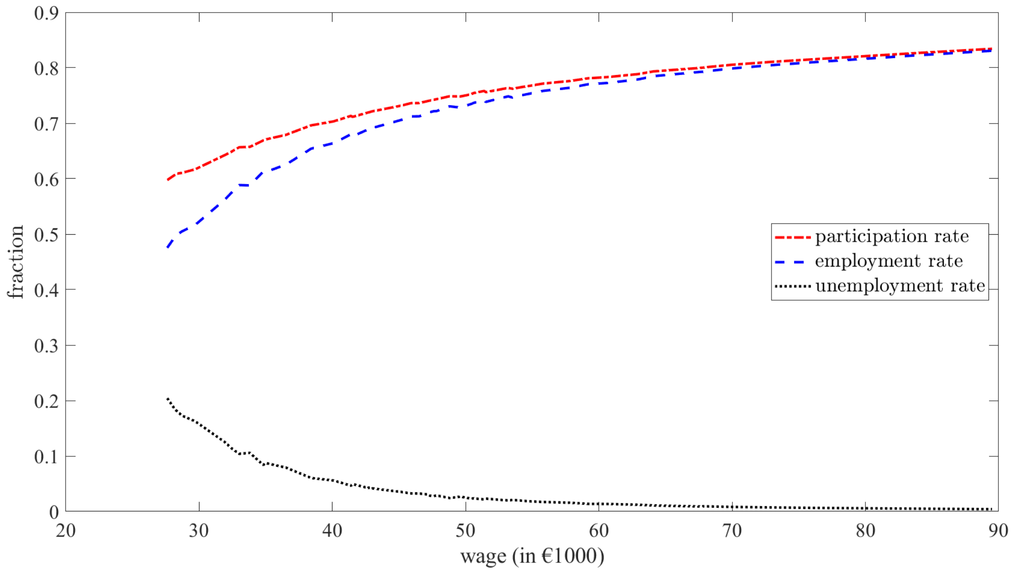


Figure 10: Participation, employment and unemployment rates by income

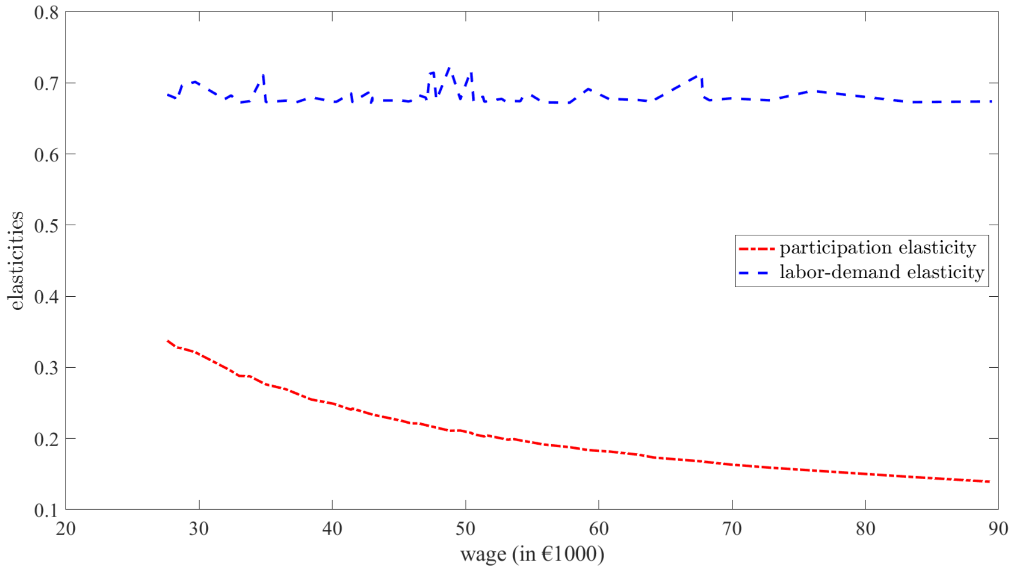


Figure 11: Participation elasticity and labor-demand elasticity by income

Online Appendix: Optimal Income Taxation in Unionized Labor Markets

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In this Appendix, we investigate the robustness of our main theoretical results, we provide an elaborate description of the construction of our data set, and we report the sensitivity analyses of our numerical simulations. In particular, we start by considering a version where unions respond to marginal tax rates, by relaxing the assumption of efficient rationing (Assumption 2 in the main text) and by allowing for endogenous occupational choice. In addition, we analyze two alternative bargaining structures: one in which a single, national union bargains with firm-owners over the entire *distribution* of wages, and one in which sectoral unions bargain with firms over wages *and* employment, as in the efficient bargaining model of McDonald and Solow (1981). Then, this Appendix documents the construction of the data set, and it provides some robustness checks of our empirical analysis. The final section presents the results from the sensitivity analysis of our simulations.

1 Union responses to marginal tax rates

This Section derives how our main results are affected if unions respond to marginal tax rates. So far, we have assumed that a union in sector i treats the tax liability T_i for its employed members as given. However, if the government sets a tax *schedule* $T(w_i)$, rather than a tax liability T_i in each sector, unions will anticipate that a higher wage affects the tax liability. Hence, the marginal tax rate will also determine wage demands of the union. To study the implications of union responses to marginal tax rates for optimal income taxation and the desirability of unions, it is convenient to reformulate our model and work with a continuum rather than a discrete set of sectors (or occupations). As before, within each sector, workers are represented by a union that maximizes the expected utility of its members. Sectors are indexed by $i \in \mathcal{I} = [0, 1]$ and ordered in such a way that wages $w(i)$ are increasing in i. $H(i)$ denotes the distribution of workers across sectors with density $h(i)$. Because we work with a continuum, we index sector i by a function argument instead of a subscript. The total measure of workers is normalized to one and the measure of (identical) firm-owners is $1/N$.[1] Within each

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[1]It is slightly more convenient to normalize the measure of workers and not, as in our baseline, the measure of firm-owners to one.

sector, workers differ in their unobservable participation costs $\varphi \in [\underline{\varphi}, \overline{\varphi}]$, which are distributed according to a cumulative distribution $G(\varphi)$ that, for simplicity, is assumed to be common across sectors.

To maintain tractability, we assume that workers in each sector produce the final consumption good directly, rather than assuming labor inputs of different types are combined to produce a final consumption good. This guarantees the absence of spillover effects between different sectors. Total output in sector i is thus given by

$$Y(i) = a(i)y(h(i)E(i)). \tag{1}$$

Here, $a(i)$ is an index of productivity, $E(i)$ denotes the employment rate of workers in sector i, and $y(\cdot)$ is a production function that maps total employment $L(i) = h(i)E(i)$ in sector i into units of the final consumption good.

Firms maximize profits by choosing how much labor to hire. The labor-demand curve is, for each i:

$$w(i) = a(i)y'(h(i)E(i)). \tag{2}$$

If $y(\cdot)$ is strictly concave, each union faces a downward-sloping labor-demand curve and firms make profits, which are subject to a non-distortionary profit tax T_f. The government also provides a benefit $-T_u$ to all workers who are not employed (voluntarily or involuntarily). In addition, the government sets a tax *schedule* $T(\cdot)$ on labor income $w(i)$. This is the key difference from our previous set-up, where it was assumed that the government sets the tax *liability* T_i in each sector directly, which unions take as given. The government chooses these instruments to maximize social welfare, subject to its budget constraint, and taking into account how labor-market outcomes are affected by changes in the tax-benefit system.

As in the baseline, we first characterize the equilibrium for any degree of union power $\rho(i) \in [0, 1]$, which is allowed to vary across sectors. Under efficient rationing (Assumption 2 in the main text), workers with participation costs $\varphi \in [\underline{\varphi}, \hat{\varphi}(i)]$ become employed, where $\hat{\varphi}(i) = G^{-1}(E(i))$. By contrast, workers with participation costs $\varphi \in [\hat{\varphi}(i), \overline{\varphi}]$ are not employed (voluntary or involuntary). The union's objective is then given by

$$\Lambda(i) = \int_{\underline{\varphi}}^{\hat{\varphi}(i)} u(c(i) - \varphi)\mathrm{d}G(\varphi) + \int_{\hat{\varphi}(i)}^{\overline{\varphi}} u(c_u)\mathrm{d}G(\varphi), \tag{3}$$

where $c(i) = w(i) - T(w(i))$ is consumption of an employed worker, and $c_u = -T_u$ denotes consumption of an unemployed worker.

If the union representing workers from sector i is a monopoly union (MU), i.e., $\rho(i) = 1$, then it sets the wage $w(i)$ that maximizes the objective (3) subject to $\hat{\varphi}(i) = G^{-1}(E(i))$ and the labor-demand equation (2). The first-order conditions can be combined to find the wage-demand equation:

$$1 - T'(w(i)) = \varepsilon(i)\frac{u(\hat{c}(i)) - u(c_u)}{\overline{u'(c(i))}w(i)}, \tag{4}$$

where $\varepsilon(i) = -y'(L(i))/(L(i)y''(L(i))) > 0$ is the labor-demand elasticity and $\hat{c}(i) = w(i) - T(w(i)) - G^{-1}(E(i))$ is the consumption net of participation costs of the marginally employed worker, i.e., the employed worker with the highest costs of participation. This condition is very similar to equation (6) from the main text, except that the left-hand side is multiplied by the net-of-tax rate $1 - T'(w(i))$. Intuitively, unions only care about demanding a higher wage if it yields a higher after-tax income.

If labor markets are perfectly competitive, i.e., if $\rho(i) = 0$, workers continue to supply labor until the marginally employed worker is indifferent between working and not working: $\hat{c}(i) = c_u$, and, hence, $\hat{\varphi}(i) = w(i) - T(w(i)) + T_u$. In this case, there is no involuntary unemployment. Furthermore, because labor-supply responses are only concentrated on the extensive margin, a local increase in the marginal tax rate at income $w(i)$ that leaves the tax liability unaffected has no impact on labor-market outcomes in sector i.

Following a similar approach as in Section 3.3 of the main text, we can characterize labor-market equilibrium in sector i for any degree of union power $\rho(i) \in [0, 1]$ by combining the labor-demand equation (2) with the following modified wage-demand equation:

$$\rho(i)(1 - T'(w(i))) = \varepsilon(i)\frac{u(\hat{c}(i)) - u(c_u)}{\overline{u'(c(i))}w(i)}. \tag{5}$$

This condition is analogous to equation (8) from the main text, except that the left-hand side is multiplied by the net-of-tax rate. Clearly, if $\rho(i) = 0$, the competitive equilibrium (CE) prevails, as there is no involuntary unemployment: $u(\hat{c}(i)) - u(c_u) = 0$. By contrast, if $\rho(i) = 1$, equations (5) and (4) coincide and the equilibrium corresponds to the monopoly union (MU) outcome. By varying the degree of union power $\rho(i) \in [0, 1]$, we can obtain any equilibrium from the RtM-model.

There is one key difference between the current formulation and the baseline. In the latter, unions treat the tax liability as given. Consequently, a local increase in the marginal tax rate at income level $w(i)$ that leaves the tax liability unaffected has no impact on labor-market outcomes in sector i. However, in the current setup, unions bargain taking the tax *schedule* $T(\cdot)$ as given. As a result, the equilibrium wage and employment rate in sector i also depend on the marginal tax rate $T'(w(i))$, see equation (5). We demonstrate formally in Appendix A.1 that a local increase in the marginal tax rate $T'(w(i))$ at income level $w(i)$ lowers the equilibrium wage $w(i)$, and, through the labor-demand equation (2), raises the equilibrium employment rate $E(i)$.[2] Intuitively, a higher marginal tax rate lowers the benefits of demanding a higher wage, which induces unions to lower their wage demands, and firms to hire more workers, cf. Hersoug (1984). This effect is referred to in the literature as the wage-moderating effect of a higher marginal tax rate.[3,4]

[2] We also show in Appendix A.1 that, as in the baseline, a higher tax burden or unemployment benefits leads to a higher wage and a reduction in the employment rate.

[3] The wage-moderating effect of a higher marginal tax rate is a robust prediction in models with labor market imperfections. It is derived in the context of unions by Hersoug (1984), but also holds in the context of matching frictions (Pissarides, 1985) and efficiency wages (Pisauro, 1991). See Lehmann et al. (2016) for empirical evidence and Kroft et al. (2020) and Hummel (2021) for a discussion of the implications for optimal income taxation.

[4] Sometimes, this effect is referred to as the wage-moderating effect of 'tax progressivity'. Indeed, if marginal tax rates increase, while average tax rates remain fixed, a higher marginal tax rate also raises the progressivity of the tax system, since a tax system is progressive only if the average tax rate increases in income.

We characterize the optimal tax schedule $T(\cdot)$ using the tax perturbation approach.[5] To do so, we study the welfare effects of a uniform increase in the tax burden $T(w)$ paid by *all* employed workers, a *local* increase in the marginal tax rate $T'(w')$ at some income level w', an increase in the profit tax T_f, and a reduction in the unemployment benefit $-T_u$. If the tax system is optimized, none of these reforms should have an impact on social welfare. This leads to the following Proposition.

Proposition 1. *Suppose Assumptions 1 (independent labor markets), 2 (efficient rationing), and 3 (no income effects at the union level) hold. In addition, suppose the government optimizes a tax schedule $T(\cdot)$ and unions respond to marginal tax rates, cf. equation (5). Then, the optimal tax schedule $T(\cdot)$, unemployment benefits $-T_u$, and profit taxes T_f are determined by:*

$$\omega_u b_u + \int_{\underline{w}}^{\overline{w}} b(w)k(w)\mathrm{d}w = 1, \tag{6}$$

$$b_f = 1, \tag{7}$$

$$\begin{aligned} &\left[\left(\frac{t(w')+\tau(w')}{1-t(w')}\right)\eta_{T'} + (b(w')-1)(1-T'(w'))\kappa_{T'}\right]k(w') \\ &+ \int_{w'}^{\overline{w}} \left[(1-b(w)) - \left(\frac{t(w)+\tau(w)}{1-t(w)}\right)\eta_T + (b(w)-1)(1-T'(w))\kappa_T\right]k(w)\mathrm{d}w = 0, \quad \forall w', \end{aligned} \tag{8}$$

where $b(w)$, $\tau(w)$, $t(w)$ and $\tilde{E}(w)$ denote the social welfare weight, union wedge, participation tax rate, and employment rate at wage w. Moreover, $k(w)$ is the density of the wage distribution, and $\kappa_T \equiv \frac{\partial w}{\partial T}$, $\kappa_{T'} \equiv \frac{\partial w}{\partial T'}$, $\eta_T \equiv -\frac{\partial E}{\partial T}\frac{w(1-t(w))}{\tilde{E}(w)}$, and $\eta_{T'} \equiv \frac{\partial E}{\partial T'}\frac{(1-t(w))w}{\tilde{E}(w)}$ are the elasticities of wages and employment with respect to an increase in the marginal tax rate and total tax burden.

Proof. See Appendix A.2. □

The first two results are the same as in the baseline (see Proposition 1 in the main text), and, hence, their explanation is not repeated here.[6]

The third result is obtained from considering a local increase in the *marginal* tax rate at income level w'. Compared to the baseline, the optimal tax formula (8) is modified in two substantive ways. Both effects are captured on the first line.

First, a higher marginal tax rate results in a higher employment rate, since wage demands are reduced: $\eta_{T'} > 0$. The wage-moderation effect of a higher marginal tax rate at w' alleviates labor-market distortions from the explicit tax $t(w')$ on labor participation, and the implicit tax $\tau(w')$ from unions bidding up wages above the market-clearing level. This is captured by the first term on the first line. Intuitively, if unions moderate wage demands in response to a higher marginal tax rate, and employment increases, social welfare increases if employment is distorted downwards, i.e., if $t(w') + \tau(w') > 0$.[7]

[5] The tax-perturbation approach is also employed by, among others, Saez (2001), Golosov et al. (2014), Gerritsen (2016), and Jacquet and Lehmann (2021).

[6] The only differences are that the current extension features a continuum (rather than a discrete number) of types and equation (6) integrates over the income (as opposed to the type) distribution.

[7] A similar term appears in Hummel (2021), who characterizes the optimal tax schedule in a directed search

Second, a higher marginal tax rate reduces the equilibrium wage: $\kappa_{T'} < 0$. As a result, income is redistributed among workers, firm-owners, and the government. In particular, if wages are lowered, firm-owners receive higher profits, workers see their after-tax income reduced, and the government experiences a reduction in tax revenue (provided that $T'(w') > 0$). The reduction in the wage transfers income from workers, whose social welfare weight is $b(w')$, to firm-owners, whose social welfare weight is $b_f = 1$. The reduction in tax payments yields a welfare effect equal to the change in the wage multiplied with $T'(w')(b(w') - 1)$, where $b(w')$ represents the increase in social welfare if the worker pays one unit of income less in tax, while 1 stands for the loss in social welfare if the government receives less tax revenue. The sum of both welfare effects is proportional to $(1 - b(w'))(1 - T'(w'))$, as captured by the second term on the first line. Hence, there is a redistributional gain (loss) due to wage moderation at w' if $b(w') < 1$ ($b(w') > 1$). Note that both the wage and employment effects are proportional to the density $k(w')$ of the wage distribution. The density $k(w')$ is the measure of workers who experience a decrease in the wage or an increase in employment if the marginal tax rate at income w' is increased.

Turning to the second line of equation (8), a higher marginal tax rate not only generates wage-moderation and employment effects at point w' in the income distribution, where the marginal tax is levied, but it also raises tax liabilities for all income levels $w > w'$. This mechanically transfers income from these workers to the government, as captured by the first term $1 - b(w)$. This is the standard mechanical effect of a higher marginal tax rate in all non-linear income tax models, cf. Mirrlees (1971), Diamond (1998), and Saez (2001). Moreover, as in the baseline, a higher tax burden also generates upward pressure on wages and a corresponding reduction in the employment rates for all income levels $w > w'$: $\kappa_T > 0$ and $\eta_T < 0$. The associated reductions in employment are socially costly if participation of these workers is distorted downwards, i.e., if $t(w) + \tau(w) > 0$ for $w > w'$. Moreover, the wage pressure for all income levels $w > w'$ redistributes income from firm-owners to workers and to the government for exactly the same reasons as we discussed above. This generates a loss (gain) in social welfare due to wage pressure at $w > w'$ if $b(w') < 1$ ($b(w') > 1$) for income levels $w > w'$.

To see how equation (8) is linked to the optimal tax formula (18) from the main text, suppose that unions treat the tax liability as given and do not respond to changes in the marginal tax rate. In that case, $\eta_{T'} = \kappa_{T'} = 0$, and both terms on the first line of equation (8) would cancel. Moreover, because equation (8) holds for each w', the term below the integral sign must be equal to zero at each point in the income distribution. By setting $T'(w) = 0$, the result from Proposition 1 coincides with equation (18) from the main text in the special case where there are no spillover effects between different sectors.

Another way to understand how equation (8) and equation (18) from the main text are linked, is to recognize that these optimal tax rules are derived from two, distinct policy experiments. In particular, in the current extension, the increase in the marginal tax rate at w' raises i) the marginal tax rate at w', and ii) the tax liabilities $T(w)$ for all workers with higher wages, i.e., for workers with $w > w'$. By contrast, in the baseline, the optimal tax formula is

model with matching frictions. In that framework, however, there is no wedge due to involuntary unemployment (i.e., no counterpart of the union wedge), because unemployment is constrained efficient.

based on the policy experiment where the tax liability is increased at only one income level. However, if we would, instead, consider increasing the tax liability in our baseline model for all workers with $w > w'$ (like in the current extension), and, in addition, assume independent labor markets (like in the current extension), then the optimal tax formula would become the same as equation (8), but with one important difference: the first line would be zero. Hence, the main modification of the current extension compared to the baseline is to add the terms on the first line of equation (8). That is, the optimal tax formula accounts for wage and employment responses to marginal tax rates.

The wage-moderation effect of a higher marginal tax rate triggers two welfare-relevant effects: it alleviates (exacerbates) labor-market distortions if labor participation is taxed (subsidized) on a net basis, and it gives additional redistributional gains (losses) if $b(w') < 1$. These welfare effects are related. Loosely speaking, the government typically only provides transfers to employed workers that exceed the unemployment benefit, i.e., sets $t(w) < 0$, if these workers have a high social welfare weight, i.e., if $b(w) > 1$. Therefore, we conjecture that, compared to the baseline, wage-moderation effects tend to reduce (raise) optimal marginal tax rates if employment is distorted upwards (downwards) – *ceteris paribus*. However, we are not sure whether the ceteris paribus condition holds, since the optimal marginal tax schedule is dependent on all social welfare weights, the entire income distribution, and participation distortions at all income levels. Only a more elaborate quantitative analysis can shed light on the implications of wage moderation for optimal taxes, but this is beyond the scope of the current paper.

Next, we ask if unions are desirable for income redistribution if the government optimizes the non-linear tax schedule and unions respond to changes in marginal tax rates. To that end, we study the welfare effect of increasing union power at income level w. This leads to the following result.

Proposition 2. *Suppose Assumptions 1 (independent labor markets) and 2 (efficient rationing) hold. In addition, suppose that the tax-benefit system is optimized as in Proposition 1. Then, an increase in union power for workers whose wage is w raises social welfare if and only if:*

$$(b(w) - 1)(1 - T'(w)) - (t(w) + \tau(w))\tilde{\varepsilon}(w) > 0, \tag{9}$$

where $\tilde{\varepsilon}(w)$ is the labor-demand elasticity at wage w.

Proof. See Appendix A.3. □

To understand this result, consider a local increase in union power for workers who are employed at wage w. An increase in union power at income level w boosts wage demands and reduces employment at w. The increase in the equilibrium wage then transfers income from firm-owners, whose social welfare weight is $b_f = 1$, to workers, whose social welfare weight is $b(w)$. As in the baseline, the welfare effect is proportional to $b(w) - 1$. Moreover, a higher equilibrium wage also transfers income from workers to the government if $T'(w) > 0$. This explains why the first term is multiplied by the net-of-tax rate $1 - T'(w)$. Turning to the second term, the increase in the wage due to higher union power also results in a lower employment rate. By how much depends on the labor-demand elasticity $\tilde{\varepsilon}(w)$. A lower employment rate, in turn, affects social welfare through the explicit tax $t(w)$ and the implicit tax $\tau(w)$ on labor participation. These effects are captured by the second term. Equation (9) states that an increase in union power results in a welfare gain i) if participation is distorted upwards, and/or ii) if the wage increase is associated with a positive redistributional gain, which requires $b(w) > 1$.

The main difference compared to the baseline (see Proposition 2 in the main text) is that whether an increase in union power raises social welfare depends on *both* social welfare weights, $b(w)$, and net taxes on labor participation, $t(w)+\tau(w)$. Importantly, as mentioned before, these are not independent. The government typically only provides transfers to employed workers that exceed the unemployment benefit, i.e., sets $t(w) < 0$, if these workers have a high social welfare weight, i.e., if $b(w) > 1$. Therefore, we view our adjusted desirability condition as only slightly weaker. Moreover, we can derive a sufficiency condition for the desirability of unions: an increase in union power unambiguously raises social welfare if participation is subsidized on a net basis ($t(w) + \tau(w) < 0$) *and* the social welfare weights of the workers represented by the union is above-average ($b(w) > 1$). Conversely, a sufficient condition for unions not to be desirable is that workers pay positive participation taxes ($t(w) > 0$) and have a below-average social welfare weight ($b(w) < 1$).[8] Given that we empirically find that participation taxes are never negative, the desirability condition implies that a necessary condition for unions to be desirable is that the social welfare weight of the workers that are represented by the union is above average, i.e., $b(w) > 1$. Hence, Proposition 2 from the main text largely carries over to the current setting.

In the baseline without spillover effects, participation taxes and social welfare weights are tightly linked. From equation (22) in the main text, labor participation for workers with wage w is subsidized on a net basis, i.e., $t(w) + \tau(w) < 0$, if and only if these workers have an above-average social welfare weight, i.e., $b(w) > 1$. This explains why $b(w) > 1$ is both necessary and sufficient for an increase in union power to be welfare-improving in the baseline, see also Proposition 2 in the main text. Intuitively, both participation distortions and distributional effects are proportional to $1 - b(w)$. Therefore, only knowledge of social welfare weights is required to judge whether an increase in union power raises social welfare. If unions respond to marginal tax rates, however, such a tight link between social welfare weights and net taxes on participation no longer exists, since participation taxes at each income level are determined by the complete optimal non-linear tax schedule, which, in turn, depends on all social welfare weights, the income distribution, and participation distortions at all income levels. Consequently, judging whether an increase in union power raises social welfare generally requires knowledge of both participation taxes and social welfare weights.

2 Inefficient rationing

We have deliberately biased our findings in favor of unions by assuming that unemployment rationing is efficient: the burden of involuntary unemployment is borne by the workers with the highest participation costs. However, there are neither theoretical nor empirical reasons to expect that labor rationing is always efficient, see Gerritsen (2017) and Gerritsen and Jacobs

[8] This sufficiency condition only requires that participation taxes are positive, since implicit taxes from unions are always weakly positive (i.e., $\tau(w) \geq 0$). Hence, a positive participation tax is sufficient to guarantee downward distortions on participation.

(2020). In this Section, we analyze how the optimal tax formulas should be modified, and under which conditions unions are desirable, if the assumption of efficient rationing is relaxed. For analytical convenience, we assume that labor markets are independent and there are no income effects at the union level.

We follow Gerritsen (2017) and Gerritsen and Jacobs (2020) by defining the rationing schedule as a continuously differentiable function

$$e_i(E_i, \varphi_i^*, \varphi), \quad e_{iE_i}(\cdot), -e_{i\varphi_i^*}(\cdot) > 0, \tag{10}$$

which specifies the probability $e_i \in [0,1]$ that workers with participation costs $\varphi \in [\underline{\varphi}, \varphi_i^*]$, find employment in sector i for a given sectoral employment rate E_i and a given participation threshold φ_i^*. The probability $e_i(\cdot)$ of finding a job in sector i increases in employment E_i and decreases if labor participation rises, i.e., if φ_i^* is higher.[9] For all values of employment E_i and the participation cut-off φ_i^*, the following relationship must hold:

$$\int_{\underline{\varphi}}^{\varphi_i^*} e_i(E_i, \varphi_i^*, \varphi)\mathrm{d}G_i(\varphi) = E_i. \tag{11}$$

Hence, integrating over all employment probabilities of the workers in sector i (who differ in terms of their participation costs) yields sectoral employment.

Under independent labor markets and no income effects, we can describe the equilibrium using reduced-form equations $w_i = w_i(\rho_i, T_i - T_u)$ and $E_i(\rho_i, T_i - T_u)$, which pin down the equilibrium wage and employment rate in sector i as a function of union power ρ_i and the participation tax $T_i - T_u$. The following Proposition characterizes the optimal tax formulas if labor rationing is inefficient.

Proposition 3. *If Assumptions 1 (independent labor markets), 3 (no income effects at the union level) are satisfied, and labor rationing is described by the rationing schedule (10), then optimal unemployment benefits $-T_u$, optimal profit taxes T_f, and optimal participation taxes $T_i - T_u$ are determined by:*

$$\omega_u b_u + \sum_i \omega_i b_i = 1, \tag{12}$$

$$b_f = 1, \tag{13}$$

$$\left(\frac{t_i + \hat{\tau}_i}{1 - t_i}\right)\eta_{ii} - \left(\frac{\varrho_i}{1 - t_i}\right)\gamma_i = (1 - b_i) + (b_i - b_f)\kappa_{ii}, \tag{14}$$

where the union wedge is redefined as

$$\hat{\tau}_i \equiv \psi_i \int_{\underline{\varphi}}^{\varphi_i^*} e_{iE_i}(E_i, \varphi_i^*, \varphi)\left(\frac{u(w_i - T_i - \varphi) - u(-T_u)}{\lambda w_i}\right)\mathrm{d}G_i(\varphi), \tag{15}$$

[9] An example of a rationing schedule that satisfies these criteria is a uniform rationing scheme. All participating workers then face the same probability of finding a job, i.e., $e_i(E_i, \varphi_i^*, \varphi) = E_i/G_i(\varphi_i^*)$ for all values of $\varphi \in [\underline{\varphi}, \varphi_i^*]$.

and ϱ_i denotes the rationing wedge, which is defined as

$$\varrho_i \equiv \frac{\psi_i e_i(E_i, \varphi_i^*, \varphi_i^*)}{E_i/G_i(\varphi_i^*)} \int_{\underline{\varphi}}^{\varphi_i^*} \frac{e_{i\varphi_i^*}(E_i, \varphi_i^*, \varphi)}{\int_{\underline{\varphi}}^{\varphi_i^*} e_{i\varphi_i^*}(E_i, \varphi_i^*, \varphi)\mathrm{d}G_i(\varphi)} \left(\frac{u(w_i - T_i - \varphi) - u(-T_u)}{\lambda w_i} \right) \mathrm{d}G_i(\varphi) \quad (16)$$

and $\gamma_i \equiv -\frac{\partial G_i(\varphi_i^*)}{\partial(T_i - T_u)} \frac{\varphi_i^*}{G_i(\varphi_i^*)}$ captures the participation response.

Proof. See Appendix B.1. $\square$

The expressions for the optimal unemployment benefit and profit tax are identical to those stated in Proposition 1 in the main text and their explanation is not repeated here. The expression for the optimal participation tax in equation (14) equates the marginal distortionary costs of a higher participation tax (left-hand side) to the marginal distributional gains of a higher participation tax (right-hand side). The expression for the optimal participation tax is modified in two ways compared to the one with efficient rationing. First, with a general rationing scheme, the union wedge $\hat{\tau}_i$ no longer measures the monetized utility loss of a *marginal* worker losing her job, but the expected utility loss of *all rationed workers*, i.e., the workers who lose their job if the wage is marginally increased. Second, in addition to the union wedge $\hat{\tau}_i$, there is a distortion associated with the inefficiency of the rationing scheme, which is captured by the rationing wedge ϱ_i.

To understand the rationing wedge ϱ_i, consider a decrease in the participation tax $T_i - T_u$. Moreover, suppose the reduction in the participation tax is combined with an increase in union power ρ_i so that the equilibrium wage (and hence, the equilibrium employment rate) remains unaffected. More people want to participate if the participation tax is lowered. A fraction $e_i(E_i, \varphi_i^*, \varphi_i^*)$ of the workers who are at the participation margin (i.e., those who are indifferent between employment and unemployment) will succeed in finding a job. However, if employment remains constant, other workers become unemployed. Since these workers are not indifferent between work and unemployment, a welfare loss occurs. The latter is captured by the term ϱ_i, which measures the marginal welfare costs associated with an inefficient allocation of jobs over those who are willing to work. These costs are weighted by the participation response γ_i.

According to equation (14), the higher is ϱ_i, i.e., the more inefficient is the rationing scheme, the *higher* should be the optimal participation tax. The intuition is similar to Gerritsen (2017): by setting a higher participation tax, the workers who care least about finding a job opt out of the labor market. This, in turn, increases the employment prospects of the workers who experience a larger surplus from finding a job. Consequently, the government replaces involuntary unemployment by voluntary unemployment, which reduces the inefficiency of labor-market rationing.

The next Proposition gives the condition under which an increase in union power raises social welfare if rationing is no longer efficient.

Proposition 4. *If labor rationing is described by the rationing schedule (10), and taxes and transfers are set according to Proposition 3, then an increase in union power ρ_i in sector i raises social welfare if and only if*

$$b_i > 1 + \left(\frac{\varrho_i}{1 - t_i} \right) \gamma_i. \quad (17)$$

Proof. See Appendix B.2. □

To understand whether it is optimal to increase union power, consider again a policy reform starting from a situation where taxes are optimally set. We marginally raise union power ρ_i in sector i, while simultaneously reducing the participation tax $T_i - T_u$ in sector i such that the wage w_i, and hence employment E_i, is kept constant. The reduction in the participation tax is financed by an increase in the profit tax T_f to ensure that the government budget remains balanced.[10] If the tax system is optimized, the tax reform has no impact on social welfare. Therefore, any impact on social welfare must come from the increase in union power. The reform transfers income from firm-owners to workers in sector i. As before, the associated welfare effect is proportional to $b_i - 1$. By construction, there are no welfare effects associated with changes in equilibrium wages and employment. However, the increase in net earnings raises participation of workers in sector i. If some of the (previously voluntarily) unemployed workers find a job, a welfare loss occurs because – with constant employment – some participants who experience a surplus from working will not be able to find a job. For a given social welfare weight, the more inefficient is the rationing scheme, or the higher is the participation response (i.e., the higher ϱ_i or γ_i), the higher should be the social welfare weight of workers b_i for unions in sector i to be desirable. The welfare costs of inefficient rationing could be so large that they completely off-set the potential welfare gains of unions. Consequently, if rationing is inefficient, increasing union power in a sector where $b_i > 1$ does not necessarily raise social welfare.

3 Occupational choice

So far we have abstracted from an intensive margin of labor supply: each individual can only work a fixed number of hours in one particular sector. The main reason for doing so is that an intensive margin raises a number of very complicated issues that we cannot yet address. For example, which party (i.e., unions or individuals) decides on the number of hours worked? Does the incidence of unemployment fall on the intensive (hours) or extensive (participation) margin? How do unions aggregate worker preferences if they can switch between sectors? In this Section, we do not attempt to answers these difficult questions. Instead, we will demonstrate that our main insights carry over to a setting where workers can switch between occupations. This is what Saez (2002) refers to as the 'intensive margin' in discrete labor-supply models.

To model occupational choice, we assume that each of the N workers draws a vector $\boldsymbol{\varphi} \equiv (\varphi_0, \varphi_1, \cdots, \varphi_I) \in \Phi$ of participation costs according to some cumulative distribution function $G(\boldsymbol{\varphi})$. The i-th element of vector $\boldsymbol{\varphi}$ indicates how costly it is for an individual to work in sector i. Based on their participation costs, individuals choose in which sector (or: occupation) to look for a job. Without labor unions, this choice simply boils down to finding the occupation j where the net payoff from working $w_j - T_j - \varphi_j$ is maximized, provided the latter exceeds the payoff from not working $-T_u$. With labor unions, however, this problem is more complicated, because individuals may not be able to find a job if wages are set above the market-clearing level. An additional difficulty is that it is no longer clear how the union objective should be

[10] The reduction in the participation tax can also be financed by a uniform increase in the tax on all (employed and non-employed) workers. This does not matter for the outcomes.

specified if individuals can switch between sectors. To overcome these issues, we adopt a similar approach as with inefficient rationing (see Section 2). In particular, we assume that there exist reduced-form equations $p_i(\boldsymbol{\varphi}, T_1 - T_u, ..., T_I - T_u)$ that are differentiable functions of all participation taxes, which specify a probability $p_i \in [0,1]$ that an individual becomes employed if she looks for a job in sector i. If the individual is unsuccessful, she cannot move to another sector but instead becomes unemployed. Each individual then solves:

$$\max_{j \in \{0,1,...,I\}} \quad u(-T_u) + p_j(\boldsymbol{\varphi}, T_1 - T_u, ..., T_I - T_u)(u(w_j - T_j - \varphi_j) - u(-T_u)), \tag{18}$$

where occupation 0 refers to non-employment, with $w_0 = \varphi_0 = 0$, $T_0 = T_u$, and $p_0 = 1$.

As before, we assume that there are no income effects at the union level and we denote by $w_i(T_1 - T_u, ..., T_I - T_u)$ and $E_i(T_1 - T_u, ..., T_I - T_u)$ the equilibrium wage and *total* employment (as opposed to the employment rate) in sector i as a function of the participation taxes. Furthermore, let Φ_i denote the set of all individuals who look for a job in sector i (including non-employment):

$$\Phi_i \equiv \{\boldsymbol{\varphi} \in \Phi | \text{ arg max}_j \; p_j(\boldsymbol{\varphi}, T_1 - T_u, ..., T_I - T_u)(u(w_j - T_j - \varphi_j) - u(-T_u)) = i\}. \tag{19}$$

In equilibrium, the following relationship holds for all i and for all participation taxes:

$$N \int_{\Phi_i} p_i(\boldsymbol{\varphi}, T_1 - T_u, ..., T_I - T_u) \mathrm{d}G(\varphi) = E_i(T_1 - T_u, ..., T_I - T_u). \tag{20}$$

We make the following assumption regarding the functions $p_i(\cdot)$, which ensures that rationing is efficient.

Assumption 1. (Efficient rationing with occupational choice) *$p_i = 0$ on the boundary of the set Φ_i for all sectors i.*

Assumption 1 extends our notion of efficient rationing to this environment by assuming that if there is involuntary unemployment, individuals who are indifferent between choosing sector i and another sector (possibly non-employment) do not find a job. This form of rationing is efficient in the sense that individuals with the lowest surplus from working in a particular sector (compared to their second-best alternative) do not find a job if wages are set above the market-clearing level. This notion of efficient rationing is similar to Lee and Saez (2012).

The following Proposition characterizes the optimal tax system with an intensive, occupational-choice margin.

Proposition 5. *If Assumptions 3 (no income effects at the union level) and 1 (efficient rationing with occupational choice) are satisfied, and individuals optimally choose their occupation according to equation (18), then the optimal unemployment benefit $-T_u$, profit taxes T_f, and participation taxes $T_i - T_u$ are determined by:*

$$\omega_u b_u + \sum_i \omega_i b_i = 1, \tag{21}$$

$$b_f = 1, \tag{22}$$

$$\sum_j \omega_j \left(\frac{t_j + \tau_j^o}{1 - t_j}\right) \eta_{ji} = \omega_i(1 - b_i) + \sum_j \omega_j (b_j - b_f)\kappa_{ji}, \quad \forall i, \tag{23}$$

where the union wedge with endogenous occupational choice is

$$\tau_j^o \equiv \psi_j N \int_{\Phi_j} \frac{\partial p_j / \partial (T_i - T_u)}{\partial E_j / \partial (T_i - T_u)} \left(\frac{u(w_j - T_j - \varphi_j) - u(-T_u)}{\lambda w_j}\right) \mathrm{d}G(\varphi).$$

Proof. See Appendix C.1. □

The optimal tax formulas are almost identical to the ones in the model without an occupational choice and the interpretation is similar. There are a few, subtle differences between equation (23) and the expression for the optimal participation tax without an occupational choice. First, the union wedge no longer captures the utility loss of the marginal worker, but instead captures the average utility loss of all workers who lose their job if employment in sector j is marginally reduced.[11] This term is similar to the union wedge $\hat{\tau}_j$ with inefficient rationing.

A second difference is that the employment and wage responses η_{ji} and κ_{ji} not only capture 'demand interactions' (through complementarities in production), but also 'supply interactions' (through occupational choice). To illustrate this, suppose that the participation tax in sector i is increased. *Ceteris paribus* this leads to a higher wage and a lower employment rate in sector i. Without an occupational choice, employment and wages in other sectors go down if labor types are complementary factors in production. With an occupational choice, a higher participation tax in sector i might lead some individuals to switch to sector $j \neq i$. This puts further downward pressure on wages in other sectors, but mitigates (and possibly overturns) the negative impact on employment in other sectors. An occupational choice thus affects the magnitude, and possibly the sign, of wage and employment responses. However, *given* these responses, i.e., given η_{ji} and κ_{ji}, the optimal tax formulas are the same as we had before.

Our second main result on the desirability of unions also generalizes to an environment with an occupational choice.

Proposition 6. *If Assumptions 3 (no income effects at the union level) and 1 (efficient rationing with occupational choice) are satisfied, individuals optimally choose their occupation according to equation (18), and taxes and transfers are set according to Proposition 5, then an increase in union power ρ_i in sector i raises social welfare if and only if $b_i > 1$.*

Proof. See Appendix C.2. □

The key to understanding why the desirability condition from Proposition 2 from the main text also holds in the current setting with occupational choice is that labor rationing is efficient. To see this, consider again a marginal increase in union power in sector i: $\mathrm{d}\rho_i > 0$. This reform puts upward pressure on the wage in sector i, which can be off-set by lowering the income tax in sector i: $\mathrm{d}T_i < 0$. The reduction in the income tax, in turn, can be financed

[11] To see why τ_j^o captures an average welfare loss, differentiate equation (20) for $i = j$ with respect to $T_i - T_u$

$$N \int_{\Phi_j} \frac{\partial p_j(\varphi, T_1 - T_u, ..., T_I - T_u)}{\partial (T_i - T_u)} \mathrm{d}G(\varphi) = \frac{\partial E_j(T_1 - T_u, ..., T_I - T_u)}{\partial (T_i - T_u)}.$$

by raising the profit tax: $\mathrm{d}T_f > 0$. As before, the tax reform has no impact on social welfare if the tax system is optimized. Furthermore, as in the model without an occupational choice, this combined reform transfers resources from firm-owners (whose social welfare weight equals one) to workers in sector i (whose social welfare weight equals b_i). However, unlike before, the higher net income of workers in sector i could attract workers from other sectors (possibly non-employment) to look for a job in sector i. These individuals experience the smallest surplus from working in sector i compared to their second-best alternative. Under our assumption of efficient rationing, they will not find a job. Anticipating this, workers on the boundary of Φ_i will not switch between sectors following an increase in union power ρ_i. The impact on social welfare is therefore the same as without an occupational-choice margin, which explains why the desirability condition is unaffected.[12]

4 Bargaining over the wage distribution

In our baseline model, bargaining takes place at the sectoral level and wages vary only across (and not within) sectors. Each sectoral union faces a trade-off between employment and wages, but does not care about the overall *distribution* of wages. There is, however, ample empirical evidence that a higher degree of unionization is associated with lower wage inequality.[13] How do our results for optimal taxes and the desirability of unions change if unions care about the entire distribution of wages?

To answer this question, we now analyze a single union which bargains with firm-owners over *all* wages. To maintain tractability, we assume efficient rationing and we assume away income effects at the union level. The union has a utilitarian objective: it maximizes the sum of all workers' expected utilities. As in the RtM-model, wages are determined through bargaining between the national union and firms, while firms (unilaterally) determine employment. Since the utility function $u(\cdot)$ is concave, the union has an incentive to compress the wage distribution. Doing so is only possible if labor markets are interdependent, since in that case marginal productivity (and hence, the wage) for any group of workers depends on employment in other sectors. If labor markets would be independent, a national union would simply set the same wages in each sector as a sectoral union would, and our previous results apply.

We explicitly solve the Nash-bargaining problem to characterize labor-market equilibrium, where the national union's bargaining power is denoted by $\delta \in [0, 1]$. Since there is only one union, we can no longer use a sector-specific measure of union power ρ_i to analyze the union's desirability. However, under Nash-bargaining, equilibrium wages and employment also depend on profit taxes, which is not the case if we use ρ_i to parameterize union power. To maintain comparability with our previous findings, we therefore assume that firm-owners are risk neutral. This ensures that equilibrium wages and employment can be written only in terms of participation taxes, like before. In Appendix D.1, we set up the bargaining problem, characterize labor-market equilibrium, and extensively discuss its properties. Here, we only highlight the most important features.

[12] This result is similar to Lee and Saez (2012) who find that a minimum wage is desirable if and only if $b_i > 1$.

[13] See, for instance, Freeman (1980, 1993), Lemieux (1993, 1998), Machin (1997), Card (2001), DiNardo and Lemieux (1997), Card et al. (2004), Visser and Checchi (2011), and Western and Rosenfeld (2011).

First, if the union has no bargaining power at all ($\delta = 0$), the labor-market equilibrium coincides with the competitive outcome. Second, if union power δ is sufficiently high, there is at least one group of workers whose wage is raised above the market-clearing level. This follows from the assumptions that, first, the union has an incentive to compress the wage distribution and, second, labor rationing is efficient. Hence, starting from the competitive labor-market outcome, a marginal increase in the bargained wage in the sector with the lowest wage compresses the wage distribution, but entails negligible welfare losses due to involuntary unemployment. Third, it may not be in the union's best interest to raise *all* wages above the market-clearing level. This is because an increase in the wage for high-skilled workers depresses the wages for low-skilled workers. A national union may therefore refrain from demanding an above market-clearing wage for high-skilled workers.

The next proposition shows how taxes should be optimized if there is a single union, which bargains with firm-owners over the entire distribution of wages. To abstain from conflicting union and government objectives, we assume that both the government and the union maximize a utilitarian objective.

Proposition 7. *If Assumptions 2 (efficient rationing), and 3 (no income effects at the union level) are satisfied, labor markets are interdependent, and a single union bargains over all wages w_i in all sectors i, then the expressions for the optimal unemployment benefits $-T_u$, optimal profit taxes T_f, and optimal participation taxes $T_i - T_u$ are the same as in Proposition 1 from the main text.*

Proof. In the absence of income effects, the reduced-form wage and employment equations can be written as $w_i = w_i(T_1 - T_u, ..., T_I - T_u)$ and $E_i = E_i(T_1 - T_u, ..., T_I - T_u)$. Since the optimal tax formulas from Proposition 1 in the main text are derived for any relationship between tax instruments and labor-market outcomes, they remain the same. □

The reason why Proposition one generalizes to a national union bargaining over the entire wage distribution is that the optimal tax rules are expressed in terms of sufficient statistics and equilibrium wages and employment only depend on participation taxes in both cases.[14]

How is the desirability condition for unions modified if the union negotiates the wages for all workers? Once more, we can answer this question by analyzing the welfare effects of a (marginal) increase in union power δ combined with a tax reform that leaves wages and employment in all sectors unaffected. If the tax system is optimized, the tax reform has no impact on social welfare. Any effect on social welfare must then necessarily come from the increase in union power. To analyze the effects of such a reform, we need to keep track of the sectors where the wage is set above the market-clearing level. Denote by $k(\delta) \equiv \{i : G(w_i - (T_i - T_u)) > E_i\}$ the set of sectors where the wage is raised above the market-clearing level. This set $k(\cdot)$ depends – among other things – on union power $\delta \in [0, 1]$. If the union has no power ($\delta = 0$), no wage is raised above the market-clearing level, and consequently $k(\cdot)$ is empty. On the other hand, $k(\delta)$ contains at least one element if $\delta = 1$, since a utilitarian monopoly union always has an incentive to increase the wage for the workers in the sector with the lowest wage. We assume

[14] The optimal tax levels are not necessarily the same because the elasticities and wedges generally differ between the different bargaining structures.

that the set of sectors where wages are above market-clearing levels $k(\delta)$ does not change in response to a marginal increase in union power.[15]

The rise in union power puts upward pressure on the wages of workers $i \in k(\delta)$ for whom the wage already exceeds the market-clearing level (the 'direct' effect). Through spillovers in production, the wages for workers in other sectors $j \notin k(\delta)$ will be affected as well (the 'indirect' effect). Now, consider a tax reform that leaves all wages and employment levels unaffected. Such a tax reform *only* requires an adjustment in the income taxes T_i for those workers whose wage exceeds the market-clearing level, i.e., for sectors $i \in k(\delta)$. Intuitively, if the adjustment in the tax system offsets the 'direct' effects, there will also be no 'indirect' effects. As before, the marginal changes in the participation taxes can be financed by a marginal increase in the profit tax such that the government budget remains balanced. The tax reform that leaves equilibrium wages and employment constant is characterized by the solution to the following system of equations:

$$\forall i \in k(\delta): \quad \sum_{j \in k(\delta)} \frac{\partial w_i(T_1 - T_u, ..., T_I - T_u, \delta)}{\partial T_j} \mathrm{d}T_j^* + \frac{\partial w_i(T_1 - T_u, ..., T_I - T_u, \delta)}{\partial \delta} \mathrm{d}\delta = 0. \quad (24)$$

Here, the functions $w_i = w_i(T_1 - T_u, ..., T_I - T_u, \delta)$ are the reduced-form equations that solve the bargaining problem (see Appendix D.1 for details). The next Proposition derives the desirability condition for the national union.

Proposition 8. *If Assumptions 2 (efficient rationing), and 3 (no income effects at the union level) are satisfied, there is a national utilitarian union bargaining with firm-owners over all wages, and the tax-benefit system is optimized according to Proposition 7, then an increase in union power δ increases social welfare if and only if*

$$\sum_{i \in k(\delta)} \omega_i (b_i - 1)(-\mathrm{d}T_i^*) > 0, \quad (25)$$

where the changes in income taxes $\mathrm{d}T_i^*$ follow from equation (24) and $k(\delta) \equiv \{i : G(w_i - (T_i - T_u)) > E_i\}$.

Proof. See Appendix D.3 □

Proposition 8 is an intuitive counterpart of Proposition 2 from the main text: an increase in union power raises social welfare if and only if doing so allows the government to increase the incomes of workers with an above-average social welfare weight. By the same logic as before, the joint increase in union power and the tax reform leaves all labor-market outcomes unaffected, while raising the *net* incomes for the low-skilled. Therefore, increasing union power raises social welfare if and only if the weighted average social welfare weight of workers whose wage is above the market-clearing level exceeds the average social welfare weight of all (employed and unemployed) workers. The weight depends on the share ω_i of workers in sector i and on the change in the income taxes $-\mathrm{d}T_i^*$ in the policy reform.

[15] Assuming $k(\delta)$ does not change following a marginal change in δ is without loss of generality, since there is a discrete number of sectors.

Since the desirability condition remains unaltered, the union's desire to compress the wage distribution does not provide an *additional* reason why a welfarist government would like to raise union power. As was the case with a restriction on profit taxes, the government can achieve the same wage compression as the labor union through the tax-transfer system, without creating involuntary unemployment. Hence, unions cannot redistribute income via wage compression any better than the government can.

5 Efficient bargaining

Up to this point, we have assumed that bargaining takes place in a right-to-manage setting. This bargaining structure generally leads to outcomes that are not Pareto efficient, because firm-owners – who take wages as given – do not take into account the impact of their hiring decisions on the union's objective (McDonald and Solow, 1981). This inefficiency can be overcome if unions and firm-owners bargain over both wages *and* employment. This Section explores whether our results generalize to a setting with efficient bargaining (EB), as in McDonald and Solow (1981). For analytical convenience we do impose the assumptions of independent labor markets, efficient rationing, and no income effects at the union level.

We would like to emphasize from the outset that we consider the EB-model less appealing for two main reasons. First, the assumption that firms and unions can write contracts on both wages *and* employment is problematic with national or sectoral unions, since individual firm-owners then need to commit to employment levels that are not profit-maximizing (Boeri and Van Ours, 2008). Oswald (1993) argues that firms unilaterally set employment, even if bargaining takes place at the firm level. Second, employment is higher in the EB-model compared to the competitive outcome, since part of firm profits are converted into jobs. This property of the EB-model is difficult to defend empirically. Therefore, we maintain the RtM-model as our baseline.

The key feature of the EB-model is that any potential equilibrium (w_i, E_i) in sector i lies on the *contract curve*, which is the line where the union's indifference curve and the firm's iso-profit curve are tangent:

$$\frac{u(w_i - T_i - \hat{\varphi}_i) - u(-T_u)}{E_i \overline{u'(w_i - T_i - \varphi)}} = \frac{w_i - F_i(\cdot)}{E_i}. \tag{26}$$

Intuitively, if the equilibrium wage and employment level are on the contract curve, then it is impossible to raise either union i's utility while keeping firm profits constant, or vice versa.

The contract curve defines a set of potential labor-market equilibria (w_i, E_i) in sector i. Which contract is negotiated depends on the power of union i relative to that of the firm. We model union i's power as its ability to bargain for a wage that exceeds the marginal product of labor. In particular, let υ_i denote the power of union i. We select the equilibrium in labor market i using the following rent-sharing rule:

$$w_i = (1 - \upsilon_i) F_i(\cdot) + \upsilon_i \phi_i(E_i), \tag{27}$$

where $\phi_i(E_i) \equiv \frac{\hat{\phi}_i(N_i E_i)}{N_i E_i}$ is the average productivity of a worker in sector i and $\hat{\phi}_i$ is the contribution of sector i to total output:[16]

$$\hat{\phi}_i(N_iE_i) \equiv F(K, N_1E_1, \cdots, N_iE_i, \cdots, N_IE_I) - F(K, N_1E_1, \cdots, 0, \cdots, N_IE_I). \tag{28}$$

If unions have zero bargaining power, i.e., $\upsilon_i = 0$, the outcome in the EB-model coincides with the competitive equilibrium: $w_i = F_i(\cdot)$. Efficiency then requires $\hat{\varphi}_i = w_i - (T_i - T_u) = \varphi_i^*$. If, on the other hand, union i has full bargaining power, i.e., $\upsilon_i = 1$, it can offer a contract which leaves no surplus to firm-owners. In the latter case, the wage equals average labor productivity and the firm makes zero profits from hiring workers in sector i: $w_iN_iE_i = \hat{\phi}_i(\cdot)$. We refer to this outcome as the full expropriation (FE) outcome.

The characterization of labor-market equilibrium is graphically illustrated in Figure 1. As in

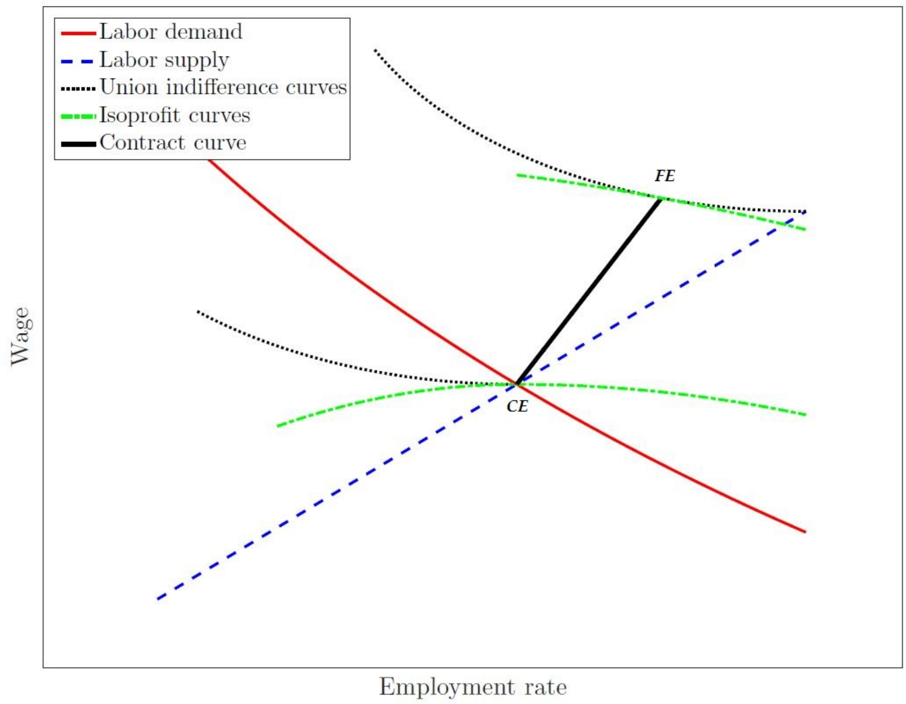

Figure 1: Labor-market equilibria in the efficient bargaining model

the RtM-model, the equilibrium coincides with the competitive outcome if the union has zero bargaining power. If union power increases, the equilibrium moves along the contract curve towards the FE-equilibrium, where the union has full bargaining power. Which equilibrium is selected depends on union power υ_i.

Figure 1 provides three important insights. First, as in the RtM-model, there is involuntary unemployment if union power υ_i is positive. Without involuntary unemployment, unions are marginally indifferent to changes in employment, since labor rationing is efficient. Hence, unions are always willing to bargain for a slightly higher wage and accept some unemployment. Second, in contrast to the RtM-model, there is also a labor-demand distortion: the wage exceeds the marginal product of labor if $\upsilon_i > 0$, see equation (27). Consequently, the labor-market equilibrium is no longer on the labor-demand curve. Intuitively, if the wage equals the marginal

[16]It should be noted that ϕ_i is different from the one used in Section 8.1 of the main text, where it denotes the wage share of sector i in aggregate labor income.

product of labor, firms are indifferent to changes in employment, whereas unions are generally not. Hence, it is possible to negotiate a labor contract with a lower wage and higher employment, which benefits both parties. As a result, efficient bargaining results in implicit subsidies on labor demand. Third, and in stark contrast to the RtM-model, an increase in union power will not only result in a higher wage, but also in *higher* employment. As illustrated in Figure 1, the contract curve is upward sloping. The higher is union power, the larger is the share of the bargaining surplus that accrues to union members. Due to the concavity of the utility function $u(\cdot)$, this surplus is translated partly into higher wages, and partly into higher employment.

In the absence of income effects at the union level, and assuming independent labor markets, the contract curve (26) and the rent-sharing rule (27) jointly determine the equilibrium wage w_i and employment E_i in sector i solely as a function of the participation taxes $T_i - T_u$. If the participation tax increases, fewer workers want to participate. In terms of Figure 1, the labor-supply schedule shifts upward. As a result, the equilibrium wage (employment rate) will be higher (lower) following the increase in the participation tax. Therefore, the comparative statics are qualitatively the same as in the RtM-model. We replicate Lemma 1 from the main text for the EB-model in Appendix E.1. The following Proposition characterizes optimal taxes.

Proposition 9. *If Assumptions 1 (independent labor markets), 2 (efficient rationing), and 3 (no income effects at the union level) are satisfied, and the efficient-bargaining equilibrium in labor market i is determined by the contract curve (26) and the rent-sharing rule (27), then optimal unemployment benefits $-T_u$, profit taxes T_f, and participation tax rates t_i are determined by:*

$$\omega_u b_u + \sum_i \omega_i b_i = 1, \tag{29}$$

$$b_f = 1, \tag{30}$$

$$\left(\frac{t_i + \tau_i - m_i}{1 - t_i}\right)\eta_{ii} = (1 - b_i) + (b_i - 1)\kappa_{ii}, \tag{31}$$

where $m_i \equiv \frac{w_i - F_i}{w_i} = \upsilon_i\left(\frac{\phi_i - F_i}{w_i}\right)$ is the implicit subsidy on labor demand. The wage and employment elasticities with respect to the participation tax rate t_i are given by:

$$\kappa_{ii} = \frac{u'_u w_i(1-t_i)\left(\frac{(1-m_i)(1-\upsilon_i)}{\varepsilon_i} + m_i\right)}{\frac{\hat{u}'_i E_i}{G'_i(\hat{\varphi}_i)} + u'_u w_i(1-t_i)\left(\frac{(1-m_i)(1-\upsilon_i)}{\varepsilon_i} + m_i\right) + (\hat{u}_i - u_u)\left(\frac{(1-m_i)}{m_i}\frac{(1-\upsilon_i)}{\varepsilon_i} - 1 + \frac{(\hat{u}'_i - \overline{u'_i})}{\overline{u'_i}}\right)} > 0, \tag{32}$$

$$\eta_{ii} = \frac{-u'_u w_i(1-t_i)}{\frac{\hat{u}'_i E_i}{G'_i(\hat{\varphi}_i)} + u'_u w_i(1-t_i)\left(\frac{(1-m_i)(1-\upsilon_i)}{\varepsilon_i} + m_i\right) + (\hat{u}_i - u_u)\left(\frac{(1-m_i)}{m_i}\frac{(1-\upsilon_i)}{\varepsilon_i} - 1 + \frac{(\hat{u}'_i - \overline{u'_i})}{\overline{u'_i}}\right)} > 0. \tag{33}$$

Proof. See Appendix E.2. □

The optimality conditions in the EB-model are very similar to their counterparts in the RtM-model. Except from differences in the definitions of the elasticities, the main difference is the implicit subsidy on labor demand m_i in the expression for the optimal participation tax rate t_i in equation (31). Since the equilibrium wage exceeds the marginal product of labor, a decrease in employment in sector i positively affects the firm's profits, which the government can tax without generating distortions. The higher is the implicit subsidy on labor demand m_i, the higher should optimal participation tax rates be set – *ceteris paribus*.

The optimal participation tax aims to redistribute income and to counter the implicit taxes on labor participation τ_i and the implicit subsidies on labor demand m_i. The equilibrium is neither on the labor-supply nor on the labor-demand curve if the union has some bargaining power. On the one hand, employment is too low, because unions generate involuntary unemployment (as captured by the union wedge τ_i), which calls for lower participation tax rates. On the other hand, employment is too high, because unions generate implicit subsidies on labor demand (as captured by m_i), which calls for higher participation tax rates. Hence, it is no longer unambiguously true that participation taxes should optimally be lower in unionized labor markets. This result contrasts with our finding from the RtM-model.

How is the desirability condition for unions affected if we assume efficient bargaining? The next Proposition answers this question.

Proposition 10. *If Assumption 2 (efficient rationing) is satisfied, the equilibrium in labor market i is determined by the contract curve (26) and the rent-sharing rule (27), and taxes and transfers are set according to Proposition 9, then increasing union power υ_i in sector i raises social welfare if and only if $b_i > 1$.*

Proof. See Appendix E.3. □

According to Proposition 10, the condition under which an increase union power in sector i is desirable is the same as in the RtM-model. Therefore, the question whether unions are desirable or not does not depend on the bargaining structure. This might seem surprising, given that – unlike in the RtM-model – employment increases in union power in the EB-model. However, also *unemployment* increases in union power, since the contract curve is steeper than the labor-supply curve. Intuitively, the union trades off employment and wages, which is not the case at the individual level. Only the effect on unemployment is critical to assess the desirability of unions. Stronger unions still generate more involuntary unemployment. Hence, an increase in union power is desirable only if there is too much employment as a result of net subsidies on participation. Therefore, the intuition for the desirability of unions in the RtM-model carries over to the EB-model: unions are only useful only if net participation subsidies lead to overemployment.

6 Data

6.1 Union data

For data on union density by sector, we draw on the "Jelle Visser database", which is officially referred to as the Institutional Characteristics of Trade Unions, Wage Setting, State Intervention and Social Pacts (Visser, 2019). This database forms the basis of the OECD Bargaining and Trade Union Data. We use database version 6.1 from 2019.[17] The ICTWSS-V6.1 is an unbalanced panel data set spanning 55 countries over the time-period 1960-2018. It contains 234 union-related variables of which we use union density at the sectoral level (variables 202-220 in the database). The union density is defined as total, net union membership as a proportion of all wage and salary earners in employment. Net union membership is defined as the total number of union members minus union members that are outside the active, dependent and employed labor force (i.e., retired workers, independent workers, students, and unemployed workers).

We focus our analysis on the latest year in our database for which we have the most comprehensive coverage of union density data. The union database contains many missing observations, because union densities are not measured every year, not for every country, and not for every sector. To obtain a more complete data set, we pool the observations on union membership for each country-sector over a 10-year time window. This procedure rests on the assumption that union membership rates are only slow-changing over time.[18] Doing so gives us a coverage of union densities at the sectoral level of approximately 75%.

6.2 Wage data

We use data on gross earnings per worker in local currency units at the sectoral level for the (latest) year where we have observations. To do so we exploit three data sources.

First, for most countries in our sample (Austria, Canada, Germany, Denmark, Spain, Finland, France, United Kingdom, Hungary, Ireland, Italy, Latvia, Netherlands, Norway, Slovakia, Sweden, and United States) we draw on the STAN (Structural Analysis) industry database, which is collected by the Organisation for Economic Co-operation and Development (OECD, 2022b). The STAN database is a panel data set containing information on output, value added, and its underlying components, as well as labor input, investment, and capital stocks at the sectoral level. The STAN database covers sectoral data on all OECD countries at the International Standard Industrial Classification of All Economic Activities, version 4 (ISIC4), at the 2-digit level from 1970-2021. From this database, we extract the wage (WAGE), employment (EMPN), full-time equivalent employment (FTEN) variables. Wage refers to gross wages and salaries for employees, *excluding* employer contributions, for example for social insurance and pensions. The total wage bill is the corresponding item in each country's National Accounts. Moreover, by focusing on the wage bill minus employer contributions, this wage measure most closely corresponds to the gross earnings variable in the OECD tax-benefit calculator, which is used to compute participation tax rates. Employment refers to the total number of persons engaged in domestic production, including the self-employed. Full-time equivalent employment is employment in persons corrected for hours worked. The wage per full-time equivalent worker is calculated as the total sectoral wage bill divided by the total number of full-time equivalent workers.

Second, we rely on the Statistics on Wages Database of the International Labor Organization (ILO, 2022e) for Switzerland, Japan, (South) Korea, New Zealand, and Turkey, since the OECD

[17] The most recent version (6.2) dates to 2021. The latter, however, no longer contains sectoral data on union densities.

[18] We confirm that union densities are slow-changing by inspecting sectoral union densities over time for countries that have more comprehensive data coverage over time.

STAN database does not contain sectoral wage data for these countries. Furthermore, the STAN wage data cover fewer sectors than the ILO data for Australia. Therefore, we also use the ILO wage data for Australia. The ILO database contains mean monthly gross earnings of employees measured in local currency units at the ISIC4 1-digit level. This unbalanced panel data set covers 187 countries and spans the time-period 1969-2021. Gross earnings are defined as monthly gross remuneration in cash and in-kind paid to employees, as a rule at regular intervals, for time worked or work done together with remuneration for time not worked, such as annual vacation, other type of paid leave or holidays. Monthly earnings data are converted to yearly earnings by multiplication with 12.

To merge the earnings data with the union data, we chose the year of the wage data that matched with the latest year for which we had the most comprehensive coverage of union data. This was possible for all countries, except for Switzerland. Here, we substituted wage data for 2016, since sectoral wage data were not available in the ILO data for 2015. Table 1 reports the coverage of our data.

Third, we calculate wages per full-time equivalent worker using data from the STAN database and the OECD. The STAN data contain information on full-time equivalent employment for the following 7 countries: Austria, Spain, France, Italy, Netherlands, Norway, and the United States. For the 16 remaining countries, only data on total employment are available (Australia, Canada, Germany, Denmark, Finland, Hungary, Ireland, Japan, (South) Korea, Latvia, New Zealand, Slovakia, Sweden, Switzerland, Turkey, and United Kingdom). Therefore, we calculate full-time equivalent employment ourselves by means of a country-sector specific part-time factor, which is defined as the ratio of average weekly hours worked relative to the statutory length of the working week in that country. We divide the wage per worker by the part-time factor to obtain the wage per full-time equivalent worker. Data on weekly hours worked come from the ILO (2022d). Data on the statutory working week are taken from the Employment Outlook of the OECD (2021). The statutory length of the working week is taken to be standard working week. Due to data availability, for the following countries we used negotiated hours rather than statutory hours: Denmark, Germany, and Switzerland. No data on the standard working week were available for Ireland and the UK. For these countries, we impute the working week at 40 hours.

6.3 Merging union and wage data

To merge the sectoral union densities from the ICTWSS-database and the sectoral wage data from the STAN and ILO-databases, we make a concordance between the sectoral classification of each database, since the sectoral division in each data set is based on a different industry classification. Table 2 shows the sectoral mapping between all datasets. The baseline sectoral classification is the one from the ICTWSS (union) data.

We exactly map the sectoral wage data from the STAN database onto the sectoral classification of the union data by aggregating and disaggregating a number of sectors in the STAN database for each country-year observation, see Table 2. In particular, we construct the Manufacturing sector to exclude the Metal sector, which is taken as a separate sector. In addition, we merge the Transport and communication sectors. Further, we create the aggregate sector

Table 1: Mapping of years in ICTWSS, STAN and ILO data

Country	ICTWSS	STAN	ILO
1. Australia	2016	2016	
2. Austria	2016	2016	
3. Canada	2017	2017	
4. Germany	2016	2016	
5. Denmark	2016	2016	
6. Spain	2016	2016	
7. Finland	2016	2016	
8. France	2016	2016	
9. Hungary	2015	2015	
10. Ireland	2016	2016	
11. Italy	2014	2014	
12. Japan	2014	2014	
13. Korea	2013		2013
14. Latvia	2016		2016
15. Netherlands	2016	2016	
16. Norway	2017	2017	
17. New Zealand	2017		2017
18. Slovakia	2016	2016	
19. Sweden	2017	2017	
20. Switzerland	2015		2016
21. Turkey	2016		2016
22. United Kingdom	2018	2018	
23. United States	2018	2018	

'Industry' by aggregating the underlying sectors. For the countries for which we rely on the ILO wage data, which are only available at the 1-digit ISIC level, we map the sectoral division in the ILO data directly onto the ICTWSS data. We could not do this for the Metal sector and the (aggregate) sector Commercial services. Due to mismatches between the sector definitions in the union data and the ILO data we drop the ILO-sectors E. Water supply; sewerage, waste management and remediation activities, J. Information and communication, and N. Administrative and support service activities. This merge of data leaves us with (potentially) 19 different sectors, of which 3 are aggregated sectors.

Our final sample contains 23 countries: Australia, Austria, Canada, Switzerland, Germany, Denmark, Spain, Finland, France, United Kingdom, Hungary, Ireland, Italy, Japan, Latvia, (South) Korea, Netherlands, Norway, New Zealand, Slovakia, Sweden, Turkey, and United States. This sample is based on the joint availability of sectoral union density data, sectoral wage data from the OECD and the ILO, and the tax-benefit calculator of the OECD for these countries. Our sample ultimately consists of data during the time period 2014-2018. For the sector Metal we have only one observation for the union density in Australia, but no corresponding wage data. Therefore, we are left with 18 sectors. Given that the coverage of sectoral union densities and wage data is incomplete, our final sample contains 294 observations spread out over 23 countries and 18 sectors.

Table 2: Sector concordance between union and earnings data

Merged data	ICTWSS	STAN	ILO
Agriculture	agr	D01T03	A. Agriculture; forestry and fishing
Industry*	ind	D05T44	
Services*	serv	D45T99	
Mining	mining	D05T09	B. Mining and quarrying
Manufacturing	manuf	D10T33 - D24T25	C. Manufacturing
Metal	metal	D24T25	
Utilities	util	D35T39	D. Electricity; gas, steam and air conditioning supply
Construction	constr	D41T43	F. Construction
Trade	trade	D45T47	G. Wholesale and retail trade; repair of motor vehicles and motorcycles
Transport and communication	transport	D49T53 + D58T63	H. Transportation and storage
Hotels and restaurants	hotels	D55T56	I. Accommodation and food service activities
Finance	finance	D64T66	K. Financial and insurance activities
Real estate and business services	business	D68T82	L. Real estate activities
Commercial services*	commercial	D45T82	
Social services	socialserv	D87T88	
Public administration	publadmin	D84	O. Public administration and defence; compulsory social security
Education	educ	D85	P. Education
Health care	health	D86	Q. Human health and social work activities
Other services	otherserv	D90T99	S. Other service activities

* Denotes an aggregate sector

6.4 Tax data

We employ the OECD tax-benefit web calculator to manually compute participation tax rates for all 294 country-sector observations in our data set (OECD, 2022a). This tax-benefit calculator computes the gross-net income trajectory for pre-specified income levels and demographics of households. The tax-benefit calculator is available for many years and we pick the year for which we used the union data, see also Table 1.

To determine participation taxes, we first calculate the sum of taxes paid minus transfers received at the household level if the primary earner is full-time employed at the sectoral wage. Subsequently, we calculate the sum of taxes paid minus transfers received at the household level when the primary earner is unemployed and entitled to social assistance-benefits (in the baseline) or unemployment benefits (in the robustness check).[19] In line with our theoretical definition, the participation tax rate is then defined as the difference between taxes paid minus transfers received when the primary earner is employed and unemployed, expressed as a fraction of gross earnings of the primary earner. The total net tax burden in work is the sum of the income tax and social-security contributions minus family benefits, and in-work tax credits.[20] The total tax burden for households where the primary earner is out of work is based on the same tax items except that we account for social-assistance benefits (in the baseline) or unemployment benefits (in the robustness check).

[19]Because our theoretical model is static, it is not obvious if the empirical counterpart of income in non-employment includes only social assistance or also unemployment benefits, which only have a limited duration. Therefore, we decided to calculate the participation tax rate at each country-sector observation twice.

[20]We set the housing benefits (e.g., rent assistance) to zero, since we do not want to distinguish between renters and home-owners.

We focus on a two-earner couple with two dependent children (the default setting in the OECD tax-benefit web calculator). The earnings of the primary earner are taken to be the sector-specific yearly full-time equivalent wage. Regarding the secondary earner, we assume positive assortative mating, such that there is a perfect correlation between earnings of primary and secondary earners. We then calculate the secondary earner's income by multiplying the primary earner's income with a country-specific ratio that measures the earnings differential between primary and secondary earners. In particular, the ratio is calculated as the product of average monthly earnings of females multiplied by total female employment divided by the product of average monthly earnings of males multiplied by total male employment, using data from ILO (2022a,b,c). In our data set, this fraction is always between 0 and 1, see Table 3. It captures differences in labor participation, unemployment rates, working hours and hourly wages between females and males (e.g., due to labor-market discrimination). For all other choices, we use the default settings of the tax-benefit calculator.

Table 3: Employment and earnings males and females

Country	Year	Employment male	Employment female	Earnings male	Earnings female	Ratio
Australia	2016	5189	4483	3958	2651	0.58
Austria	2016	1922	1719	3836	2447	0.57
Canada	2017	7916	7245	3605	2767	0.70
Denmark	2016	1213	1074	4709	3882	0.73
Finland	2016	1101	1020	3647	2923	0.74
France	2016	12393	11736	3786	3110	0.78
Germany	2016	19289	16985	5353	4371	0.72
Hungary	2015	2106	1788	1913	1586	0.70
Ireland	2016	983	841	3950	3374	0.73
Italy	2014	12032	8848	3228	2662	0.61
Japan	2014	29362	22026	3019	2180	0.54
Korea	2013	13107	9000	3633	2347	0.44
Latvia	2016	392	408	1707	1445	0.88
Netherlands	2016	3758	3209	3631	2281	0.54
New Zealand	2017	1083	978	3518	2474	0.63
Norway	2017	1173	1060	4564	3975	0.79
Slovakia	2016	1263	1048	1987	1550	0.65
Spain	2016	9465	7897	2889	2312	0.67
Sweden	2017	2249	2062	3764	3343	0.81
Switzerland	2015	2043	1758	5821	4805	0.71
Turkey	2016	15678	6749	1296	1203	0.40
United Kingdom	2018	14447	12912	3726	2459	0.59
United States	2018	67672	59203	4618	3521	0.67

6.5 Robustness check: unemployment benefits

In the robustness exercise, we assume that households collect unemployment benefits when the primary earner becomes unemployed. Table 4 shows the descriptive statistics of participation tax rates based on unemployment benefits by country, while Table 5 shows the descriptive statistics by sector.

Table 4: Participation tax rates based on unemployment benefits by country

Country	No. sectors	Participation tax rate Mean	Std. dev.	Min.	Max.
Sample	294	67.75	15.87	30.19	151.4
Australia	13	47.05	4.54	39.89	54.48
Austria	5	77.54	12.57	71.47	100.0
Canada	18	69.01	8.14	55.52	81.89
Denmark	14	75.46	5.68	66.27	86.95
Finland	12	79.24	1.23	76.19	81.53
France	15	74.00	6.57	51.74	78.81
Germany	7	84.09	5.65	71.91	89.84
Hungary	18	66.32	6.32	52.79	78.14
Ireland	16	57.79	25.66	46.58	151.4
Italy	9	80.89	4.59	70.59	86.44
Japan	11	79.75	4.75	73.17	87.34
Korea	12	56.40	11.84	38.05	74.32
Latvia	10	88.43	2.24	82.16	89.79
Netherlands	17	73.47	3.48	65.08	78.19
New Zealand	13	35.60	3.04	30.19	39.99
Norway	13	75.51	2.87	66.33	77.05
Slovakia	10	76.25	3.56	69.34	79.75
Spain	17	76.40	15.95	57.43	131.8
Sweden	15	67.74	6.21	59.71	78.63
Switzerland	7	84.50	0.39	84.12	85.21
Turkey	6	64.46	4.79	60.79	73.00
United Kingdom	18	50.02	11.55	35.80	67.00
United States	18	57.55	12.41	47.76	80.78

Not surprisingly, participation tax rates are substantially higher once we take unemployment benefits into account: on average 68% based on unemployment benefits, compared to an average of 37% in the baseline based on social-assistance benefits, see also Figure 2.

Participation tax rates based on unemployment benefits also feature quite some cross-country heterogeneity, and generate a different country ranking than based on social assistance, because unemployment benefit systems differ a lot across countries. The countries with the highest participation tax rates based on unemployment benefits are: Latvia (88%), Switzerland (84%), and Germany (84%). The countries with the lowest participation tax rates are: New Zealand (36%), Australia (47%), and United Kingdom (48%).

The participation tax rates based on unemployment benefits show a bit more variation across sectors, but are generally in the order of 60-70%, with Agriculture again being an outlier, see Figure 2.

Figure 3 gives the scatter plot of participation tax rates against union densities. This scatter plot shows the same pattern as in the main text. A simple regression of participation tax rates on union density returns a positive coefficient of 0.076 (s.e. 0.041), which is significant at the 10-percent level.

Table 5: Participation tax rates based on unemployment benefits by sector

Sector	No. countries	Participation tax rate Mean	Std. dev.	Min.	Max.
Sample	294	67.75	15.87	30.19	151.4
Agriculture	19	78.84	26.38	36.12	151.4
Commercial	17	71.54	12.70	46.58	89.21
Construction	21	69.67	13.44	37.31	89.45
Education	17	64.30	16.09	34.39	88.56
Finance	17	58.48	15.71	30.19	85.21
Health care	10	65.45	13.83	45.98	89.34
Hotels and restaurants	15	69.87	13.32	35.26	81.89
Industry	15	71.00	13.05	47.43	89.34
Manufacturing	22	66.10	14.94	36.40	88.53
Mining	8	56.59	15.12	35.80	81.93
Other services	15	70.84	12.72	39.99	86.90
Public administration	19	66.58	16.81	33.07	89.79
Real estate and business services	14	63.81	13.34	35.75	78.90
Services	16	71.43	12.57	47.53	89.27
Social services	23	73.14	13.01	38.77	89.84
Trade	17	66.84	13.26	39.14	84.12
Transport and communication	18	66.00	15.17	36.11	87.34
Utilities	11	55.28	17.00	30.36	79.75

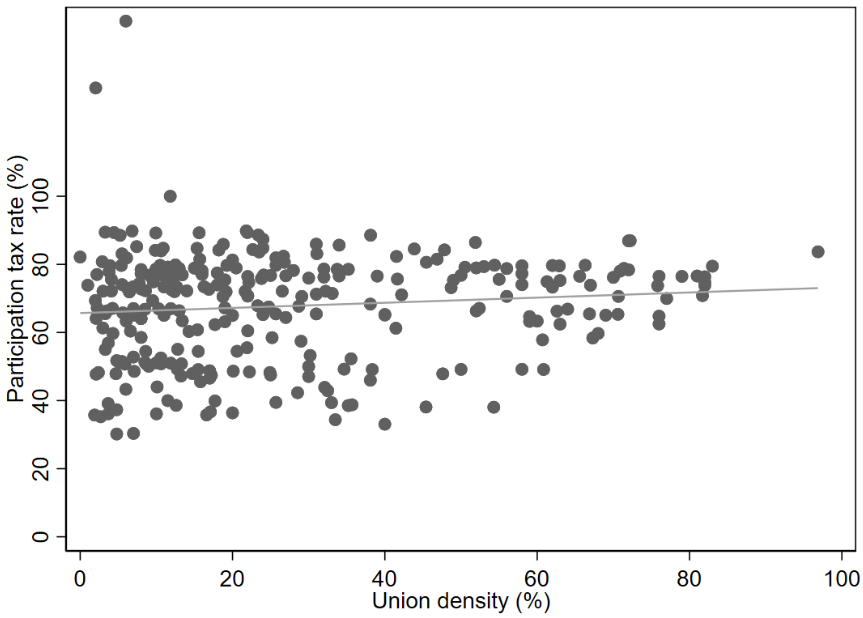

Figure 3: Participation tax rates and union densities

Finally, Table 6 presents the regression results of a fixed-effects regression of participation tax rates on union densities. This regression strengthens the baseline results.

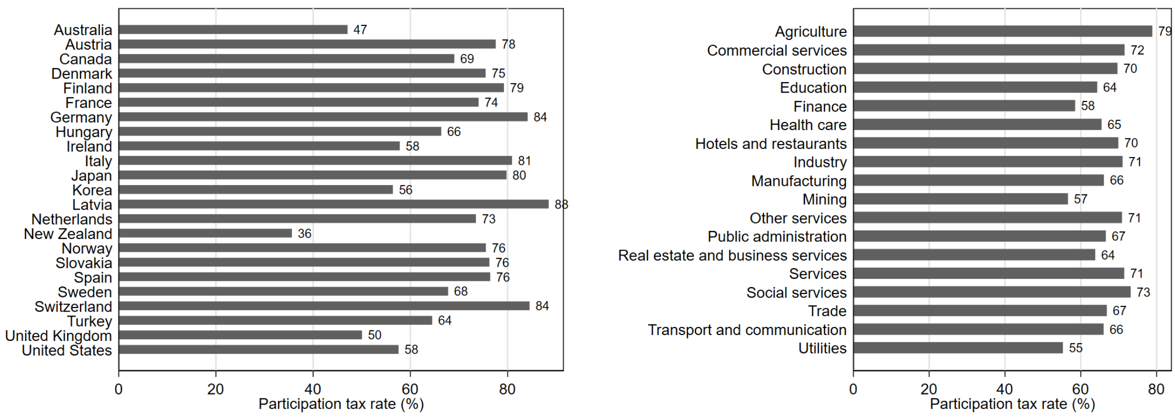

Figure 2: Average participation tax rates across countries and sectors based on unemployment benefits

Table 6: Fixed-effects regressions of participation tax rates on union densities

Variable	Coefficient	Standard error	t-value
Union density	-0.172	0.042	-4.11
Constant	59.49	2.28	26.0
R^2	0.67	R^2 adj.	0.64

Country-fixed effects included, United States is the reference country

7 Simulations: sensitivity analysis

This section analyzes how the results from our simulations are affected if some of the key parameters of our model are changed. For each of the robustness checks, we change one of the parameters and recalibrate the model to match the same empirical targets as in the baseline.[21]

7.1 Labor-demand elasticity

We first examine how our results are affected if we consider different values for the labor-demand elasticity. This elasticity ultimately determines the trade-off between wages and employment for the union. Figures 4 and 5 (6 and 7) show the optimal participation tax rates and social welfare weights if the average labor-demand elasticity is doubled to $\bar{\varepsilon} = 1.4$ or cut in half to $\bar{\varepsilon} = 0.35$. The implied elasticity of substitution in the production then equals $\sigma = 1.354$ and $\sigma = 0.355$, respectively. The average participation tax rate at the optimal tax system with unions is comparable to the baseline scenario as it ranges from 58.0% to 59.0%, depending on the elasticity of labor demand.

We confirm our finding that optimal participation tax rates are significantly lower in unionized labor markets: optimal participation tax rates in competitive labor markets are on average between 7.4 and 7.5 percentage points higher, depending on the elasticity of labor demand. Furthermore, we find that if the tax system is optimized, an increase in union power does not

[21]When we compare the optimal tax system with and without unions, we do *not* recalibrate the model, but instead conduct a comparative statics exercise by setting $\rho = 0$.

improve social welfare in both cases: the social welfare weight for all employed workers remains below the average of one.

It might be surprising that the impact of unions on the average participation tax rate is so similar for different labor-demand elasticities. The explanation for this finding is that the degree of union power ρ is recalibrated to make sure that the unemployment rate in the calibrated economy corresponds to the actual unemployment rate of 6.9%. A higher labor-demand elasticity raises the costs of demanding higher wages, and, hence, requires higher union power to match the unemployment rate observed in the data. Hence, the impact of a larger labor-demand elasticity on the union wedge is countered by larger union power.

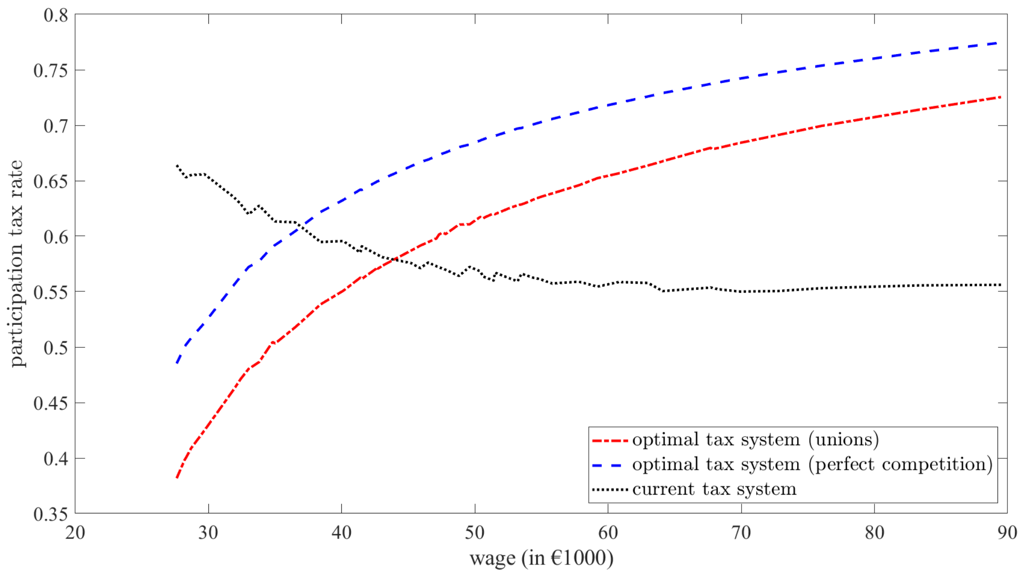

Figure 4: Optimal participation tax rates (high labor-demand elasticity)

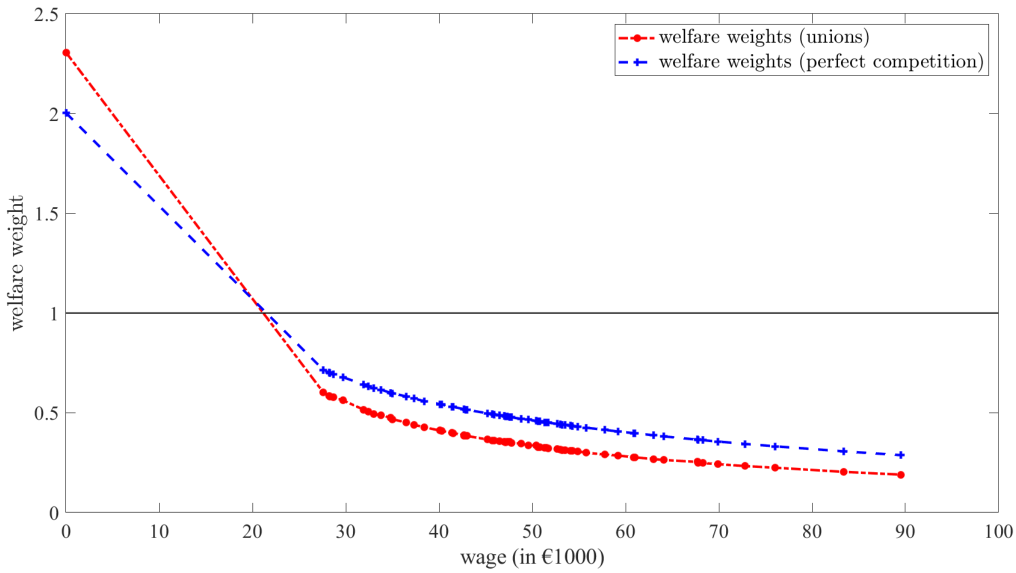

Figure 5: Social welfare weights (high labor-demand elasticity)

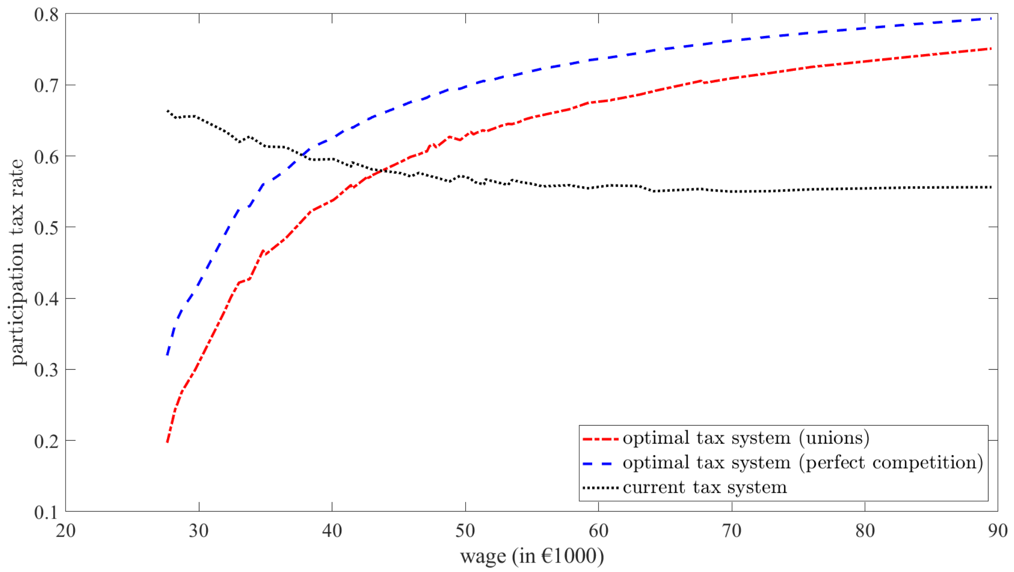

Figure 6: Optimal participation tax rates (low labor-demand elasticity)

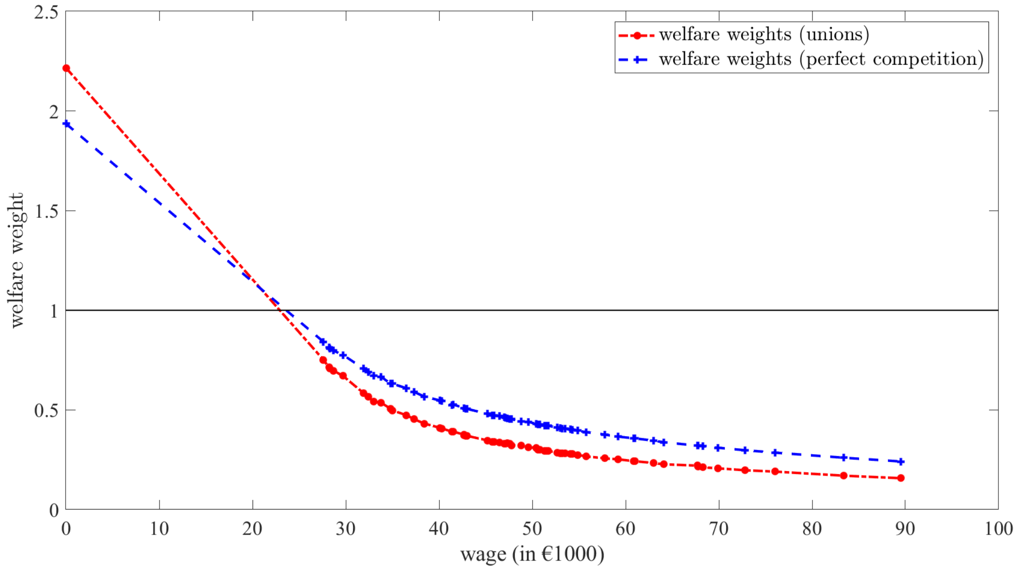

Figure 7: Social welfare weights (low labor-demand elasticity)

7.2 Union power

In this robustness check, we investigate how our results are affected if the degree of union power increases, see Figures 8 and 9. To that end, we calibrate the degree of union power at $\rho = 0.330$ to match an involuntary unemployment rate of 13.8%, which is twice as high as the rate of 6.9% in the baseline year 2015. Not surprisingly, the impact of unions on the optimal participation tax rates is larger. On average, the optimal participation tax rate with unions is approximately 10.5 percentage points below the optimal participation tax rate with perfectly competitive labor markets (compared to 7.4 percentage points in the baseline). Furthermore, we confirm our baseline finding that an increase in union power does not raise social welfare:

all social welfare weights for employed workers are below the average of one if the tax system is optimized.

Figures 10 and 11 plot the optimal participation tax rates and social welfare weights if the degree of union power is calibrated at $\rho = 0.125$, to match an unemployment rate of 3.45%, which is half the actual unemployment rate in the year 2015. This could capture, for instance, that only a fraction of involuntary unemployment is driven by unions demanding above market-clearing wages. We again find that an increase in union power reduces social welfare. Furthermore, unions lead to lower optimal participation tax rates compared to the competitive benchmark, but the difference is less pronounced (4.7 percentage points versus 7.4 percentage points in the baseline).

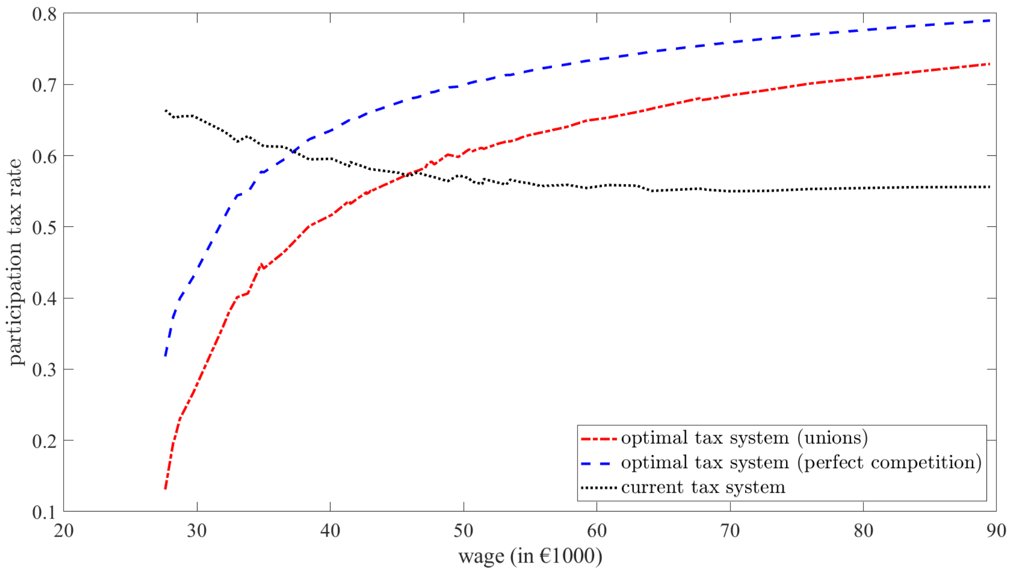

Figure 8: Optimal participation tax rates (strong unions)

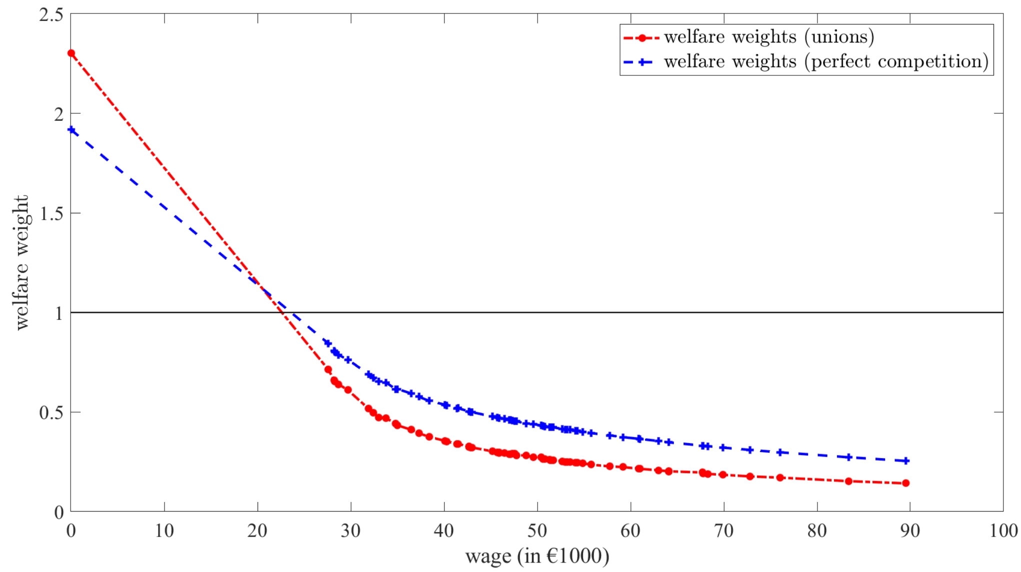

Figure 9: Social welfare weights (strong unions)

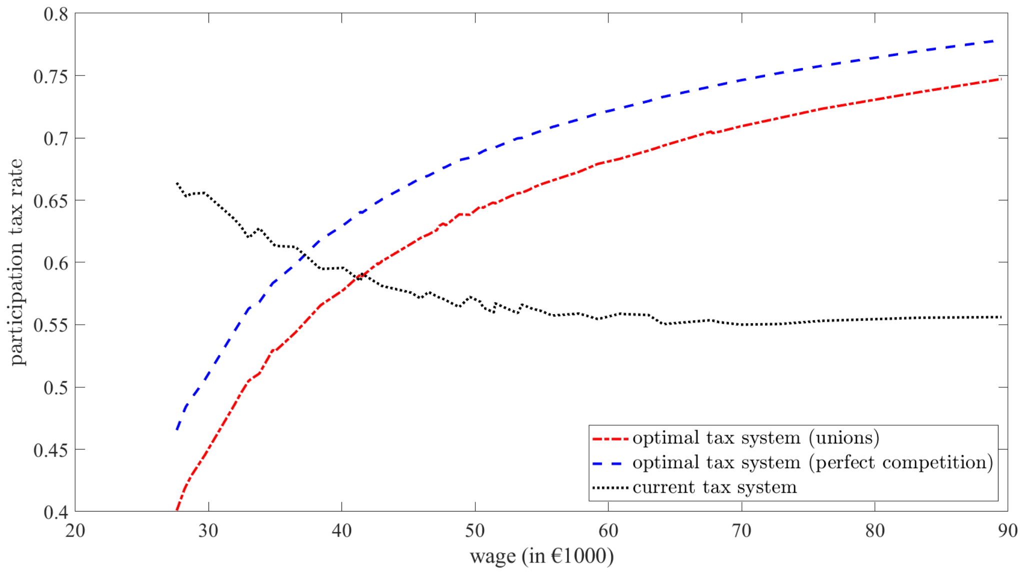

Figure 10: Optimal participation tax rates (weak unions)

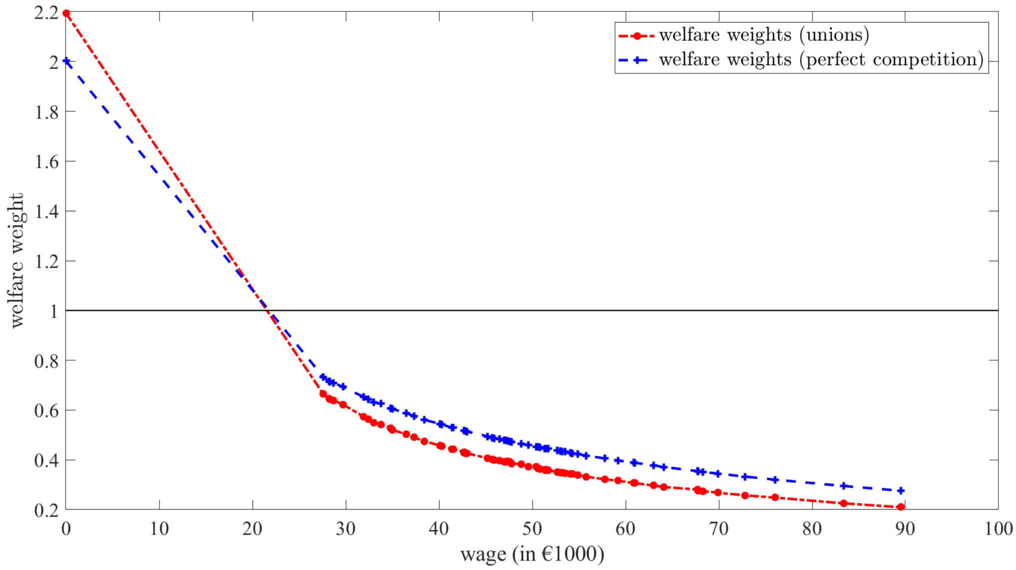

Figure 11: Social welfare weights (weak unions)

7.3 Participation elasticity

Next, we increase the average participation elasticity in the calibrated economy from its value of $\bar{\pi} = 0.25$ in the baseline to a value of $\bar{\pi} = 0.50$. Figures 12 and 13 plot the optimal participation tax rates and the social welfare weights at the optimal tax system with and without unions. In line with the theoretical findings from Diamond (1980), optimal participation tax rates are lower than before, as can be seen by comparing Figures 7 and 12. The difference is most pronounced for low- and middle-income groups. The reason is that the participation elasticity is declining in income, cf. equation (34). Targeting a higher average participation elasticity, in turn, leads to larger increases in the participation elasticity at lower levels of income. Consequently, compared to the baseline, optimal participation tax rates are lowered especially for these workers.

Interestingly, the impact of unions on the optimal tax system is less pronounced if the participation elasticity is increased. Optimal participation tax rates with unions are on average only 3.6 percentage points below the optimal participation tax rates with competitive labor markets (compared to a difference of 7.4 percentage points in the baseline). Intuitively, if the participation elasticity is large, an increase in the wage above the market-clearing level leads to a sharp increase in involuntary unemployment. Consequently, the degree of union power that is required to match the unemployment rate of 6.9% in the data is lower than in the baseline. The reduction in union power, in turn, lowers the union wedge. As a result, the impact of unions on the optimal tax-benefit system is smaller. Lastly, we again find that the social welfare weights for all employed workers are below the average of one, which according to Proposition 2 implies that an increase in union power lowers social welfare.

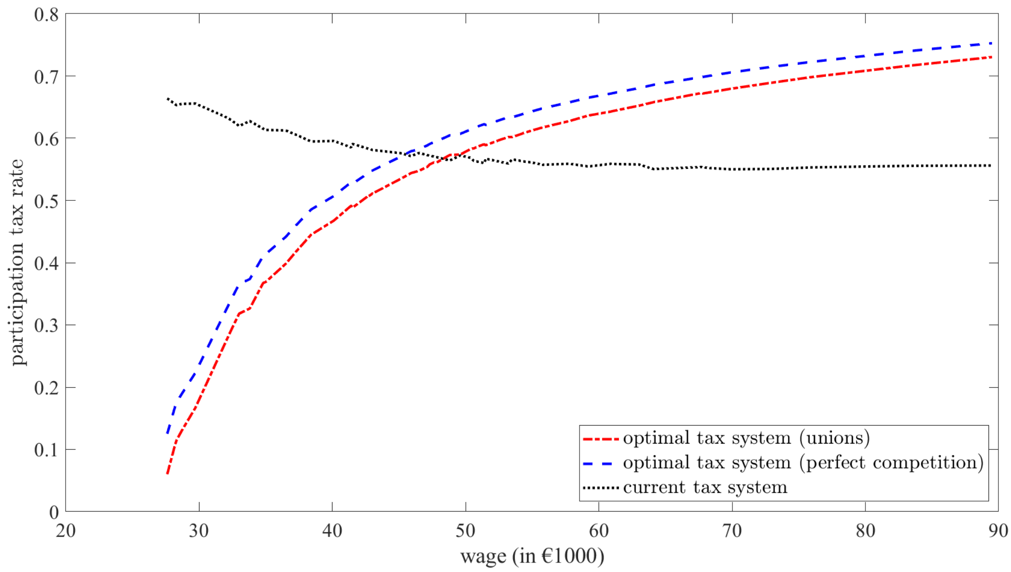

Figure 12: Optimal participation tax rates (high participation elasticity)

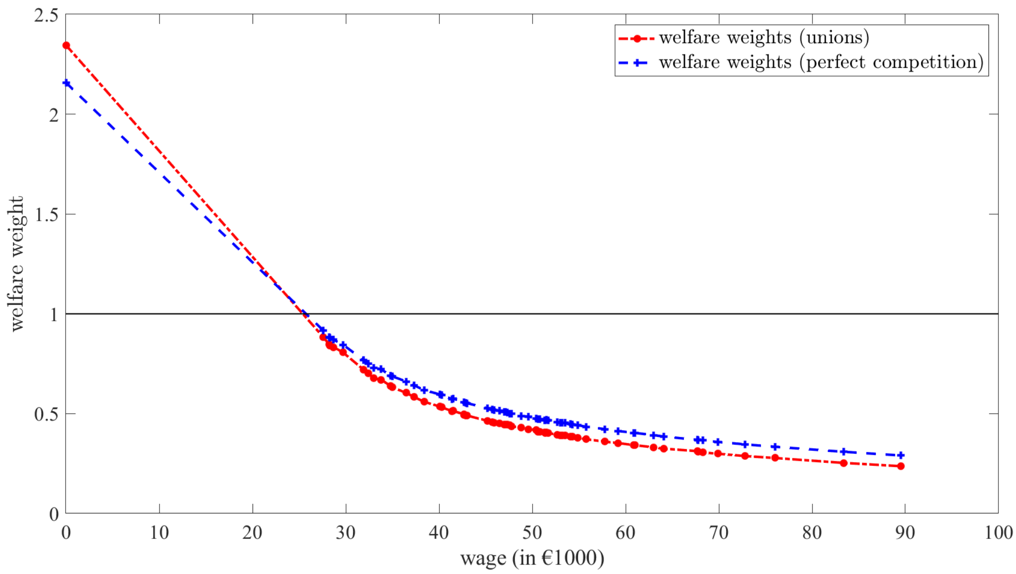

Figure 13: Social welfare weights (high participation elasticity)

7.4 Inequality aversion

We significantly decrease inequality aversion by reducing the coefficient of absolute risk aversion to $\theta = 0.016$. At this value of θ, the coefficient of *relative* risk-aversion is 0.50 for an individual with zero participation costs who earns the average wage. The optimal participation tax rate with unions equals approximately 27.8% on average, which is less than half the current rate of 58.3%. With a lower degree of inequality aversion, the government redistributes less income towards the unemployed and more towards low-income workers. In particular, the optimal participation tax rate at the bottom of the income distribution is now *negative*, as can be seen from Figure 14. Hence, low-income workers receive a subsidy of approximately €8,228, which exceeds the non-employment benefit of €5,323 at the optimal tax system with unions (which is much lower than the value of €12,560 in the baseline).

The finding that participation is optimally subsidized for low-skilled workers has an important implication: the social welfare weight of low-skilled workers exceeds the average of one, see Figure 15. Therefore, according to Proposition 2, an increase in union power for low-skilled workers *raises* social welfare – which does not occur in the baseline. This is true for individuals whose current earnings are below €28,300, where participation is subsidized at the optimal tax system.[22] Hence, an increase in union power alleviates these upward distortions in labor participation.

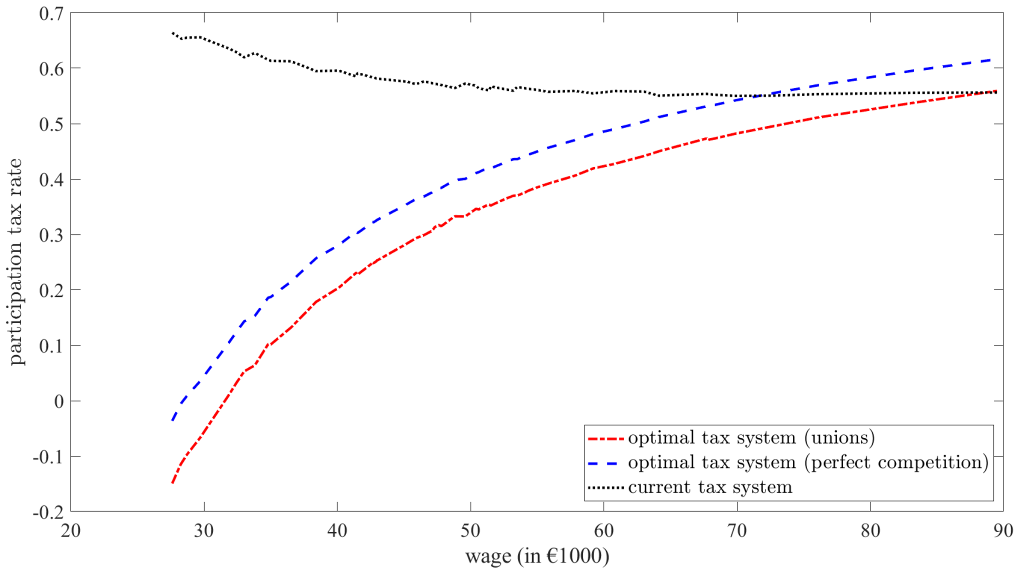

Figure 14: Optimal participation tax rates (low inequality aversion)

[22]At the *current* tax system, however, participation taxes for these workers are positive. This explains why in the analysis from Section 7 in the main text we do not find that an increase in union power for these workers raises social welfare.

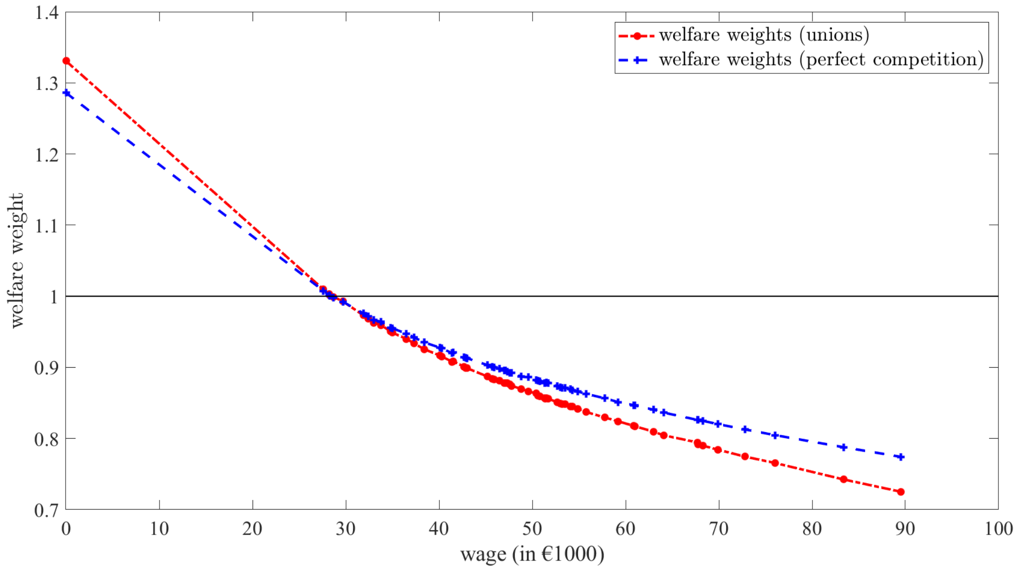

Figure 15: Social welfare weights (low inequality aversion)

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A Union responses to marginal tax rates

A.1 Comparative statics

This Appendix studies how a (local) increase in the unemployment benefit $-T_u$, the tax burden $T(w(i))$, the marginal tax rate $T'(w(i))$ at income level $w(i)$, and union power $\rho(i)$ affects the equilibrium wage $w(i)$ and employment rate $E(i)$ of workers in sector i. To do so, we, first, use $\hat{\varphi}(i) = G^{-1}(E(i))$ and the labor-demand equation (2) to substitute for $w(i)$ in the wage-demand equation (5). Second, we introduce tax reform parameters ν and ξ and define[23]

$$
\begin{aligned}
&\Upsilon(E(i), T_u, \nu, \xi, \rho(i)) \equiv \rho(i)(1 - T'(a(i)y'(h(i)E(i))) - \xi) \\
&\times \int_{\underline{\varphi}}^{G^{-1}(E(i))} u'(a(i)y'(h(i)E(i)) - T(a(i)y'(h(i)E(i))) - \nu - \varphi)\mathrm{d}G(\varphi) \times a(i)h(i)y''(h(i)E(i)) \\
&+ \Big[u(a(i)y'(h(i)E(i)) - T(a(i)y'^{-1}(E(i))) - u(-T_u)\Big] = 0.
\end{aligned} \tag{34}
$$

This equation pins down the equilibrium employment rate $E(i)$ for workers in sector i as a function of the unemployment benefit $-T_u$, union power $\rho(i)$, and the reform parameters ν and ξ. The parameter ν can be used to study how a local increase in the tax burden $T(w(i))$ affects the equilibrium employment rate. The parameter ξ can be used to study the effect of locally increasing the marginal tax rate $T'(w(i))$ at income level $w(i) = a(i)y'(h(i)E(i))$. The impact on $E(i)$ follows from applying the implicit function theorem, and evaluating the resulting expressions at $\nu = \xi = 0$. With a slight abuse of notation, we denote by $E_T(i) = \partial E(i)/\partial T(w(i))$ the impact of a higher tax burden on the employment rate. The latter is given by

$$
\begin{aligned}
E_T(i) &= \frac{\partial E(i)}{\partial T(w(i))} = -\frac{\Upsilon_\nu(E(i), T_u, 0, 0, \rho(i))}{\Upsilon_E(E(i), T_u, 0, 0, \rho(i))} = \frac{u'^{-1}(E(i)))}{\Upsilon_E(E(i), T_u, 0, 0, \rho(i))} \\
&+ \left[\frac{\rho(i)(1 - T'(w(i)))a(i)h(i)y''(h(i)E(i)) \int_{\underline{\varphi}}^{G^{-1}(E(i))} u''(w(i) - T(w(i)) - \varphi)\mathrm{d}G(\varphi)}{\Upsilon_E(E(i), T_u, 0, 0, \rho(i))}\right] < 0.
\end{aligned} \tag{35}
$$

The sign follows from the concavity of $y(\cdot)$ and $u(\cdot)$, and concavity of the union objective implies that $\Upsilon_E < 0$.

The impact of a local increase in the marginal tax rate on the equilibrium employment rate

[23] See, for example, Jacquet et al. (2013) and Jacobs et al. (2017), among many others.

is:

$$E_{T'}(i) = \frac{\partial E(i)}{\partial T'(w(i))} = -\frac{\Upsilon_{\xi}(E(i), T_u, 0, 0, \rho(i))}{\Upsilon_E(E(i), T_u, 0, 0, \rho(i))} \tag{36}$$
$$= \left[\frac{\rho(i)a(i)h(i)y''(h(i)E(i)) \int_{\underline{\varphi}}^{G^{-1}(E(i))} u'(w(i) - T(w(i)) - \varphi)\mathrm{d}G(\varphi)}{\Upsilon_E(E(i), T_u, 0, 0, \rho(i))}\right] \geq 0,$$

with a strict inequality if $\rho(i) > 0$. Again, the sign follows from the assumptions on the utility and production function and concavity of the union objective.

The effect of lowering the unemployment benefit (i.e., increasing T_u) is

$$E_{T_u}(i) = \frac{\partial E(i)}{\partial T_u} = -\frac{\Upsilon_{T_u}(E(i), T_u, 0, 0, \rho(i))}{\Upsilon_E(E(i), T_u, 0, 0, \rho(i))} = \frac{-u'(-T_u)}{\Upsilon_E(E(i), T_u, 0, 0, \rho(i))} > 0. \tag{37}$$

Lastly, the impact of a local increase in union power $\rho(i)$ is:

$$E_{\rho}(i) = \frac{\partial E(i)}{\partial \rho(i)} = -\frac{\Upsilon_{\rho}(E(i), T_u, 0, 0, \rho(i))}{\Upsilon_E(E(i), T_u, 0, 0, \rho(i))} \tag{38}$$
$$= \frac{-(1 - T'(w(i)))a(i)h(i)y''(h(i)E(i)) \int_{\underline{\varphi}}^{G^{-1}(E(i))} u'(w(i) - T(w(i)) - \varphi)\mathrm{d}G(\varphi)}{\Upsilon_E(E(i), T_u, 0, 0, \rho(i))} < 0.$$

From equations (36) and (38) follows that the impact of union power and the marginal tax rate are closely related: $E_{T'}(i) = -\frac{\rho(i)}{1-T'(w(i))}E_{\rho}(i)$. Intuitively, by making wage increases more attractive, a lower marginal tax rate has a similar impact as an increase in union power: both lead to an increase in the wage and a reduction in the employment rate.

To summarize, a (local) increase in the tax burden $T(w(i))$ (captured by $\mathrm{d}\nu > 0$), unemployment benefit $-T_u$, or union power $\rho(i)$ has a negative impact on the employment rate, whereas a local increase in the marginal tax rate $T'(w(i))$ (captured by $\mathrm{d}\xi > 0$) positively affects the employment rate $E(i)$. The impact on the equilibrium wage in sector i then follows directly from the labor-demand equation (2). Because the latter is downward-sloping, a higher tax burden, union power or unemployment benefit positively affects the wage $w(i)$ of workers in sector i, whereas a higher marginal tax rate leads to a lower equilibrium wage $w(i)$.

A.2 Optimal taxation

In the current framework with a continuum of types, the government's objective is given by

$$\mathcal{W} = \int_0^1 \psi(i)\Big[\int_{\underline{\varphi}}^{G^{-1}(E(i))} u(w(i) - T(w(i)) - \varphi)\mathrm{d}G(\varphi) + \int_{G^{-1}(E(i))}^{\overline{\varphi}} u(-T_u)\mathrm{d}G(\varphi)\Big]h(i)\mathrm{d}i$$
$$+ N^{-1}\psi_f u\bigg(N\bigg(\int_0^1 a(i)y(h(i)E(i))\mathrm{d}i - \int_0^1 w(i)E(i)h(i)\mathrm{d}i - T_f\bigg)\bigg). \tag{39}$$

Recall that the measure of workers is normalized to one and the measure of firm-owners is $1/N$. The government's budget constraint, in turn, is

$$\int_0^1 \Big[E(i)T(w(i)) + (1 - E(i))T_u + T_f - R\Big] h(i)\mathrm{d}i = 0. \tag{40}$$

For each i, the equilibrium wage and employment rate are pinned down by

$$w(i) = a(i)y'(h(i)E(i)), \tag{41}$$

$$\begin{aligned}
&\rho(i)(1 - T'(w(i))) \times \int_{\underline{\varphi}}^{G^{-1}(E(i))} u'(w(i) - T(w(i)) - \varphi)\mathrm{d}G(\varphi) \times a(i)h(i)y''(h(i)E(i)) \\
&+ \Big[u(w(i) - T(w(i)) - G^{-1}(E(i))) - u(-T_u)\Big] = 0.
\end{aligned} \tag{42}$$

Equation (41) is the labor-demand equation and equation (42) the modified wage-demand equation. The government's problem is to find the tax schedule $T(\cdot)$ that maximizes social welfare (39) subject to the budget constraint (40), taking into account the impact on equilibrium wages and employment rates in each sector i as determined by the labor-market equilibrium conditions (41)–(42).

The Lagrangian of the government's problem is given by

$$\begin{aligned}
\mathcal{L} = &\int_0^1 \psi(i)\Big[\int_{\underline{\varphi}}^{G^{-1}(E(i))} u(w(i) - T(w(i)) - \varphi)\mathrm{d}G(\varphi) + \int_{G^{-1}(E(i))}^{\overline{\varphi}} u(-T_u)\mathrm{d}G(\varphi)\Big] h(i)\mathrm{d}i \\
&+ N^{-1}\psi_f u\Big(N\Big(\int_0^1 a(i)y(h(i)E(i))\mathrm{d}i - \int_0^1 w(i)E(i)h(i)\mathrm{d}i - T_f\Big)\Big) \\
&+ \lambda \int_0^1 \Big[E(i)T(w(i)) + (1 - E(i))T_u + T_f - R\Big] h(i)\mathrm{d}i,
\end{aligned} \tag{43}$$

where λ is the multiplier on the government budget constraint.

To solve this problem, we proceed in a similar way as Jacquet and Lehmann (2021). Specifically, we start by replacing the tax schedule $T(w)$ by a perturbed tax schedule $T(w) + m\tilde{R}(w)$. Under the perturbed tax schedule, the equilibrium wage w and employment rate E in sector i are pinned down by

$$w = a(i)y'(h(i)E), \tag{44}$$

$$\begin{aligned}
&\rho(i)(1 - T'(w) - m\tilde{R}'(w)) \times \int_{\underline{\varphi}}^{G^{-1}(E)} u'(w - T(w) - m\tilde{R}(w) - \varphi)\mathrm{d}G(\varphi) \times a(i)h(i)y''(h(i)E) \\
&+ \Big[u(w - T(w(i)) - m\tilde{R}(w) - G^{-1}(E)) - u(-T_u)\Big] = 0,
\end{aligned} \tag{45}$$

which are the counterparts of equations (41)–(42) under the perturbed tax schedule. Denote by $w^R(i, m)$ and $E^R(i, m)$ the equilibrium wage and employment rate in sector i under the perturbed tax schedule, so that $w^R(i, 0) = w(i)$ and $E^R(i, 0) = E(i)$.

Assuming that the tax function is twice differentiable, and equations (44)–(45) pin down a unique solution, we can apply the implicit function theorem to derive[24]

$$\frac{\partial E^R(i,0)}{\partial m} = E_T(i)\tilde{R}(w(i)) + E_{T'}(i)\tilde{R}'(w(i)), \tag{46}$$

$$\frac{\partial w^R(i,0)}{\partial m} = w_T(i)\tilde{R}(w(i)) + w_{T'}(i)\tilde{R}'(w(i)), \tag{47}$$

where $E_T(i) < 0$ and $E_{T'}(i) > 0$ are as defined in equations (35)–(36). Furthermore, from the labor-demand equation (2), $w_T(i)$ and $w_{T'}(i)$ satisfy

$$w_T(i) = E_T(i)a(i)y''(h(i)E(i))h(i) > 0, \quad w_{T'}(i) = E_{T'}(i)a(i)y''(h(i)E(i))h(i) < 0, \tag{48}$$

which capture the impact of a local increase in the tax burden and marginal tax rate on the equilibrium wage, respectively.[25]

The government's Lagrangian under the perturbed tax schedule is

$$\begin{aligned}\tilde{\mathcal{L}}(m) = \int_0^1 \psi(i)\Big[\int_{\underline{\varphi}}^{G^{-1}(E^R(i,m))} & u(w^R(i,m) - T(w^R(i,m)) - m\tilde{R}(w^R(i,m)) - \varphi)\mathrm{d}G(\varphi) \\ & + \int_{G^{-1}(E^R(i,m))}^{\overline{\varphi}} u(-T_u)\mathrm{d}G(\varphi)\Big]h(i)\mathrm{d}i \\ & + N^{-1}\psi_f u\Big(N\Big(\int_0^1 a(i)y(h(i)E^R(i,m))\mathrm{d}i - \int_0^1 w^R(i,m)E^R(i,m)h(i)\mathrm{d}i - T_f\Big)\Big) \\ & + \lambda\int_0^1 \Big[E^R(i,m)(T(w^R(i,m)) + m\tilde{R}(w^R(i,m))) + (1-E^R(i,m))T_u + T_f - R\Big]h(i)\mathrm{d}i.\end{aligned} \tag{49}$$

This is the Lagrangian (43) if the tax schedule is $T(w) + m\tilde{R}(w)$ and equilibrium wages and employment rates are denoted by $w^R(i,m)$ and $E^R(i,m)$.

The welfare impact of perturbing the tax schedule $T(w)$ in the direction $\tilde{R}(w)$, evaluated at $m = 0$, is given by

$$\begin{aligned}\frac{\partial\tilde{\mathcal{L}}(0)}{\partial m} = \int_0^1 \Big[& -\psi(i)\int_{\underline{\varphi}}^{G^{-1}(E(i))} u'(w(i) - T(w(i)) - \varphi)\mathrm{d}G(\varphi)\tilde{R}(w(i)) \\ & + \psi(i)\int_{\underline{\varphi}}^{G^{-1}(E(i))} u'(w(i) - T(w(i)) - \varphi)\mathrm{d}G(\varphi)(1 - T'(w(i)))\frac{\partial w^R(i,0)}{\partial m} \\ & + \psi(i)\Big[u(w(i) - T(w(i)) - G^{-1}(E(i))) - u(-T_u)\Big]\frac{\partial E^R(i,0)}{\partial m}\Big]h(i)di \\ & + \psi_f u'\Big(N\Big(\int_0^1 a(i)y(h(i)E(i))\mathrm{d}i - \int_0^1 w(i)E(i)h(i)\mathrm{d}i - T_f\Big)\Big) \\ & \times\Big[\int_0^1 a(i)y'(h(i)E(i))\frac{\partial E^R(i,0)}{\partial m}h(i)\mathrm{d}i - \int_0^1 w(i)\frac{\partial E^R(i,0)}{\partial m}h(i)\mathrm{d}i - \int_0^1 \frac{\partial w^R(i,0)}{\partial m}E(i)h(i)\mathrm{d}i\Big] \\ & + \lambda\Big[\int_0^1 \Big(E(i)\tilde{R}(w(i)) + E(i)T'(w(i))\frac{\partial w^R(i,0)}{\partial m} + (T(w(i)) - T_u)\frac{\partial E^R(i,0)}{\partial m}\Big)h(i)\mathrm{d}i\Big].\end{aligned} \tag{50}$$

[24]See Jacquet and Lehmann (2021) for further details.

[25]Note that these effects capture the behavioral responses along the actual (and not a linearized) tax schedule, so they account for the non-linearity of the tax schedule. See Jacquet and Lehmann (2021) for further details

The first three lines capture the welfare effect associated with changes in the utility of workers, the next two lines capture the welfare effect on firm-owners, and the final line captures the impact of the tax reform on the government budget.

The right-hand side of this equation can be simplified in a number of steps. First, note that profit maximization implies that the first two terms on the fifth line cancel (this is an application of the envelope theorem). Second, define the social welfare weight of firm-owners and of workers who are employed in sector i as

$$b_f = \frac{\psi_f u'(c_f)}{\lambda}, \quad \hat{b}(i) = \frac{\psi(i)\int_{\underline{\varphi}}^{G^{-1}(E(i))} u'(w(i) - T(w(i)) - \varphi)\mathrm{d}G(\varphi)}{\lambda E(i)}, \tag{51}$$

where c_f denotes the consumption of firm-owners. Third, combine the terms with $\tilde{R}(w(i))$, $\frac{\partial w^R(i,0)}{\partial m}$ and $\frac{\partial E^R(i,0)}{\partial m}$. Rearranging gives

$$\begin{aligned}\frac{\partial\tilde{\mathcal{L}}(0)}{\partial m}\frac{1}{\lambda} = \int_0^1 \Bigg[&(1-\hat{b}(i))E(i)\tilde{R}(w(i)) + \big[\hat{b}(i)(1-T'(w(i))) - b_f + T'(w(i))\big]E(i)\frac{\partial w^R(i,0)}{\partial m} \\ &+ \left(\psi(i)\frac{u(w(i)-T(w(i))-G^{-1}(E(i))) - u(-T_u)}{\lambda} + (T(w(i)) - T_u)\right)\frac{\partial E^R(i,0)}{\partial m}\Bigg]h(i)\mathrm{d}i. \end{aligned}\tag{52}$$

Using equations (46)–(47) and collecting terms, we get

$$\begin{aligned}\frac{\partial\tilde{\mathcal{L}}(0)}{\partial m}\frac{1}{\lambda} = \int_0^1 \Bigg[&\left(1-\hat{b}(i) + \Big(\hat{b}(i)(1-T'(w(i))) - b_f + T'(w(i))\Big)w_T(i)\right)E(i)\tilde{R}(w(i)) \\ &+ \left(\psi(i)\frac{u(w(i)-T(w(i))-G^{-1}(E(i))) - u(-T_u)}{\lambda} + (T(w(i)) - T_u)\right)E_T(i)\tilde{R}(w(i)) \\ &+ \Big(\hat{b}(i)(1-T'(w(i))) - b_f + T'(w(i))\Big)E(i)w_{T'}(i)\tilde{R}'(w(i)) \\ &+ \left(\psi(i)\frac{u(w(i)-T(w(i))-G^{-1}(E(i))) - u(-T_u)}{\lambda} + (T(w(i)) - T_u)\right)E_{T'}(i)\tilde{R}'(w(i))\Bigg]h(i)\mathrm{d}i. \end{aligned}\tag{53}$$

To proceed, let $K(w)$ denote the wage distribution with associated density $k(w)$, defined over the support $[\underline{w}, \overline{w}]$, where $\underline{w} = w(0)$ is the lowest wage and $\overline{w} = w(1)$ the highest wage. As some workers are not employed, the wage distribution has a mass point at zero: $\omega_u = \int_0^1 (1 - E(i))h(i)\mathrm{d}i$. Monotonicity of wages, in turn, implies

$$K(w(i)) = \omega_u + \int_0^i E(j)h(j)\mathrm{d}j, \quad \leftrightarrow \quad k(w(i))w'(i) = E(i)h(i). \tag{54}$$

The fraction of workers with a wage below $w(i)$ contains the employed workers whose type is below i and *all* unemployed workers (irrespective of their type). Next, we change the index of all variables from i to w. Moreover, we denote by $b(w)$ the social welfare weight of workers who are paid w, so that $b(w(i)) = \hat{b}(i)$. Further, we define by $\tilde{E}(w)$ the employment rate among workers whose wage if employed equals w, so that $\tilde{E}(w(i)) = E(i)$. Then, by substituting all these definitions into equation (53), we arrive at

$$\frac{\partial \tilde{\mathcal{L}}(0)}{\partial m}\frac{1}{\lambda} = \int_{\underline{w}}^{\overline{w}} \Bigg[\bigg(1 - b(w) + \Big(b(w)(1-T'(w)) - b_f + T'(w)\Big) w_T\bigg)\tilde{R}(w) \tag{55}$$
$$+ \big(\tau(w)w + t(w)w\big)\frac{E_T}{\tilde{E}(w)}\tilde{R}(w) + \Big(b(w)(1-T'(w)) - b_f + T'(w)\Big) w_{T'}\tilde{R}'(w)$$
$$+ \big(\tau(w)w + t(w)w\big)\frac{E_{T'}}{\tilde{E}(w)}\tilde{R}'(w)\Bigg] k(w)\mathrm{d}w,$$

where, to avoid further notation, we ignored the function arguments on the partial effects $w_{T'}$, w_T, E_T and $E_{T'}$. Moreover, the union wedge and participation tax rate are defined as:

$$\tau(w(i)) \equiv \psi(i)\frac{u(w(i) - T(w(i)) - G^{-1}(E(i))) - u(-T_u)}{\lambda w(i)}, \quad t(w(i)) \equiv \frac{T(w(i)) - T_u}{w(i)}. \tag{56}$$

By integrating by parts the terms featuring $\tilde{R}(w)$, equation (55) can be rewritten as

$$\frac{\partial \tilde{\mathcal{L}}(0)}{\partial m}\frac{1}{\lambda} = \tag{57}$$
$$\int_{\underline{w}}^{\overline{w}} \left[\Big(b(w)(1-T'(w)) - b_f + T'(w)\Big) w_{T'} + \big(\tau(w)w + t(w)w\big)\frac{E_{T'}}{\tilde{E}(w)}\right] k(w)\tilde{R}'(w)\mathrm{d}w$$
$$+ \int_{\underline{w}}^{\overline{w}} \left(\int_{w}^{\overline{w}} \left[1 - b(z) + \Big(b(z)(1-T'(z)) - b_f + T'(z)\Big) w_T + \big(\tau(z)z + t(z)z\big)\frac{E_T}{\tilde{E}(z)}\right] k(z)\mathrm{d}z\right)\tilde{R}'(w)\mathrm{d}w$$
$$- \int_{\overline{w}}^{\overline{w}} \left[1 - b(z) + \Big(b(z)(1-T'(z)) - b_f + T'(z)\Big) w_T + \big(\tau(z)z + t(z)z\big)\frac{E_T}{\tilde{E}(z)}\right] k(z)\mathrm{d}z \times \tilde{R}(\overline{w})$$
$$+ \int_{\underline{w}}^{\overline{w}} \left[1 - b(z) + \Big(b(z)(1-T'(z)) - b_f + T'(z)\Big) w_T + \big(\tau(z)z + t(z)z\big)\frac{E_T}{\tilde{E}(z)}\right] k(z)\mathrm{d}z \times \tilde{R}(\underline{w}).$$

Because the upper and lower bound coincide, the term on the fourth line is equal to zero. Combining the first and second lines gives

$$\frac{\partial \tilde{\mathcal{L}}(0)}{\partial m}\frac{1}{\lambda} = \tag{58}$$
$$\int_{\underline{w}}^{\overline{w}} \Bigg[\bigg(\Big(b(w)(1-T'(w)) - b_f + T'(w)\Big) w_{T'} + \big(\tau(w)w + t(w)w\big)\frac{E_{T'}}{\tilde{E}(w)}\bigg) k(w)$$
$$+ \left(\int_{w}^{\overline{w}} \left[1 - b(z) + \Big(b(z)(1-T'(z)) - b_f + T'(z)\Big) w_T + \big(\tau(z)z + t(z)z\big)\frac{E_T}{\tilde{E}(z)}\right] k(z)\mathrm{d}z\right)\Bigg]\tilde{R}'(w)\mathrm{d}w$$
$$+ \int_{\underline{w}}^{\overline{w}} \left[1 - b(z) + \Big(b(z)(1-T'(z)) - b_f + T'(z)\Big) w_T + \big(\tau(z)z + t(z)z\big)\frac{E_T}{\tilde{E}(z)}\right] k(z)\mathrm{d}z \times \tilde{R}(\underline{w}).$$

Below we use this formula to derive a number of properties of the optimal tax schedule.

Before doing so, however, we first consider the welfare impact of raising the profit tax T_f. If the profit tax is optimized, increasing it should have no impact on social welfare. This requires

$$\frac{\partial \mathcal{L}}{\partial T_f} = -\psi_f u'(c_f) + \lambda = 0, \quad \leftrightarrow \quad b_f = \frac{\psi_f u'(c_f)}{\lambda} = 1, \tag{59}$$

which follows from differentiating the Lagrangian (43) with respect to T_f and setting the resulting expression equal to zero. Equation (59) coincides with the second result from Proposition 1.

Next, we consider a reduction in the unemployment benefit $-T_u$. Unlike the profit tax, equilibrium wages and employment rates are affected by a change in the unemployment benefit, see Appendix A.1. If the unemployment benefit is optimized, a small change leaves social welfare unaffected. Therefore,

$$\frac{\partial \mathcal{L}}{\partial T_u} = \int_0^1 \Big[-\psi(i) \int_{G^{-1}(E(i))}^{\overline{\varphi}} u'(-T_u)\mathrm{d}G(\varphi) + \lambda(1-E(i)) \Big] h(i)\mathrm{d}i, \tag{60}$$

$$+ \int_0^1 \Big[\psi(i) \int_{\underline{\varphi}}^{G^{-1}(E(i))} u'(w(i) - T(w(i)) - \varphi)\mathrm{d}G(\varphi)(1 - T'(w(i)))$$

$$- E(i)\psi_f u'(c_f) + \lambda E(i) T'(w(i)) \Big] w_{T_u}(i) h(i)\mathrm{d}i$$

$$+ \int_0^1 \Big[\psi(i) \Big[u(w(i) - T(w(i)) - G^{-1}(E(i))) - u(-T_u) \Big] + \lambda(T(w(i)) - T_u) \Big] E_{T_u}(i) h(i)\mathrm{d}i = 0.$$

which is obtained from differentiating equation (43) with respect to T_u, taking into account the impact of a higher T_u on equilibrium wages and employment rates. Here, $E_{T_u}(i)$ is as defined in equation (37) and $w_{T_u}(i) = E_{T_u}(i) \times a(i) y''(h(i)E(i))h(i) < 0$.

To proceed, divide equation (60) by λ and impose the definition for $\hat{b}(i)$, b_f, and the define the social welfare weight of the unemployed as

$$b_u = \frac{\int_0^1 \psi(i)(1 - E(i)) u'(-T_u) h(i)\mathrm{d}i}{\lambda \int_0^1 (1 - E(i)) h(i)\mathrm{d}i}. \tag{61}$$

Substitution of equation (61) into equation (60) gives:

$$\int_0^1 (1 - b_u)(1 - E(i))h(i)\mathrm{d}i + \int_0^1 \Big[\hat{b}(i)(1 - T'(w(i))) - b_f + T'(w(i)) \Big] w_{T_u}(i) E(i) h(i)\mathrm{d}i \tag{62}$$

$$+ \int_0^1 \Big[\psi(i) \frac{u(w(i) - T(w(i)) - G^{-1}(E(i))) - u(-T_u)}{\lambda} + (T(w(i)) - T_u) \Big] E_{T_u}(i) h(i)\mathrm{d}i = 0.$$

Next, note that the first term equals $(1 - b_u)\omega_u$. For the second and third term, apply the same change in indexation of variables from i to w as before, and substitute the definitions of the social welfare weight $b(w)$, the union wedge $\tau(w)$, the employment rate $\tilde{E}(w)$ at wage w, the participation tax rate $t(w)$, and the optimal profit tax $b_f = 1$ to find:

$$\omega_u(1 - b_u) + \int_{\underline{w}}^{\overline{w}} (b(w) - 1)(1 - T'(w)) w_{T_u} k(w)\mathrm{d}w \tag{63}$$

$$+ \int_{\underline{w}}^{\overline{w}} \left[\tau(w) w + t(w) w \right] \frac{E_{T_u}}{\tilde{E}(w)} k(w)\mathrm{d}w = 0,$$

where again we dropped the function arguments of the behavioral responses w_{T_u} and E_{T_u} to avoid additional notation.

To derive the first result from Proposition 1, consider equation (58). If the tax function $T(\cdot)$ is optimized, it must be that the welfare impact of perturbing the tax function in *any* direction

$\tilde{R}(w)$ equals zero. Therefore, the term on the last line must be equal to zero. Imposing $b_f = 1$ then yields

$$\int_{\underline{w}}^{\overline{w}} \left[1 - b(w) + (b(w) - 1)(1 - T'(w))w_T + \big(\tau(w)w + t(w)w\big)\frac{E_T}{\tilde{E}(w)}\right]k(w)\mathrm{d}w = 0. \tag{64}$$

Suppose that there are no income effects at the union level, cf. Assumption 3. In that case, for each worker type, $w_T = -w_{T_u}$ and $E_T = -E_{T_u}$. Combining equations (63) and (64) then gives:

$$\omega_u(1 - b_u) = \int_{\underline{w}}^{\overline{w}} (b(w) - 1)k(w)\mathrm{d}w. \tag{65}$$

Rearranging leads to the first result from Proposition 1.

To arrive at the final result from Proposition 1, consider again equation (58) and substitute $b_f = 1$. As mentioned before, if the tax function $T(\cdot)$ is optimized, the welfare impact of perturbing it in *any* direction $\tilde{R}(w)$ must be equal to zero. By the fundamental lemma of the calculus of variations, it follows that the term below the integral sign that is multiplied by $\tilde{R}'(w)$ equals zero:

$$\begin{aligned}&\left[(b(w) - 1)(1 - T'(w))w_{T'} + (t(w) + \tau(w))\frac{wE_{T'}}{\tilde{E}(w)}\right]k(w)\\&+ \int_{w}^{\overline{w}} \left[(1 - b(z)) + (b(z) - 1)(1 - T'(z))w_T + (t(z) + \tau(z))\frac{zE_T}{\tilde{E}(z)}\right]k(z)\mathrm{d}z = 0,\end{aligned} \tag{66}$$

which must hold for each $w \in [\underline{w}, \overline{w}]$. Evaluate this result at $w = w'$ and changing the index of integration from z to w, equation (66) coincides with equation (8) from Proposition 1 after making the substitutions for the wage and employment elasticities with respect to an increase in the marginal tax rate and tax burden.

To obtain an intuitive derivation of this result in the spirit of Saez (2001), consider Figure 16. The black, dotted (red, solid) line shows the tax schedule $T(w)$ before (after) the tax reform. The reform increases the marginal tax rate in the small interval $[w', w' + \Delta]$ by an amount equal do $\mathrm{d}T'$. As a result, the tax burden for individuals with earnings above $w' + \Delta$ increases by an amount $\Delta\mathrm{d}T'$. Such a reform has three welfare-relevant effects. First, the tax burden *mechanically* increases for employed individuals with earnings above $w'+\Delta$. As a result, income is transferred from these workers to the government. This mechanical effect is captured by the first term below the integral sign on the second line of equation (66). Second, for individuals with earnings above $w' + \Delta$, a higher tax burden raises the equilibrium wage and lowers the equilibrium employment rate, cf. equation (35). The welfare effects of a higher tax liability on wages and employment are captured by the second and third term (proportional to w_T and E_T, respectively) below the integral sign on the second line of equation (66). The wage effect reflects the distributional impact of a higher wage if the tax burden increases; a higher wage redistributes income from firms to workers and the government. The term associated with the employment effect reflects the impact of higher tax burdens on participation distortions. Third, the increase in the marginal tax rate by an amount $\mathrm{d}T'$ in the interval $[w', w' + \Delta]$ generates lower equilibrium wages and higher equilibrium employment rates for individuals within this

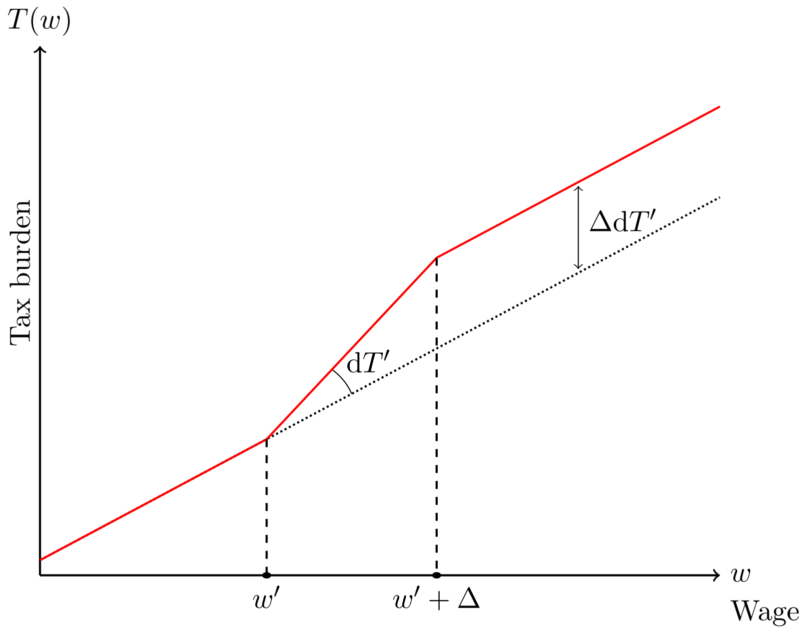

Figure 16: Tax perturbation approach

interval, which are captured by the terms on the first line of equation (66). In particular, a reduction in the negotiated wage redistributes income from workers and the government to firm-owners for individuals within this interval. Moreover, a higher marginal tax rate leads to higher equilibrium employment rates for individuals within this interval, in line with the wage-moderating effect of a higher marginal tax rate, cf. equation (36). Both effects are proportional to the density $k(w)$ of the income distribution, which determines for how many individuals the marginal tax rate is increased. Equating to zero the sum of the welfare-relevant effects leads to equation (66).

A.3 Desirability of unions

To study whether unions are desirable, suppose there is an increase in union power $\rho(i)$ for workers who are employed in sector i at wage $w(i)$. The increase in union power generates a (local) increase in the wage $w(i)$ and a (local) reduction in the employment rate $E(i)$, cf. equation (38). To determine the welfare impact of a local increase in union power, differentiate the Lagrangian (43) with respect to $\rho(i)$. With a slight abuse of notation, this gives

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \rho(i)} &= h(i)\Bigg(w_\rho\Bigg[\psi(i)\int_{\underline{\varphi}}^{G^{-1}(E(i))} u'(w(i)-T(w(i))-\varphi)\mathrm{d}G(\varphi)(1-T'(w(i))) - \psi_f u'(c_f)E(i) \\ &+ \lambda E(i)T'(w(i))\Bigg] + E_\rho\Bigg[\psi(i)(u(w(i)-T(w(i))-G^{-1}(E(i))) - u(-T_u)) + \lambda(T(w(i))-T_u)\Bigg]\Bigg) \\ &= \lambda E(i)h(i)\Bigg[w_\rho(b(w(i))-1)(1-T'(w(i))) + \frac{E_\rho}{\tilde{E}(w(i))}\Big(\tau(w(i))w(i)+t(w(i))w(i)\Big)\Bigg], \end{aligned} \tag{67}$$

where we used the property $b_f = 1$ and substituted the definitions for the social welfare weight $b(w)$, the union wedge $\tau(w)$, and the participation tax rate $t(w)$. Furthermore, $w_\rho > 0$ and

$E_\rho < 0$ capture the impact of a local increase in union power on the equilibrium wage and employment rate.

A local increase in union power at wage w positively affects social welfare if and only if equation (67) is positive. Because $\lambda E(i)h(i) > 0$, the latter requires

$$w_\rho(b(w)-1)(1-T'(w)) + \frac{E_\rho}{\tilde{E}(w)}\Big(\tau(w)w + t(w)w\Big) > 0. \quad (68)$$

To proceed, note that we can write $E_\rho = E_w w_\rho$, where E_w is the slope of the labor-demand equation (2). The latter can be obtained from implicitly differentiating $w = ay'(hE)$. Next, define the labor-demand elasticity as $\tilde{\varepsilon}(w) = -E_w w/\tilde{E}(w) > 0$ so that $\tilde{\varepsilon}(w(i)) = \varepsilon(i)$. Equation (68) then becomes:

$$w_\rho(b(w)-1)(1-T'(w)) - w_\rho\tilde{\varepsilon}(w)(t(w)+\tau(w)) > 0. \quad (69)$$

Dividing by $w_\rho > 0$ leads to equation (9) of Proposition 2.

B Inefficient rationing

B.1 Optimal taxation

To prove Proposition 3, we start by characterizing some properties of the general rationing schedule, which satisfies, for all values of E_i and φ_i^*

$$\int_{\underline{\varphi}}^{\varphi_i^*} e_i(E_i, \varphi_i^*, \varphi)\mathrm{d}G_i(\varphi) = E_i. \quad (70)$$

Differentiate equation (70) with respect to E_i and φ_i^* to obtain:

$$\int_{\underline{\varphi}}^{\varphi_i^*} e_{iE_i}(E_i, \varphi_i^*, \varphi)\mathrm{d}G_i(\varphi) = 1, \quad (71)$$

$$\int_{\underline{\varphi}}^{\varphi_i^*} e_{i\varphi_i^*}(E_i, \varphi_i^*, \varphi)\mathrm{d}G_i(\varphi) + e_i(E_i, \varphi_i^*, \varphi_i^*)G_i'(\varphi_i^*) = 0. \quad (72)$$

As stated in the main text, rather than deriving labor-market equilibrium explicitly for a general rationing scheme, we instead assume that income effects at the union level are absent and labor markets are independent. In this case, the equilibrium wage and employment rate in sector i depend only on union power ρ_i and the participation tax: $E_i = E_i(\rho_i, T_i - T_u)$ and $w_i = w_i(\rho_i, T_i - T_u)$. To derive the social welfare function, first use equation (70) to write

$$(1-E_i)u(-T_u) = u(-T_u) - \int_{\underline{\varphi}}^{\varphi_i^*} e_i(E_i, \varphi_i^*, \varphi)u(-T_u)dG_i(\varphi). \quad (73)$$

Consequently, the Lagrangian for maximizing social welfare is:

$$\mathcal{L} = \sum_i \psi_i N_i \left(u(-T_u) + \int_{\underline{\varphi}}^{\varphi_i^*} e_i(E_i, \varphi_i^*, \varphi)(u(w_i - (T_i - T_u) - T_u - \varphi) - u(-T_u))\mathrm{d}G_i(\varphi) \right)$$
$$+ \psi_f u(F(\cdot) - \sum_i w_i N_i E_i - T_f) + \lambda \left(\sum_i N_i (T_u + E_i(T_i - T_u)) + T_f - R \right). \tag{74}$$

The first-order conditions for T_u, T_f, and $T_i - T_u$ are given by:

$$T_u: \quad -\sum_i \psi_i N_i (E_i \overline{u_i'} + (1 - E_i)u_u') + \lambda \sum_i N_i = 0, \tag{75}$$

$$T_f: \quad -\psi_f u_f' + \lambda = 0, \tag{76}$$

$$T_i - T_u: \quad -N_i E_i(\psi_i \overline{u_i'} - \lambda) + N_i E_i \left[\psi_i \overline{u_i'} - \psi_f u_f' \right] \frac{\partial w_i}{\partial (T_i - T_u)}$$
$$+ N_i E_i \left[\psi_i \int_{\underline{\varphi}}^{\varphi_i^*} e_{iE_i}(u_i(\varphi) - u_u)\mathrm{d}G_i(\varphi) + \lambda (T_i - T_u) \right] \frac{\partial E_i}{\partial (T_i - T_u)}$$
$$+ N_i \left[\psi_i \int_{\underline{\varphi}}^{\varphi_i^*} e_{i\varphi_i^*}(u_i(\varphi) - u_u)\mathrm{d}G_i(\varphi) \right] \frac{\partial \varphi_i^*}{\partial (T_i - T_u)} = 0. \tag{77}$$

Here, we used the assumption that labor markets are independent. The expected utility of the employed workers in sector i is given by:

$$\overline{u_i'} \equiv \int_{\underline{\varphi}}^{\varphi_i^*} \frac{e_i(E_i, \varphi_i^*, \varphi)}{E_i} u'(w_i - T_i - \varphi)\mathrm{d}G_i(\varphi), \tag{78}$$

and $u_i(\varphi) \equiv u(w_i - T_i - \varphi)$ is the utility of the worker with participation costs $\varphi \in [\underline{\varphi}, \varphi_i^*]$ who is employed in sector i.

Equations (75) and (76) lead to the first two results in Proposition 3. Next, divide equation (77) by $N_i E_i \lambda$. Define the expected utility loss of labor rationing in sector i for those workers who lose their job if the employment rate E_i is marginally reduced as:

$$\hat{\tau}_i \equiv \psi_i \int_{\underline{\varphi}}^{\varphi_i^*} e_{iE_i}(E_i, \varphi_i^*, \varphi) \left(\frac{u(w_i - T_i - \varphi) - u(-T_u)}{\lambda w_i} \right) \mathrm{d}G_i(\varphi). \tag{79}$$

Substitute equation (79) into equation (77) and use the definition of the elasticities η_{ii} and κ_{ii} to find

$$\left(\frac{t_i + \hat{\tau}_i}{1 - t_i} \right) \eta_{ii} = (1 - b_i) + (b_i - b_f)\kappa_{ii}$$
$$+ \frac{\partial \varphi_i^*}{\partial (T_i - T_u)} \frac{1}{E_i} \left[\psi_i \int_{\underline{\varphi}}^{\varphi_i^*} e_{i\varphi_i^*}(E_i, \varphi_i^*, \varphi) \frac{(u_i(\varphi) - u_u)}{\lambda} \mathrm{d}G_i(\varphi) \right]. \tag{80}$$

Next, use equation (72) to rewrite the last part of equation (80) as:

$$
\begin{aligned}
&\frac{\partial \varphi_i^*}{\partial (T_i - T_u)} \frac{1}{E_i} \left[\psi_i \int_{\underline{\varphi}}^{\varphi_i^*} e_{i\varphi_i^*}(E_i, \varphi_i^*, \varphi) \frac{(u_i(\varphi) - u_u)}{\lambda} \mathrm{d}G_i(\varphi) \right] \\
= &- \frac{\partial \varphi_i^*}{\partial (T_i - T_u)} \frac{G_i'(\varphi_i^*)}{G_i(\varphi_i^*)} \frac{\varphi_i^*}{1 - t_i} \frac{e_i(E_i, \varphi_i^*, \varphi_i^*)}{E_i / G_i(\varphi_i^*)} \\
&\times \left[\psi_i \int_{\underline{\varphi}}^{\varphi_i^*} \frac{e_{i\varphi_i^*}(E_i, \varphi_i^*, \varphi)}{\int_{\underline{\varphi}}^{\varphi_i^*} e_{i\varphi_i^*}(E_i, \varphi_i^*, \varphi) \mathrm{d}G_i(\varphi)} \left(\frac{u_i(\varphi) - u_u}{\lambda w_i} \right) \mathrm{d}G_i(\varphi) \right].
\end{aligned} \tag{81}
$$

As a final step, define the rationing wedge as

$$
\varrho_i \equiv \frac{\psi_i e_i(E_i, \varphi_i^*, \varphi_i^*)}{E_i / G_i(\varphi_i^*)} \int_{\underline{\varphi}}^{\varphi_i^*} \frac{e_{i\varphi_i^*}(E_i, \varphi_i^*, \varphi)}{\int_{\underline{\varphi}}^{\varphi_i^*} e_{i\varphi_i^*}(E_i, \varphi_i^*, \varphi) \mathrm{d}G_i(\varphi)} \left(\frac{u(w_i - T_i - \varphi) - u(-T_u)}{\lambda w_i} \right) \mathrm{d}G_i(\varphi) \tag{82}
$$

and the participation response by

$$
\gamma_i \equiv - \frac{\partial G_i(\varphi_i^*)}{\partial (T_i - T_u)} \frac{\varphi_i^*}{G_i(\varphi_i^*)}, \tag{83}
$$

where the threshold depends on the participation tax through $\varphi_i^* = w_i(\rho_i, T_i - T_u) - (T_i - T_u)$. After substituting these definitions in equation (80), we arrive at:

$$
\left(\frac{t_i + \hat{\tau}_i}{1 - t_i} \right) \eta_{ii} - \left(\frac{\varrho_i}{1 - t_i} \right) \gamma_i = (1 - b_i) + (b_i - b_f) \kappa_{ii}. \tag{84}
$$

B.2 Desirability of unions

To study the welfare effects of the reform described in Section 2, one can differentiate the Lagrangian in equation (74) with respect to T_i and T_f under the assumptions that the reform is budget neutral, and leaves wages and employment in sector i (i.e., w_i and E_i) unaffected. The welfare effect is then:

$$
\begin{aligned}
\frac{\mathrm{d}\mathcal{W}}{\lambda} &= N_i E_i (1 - b_i) \mathrm{d}T_i + (1 - b_f) \mathrm{d}T_f \\
&+ N_i E_i \left[\psi_i \frac{1}{E_i} \int_{\underline{\varphi}}^{\varphi_i^*} e_{i\varphi_i^*}(E_i, \varphi_i^*, \varphi) \left(\frac{u_i(\varphi) - u_u}{\lambda} \right) \mathrm{d}G_i(\varphi) \right] \frac{\partial \varphi_i^*}{\partial (T_i - T_u)} \mathrm{d}T_i.
\end{aligned} \tag{85}
$$

The first term reflects the (direct) change in workers' utility in sector i following the change in the participation tax, whereas the second term reflects the change in firm-owners' utility induced by a change in the profit tax. The third term reflects the utility loss due to a change in labor participation: if T_i is lowered, more workers want to participate. If some of these workers find a job and employment remains constant, then it must be that some other workers lose their jobs and thus experience a utility loss, since rationing is not fully efficient.

Under the balanced-budget assumption, we have $N_i E_i \mathrm{d}T_i + \mathrm{d}T_f = 0$. In addition, if the government can levy a non-distortionary profit tax, then $b_f = 1$. Substituting these results in equation (85), the change in social welfare can be written as:

$$\frac{\mathrm{d}\mathcal{W}}{\lambda} = N_i E_i \left(1 - b_i + \left[\psi_i \frac{1}{E_i} \int_{\underline{\varphi}}^{\varphi_i^*} e_{i\varphi_i^*}(E_i, \varphi_i^*, \varphi) \left(\frac{u_i(\varphi) - u_u}{\lambda} \right) \mathrm{d}G_i(\varphi) \right] \frac{\partial \varphi_i^*}{\partial (T_i - T_u)} \right) \mathrm{d}T_i. \quad (86)$$

Given that T_i is lowered in the policy experiment (i.e., $\mathrm{d}T_i < 0$), the welfare effect is positive provided that the term in between brackets is negative. Using the definitions for ϱ_i and γ_i from equations (82) and (83), this is the case if:

$$b_i > 1 + \left(\frac{\varrho_i}{1 - t_i} \right) \gamma_i. \quad (87)$$

The proof is completed by the observation that if the tax system is optimized, the welfare impact of the joint reform (increasing union power ρ_i, lowering T_i and raising T_f) is driven only by the increase in union power, as changes in the tax system have no impact on social welfare.

C Occupational choice

C.1 Optimal taxation

The total labor force consists of N workers who draw a vector $\boldsymbol{\varphi} \equiv (\varphi_0, \varphi_1, \cdots, \varphi_I) \in \Phi$ of participation costs according to some distribution function $G(\boldsymbol{\varphi})$. Based on this draw, each individual chooses the occupation $j \in \{0, 1, .., I\}$ according to equation (18), where occupation 0 refers to non-employment with $w_0 = \varphi_0 = 0$ and $T_0 = T_u$. Aggregate employment in sector i is denoted by E_i and total (voluntary and involuntary) unemployment is given by E_0, so that $\sum_{i=0}^{I} E_i = N$. This notation differs from what is used in the rest of the paper, where E_i is the employment *rate*. Another difference is that, unless stated otherwise, summation over i in this Appendix means summing over $i \in \{0, 1, \cdots, I\}$ instead of summing over $i \in \{1, \cdots, I\}$.

The Lagrangian for the maximization of social welfare is:

$$\mathcal{L} = N \sum_i \psi_i \int_{\Phi_i} \Big[u(-T_u) + p_i(\boldsymbol{\varphi}, T_1 - T_u, \cdots, T_I - T_u)(u(w_i - (T_i - T_u) - T_u) - u(-T_u)) \mathrm{d}G(\varphi) \Big]$$
$$+ \psi_f u(F(\cdot) - \sum_i w_i E_i - T_f) + \lambda \Big[\sum_i E_i T_i + T_f - R \Big]. \quad (88)$$

As in the previous cases, the first-order conditions with respect to T_u and T_f imply that the average social welfare weight of all workers and firm-owners equals one. The first-order condition with respect to the participation tax $T_i - T_u$ in sector i is:

$$E_i(\lambda - \psi_i \overline{u_i'}) + \lambda \sum_{j=1}^{I} E_j(\psi_j \overline{u_j'} - \psi_f u_f') \frac{\partial w_j}{\partial (T_i - T_u)} + \lambda \sum_{j=0}^{I} T_j \frac{\partial E_j}{\partial (T_i - T_u)} \quad (89)$$
$$+ \lambda N \sum_{j=1}^{I} \psi_j \int_{\Phi_j} \frac{\partial p_j(\boldsymbol{\varphi}, T_1 - T_u, \cdots, T_I - T_u)}{\partial (T_i - T_u)} (u(w_j - T_j - \varphi_j) - u(-T_u)) \mathrm{d}G(\varphi) = 0.$$

Here, we used the property that $w_0 = 0$ and $p_0 = 1$, so they are not affected by taxation. The average marginal utility of employed workers in sector i is:

$$\overline{u'_i} = \frac{N}{E_i} \int_{\Phi_i} p_i(\boldsymbol{\varphi}, T_1 - T_u, \cdots, T_I - T_u) u'(w_j - T_j - \varphi_j) \mathrm{d}G(\varphi). \tag{90}$$

The first-order condition (89) can be simplified in a number of steps. First, because $\sum_{j=0}^{I} E_j(T_1 - T_u, .., T_I - T_u) = 1$ for all tax instruments, we can differentiate both sides with respect to $T_i - T_u$:

$$\sum_{j=0}^{I} \frac{\partial E_j}{\partial(T_i - T_u)} = 0 \quad \Leftrightarrow \quad \sum_{j=1}^{I} \frac{\partial E_j}{\partial(T_i - T_u)} = -\frac{\partial E_0}{\partial(T_i - T_u)}. \tag{91}$$

Therefore, the third term on the first line of equation (89) can be simplified to:

$$\sum_{j=0}^{I} \frac{\partial E_j}{\partial(T_i - T_u)} T_j = \sum_{j=1}^{I} \frac{\partial E_j}{\partial(T_i - T_u)} T_j + \frac{\partial E_0}{\partial(T_i - T_u)} T_u = \sum_{j=1}^{I} \frac{\partial E_j}{\partial(T_i - T_u)} (T_j - T_u). \tag{92}$$

Second, for all tax instruments, aggregate employment and the employment probabilities are related through

$$N \int_{\Phi_j} p_j(\boldsymbol{\varphi}, T_1 - T_u, \cdots, T_I - T_u) \mathrm{d}G(\varphi) \equiv E_j(T_1 - T_u, \cdots, T_I - T_u). \tag{93}$$

Differentiating both sides with respect to $T_i - T_u$ and imposing that employment probabilities are zero on the boundary of Φ_j allows us to rewrite the second line of equation (89):

$$N \int_{\Phi_j} \frac{\partial p_j(\boldsymbol{\varphi}, T_1 - T_u, \cdots, T_I - T_u)}{\partial(T_i - T_u)} \mathrm{d}G(\varphi) = \frac{\partial E_j(T_1 - T_u, \cdots, T_I - T_u)}{\partial(T_i - T_u)}. \tag{94}$$

Next, multiply and divide the final term in equation (89) by $\partial E_j / \partial(T_i - T_u)$ for each j and divide the entire expression by λ to find:

$$E_i(1 - b_i) + \sum_j E_j(b_j - b_f) \frac{\partial w_j}{\partial(T_i - T_u)} \tag{95}$$

$$\sum_{j=1}^{I} \frac{\partial E_j}{\partial(T_i - T_u)} \left[(T_j - T_u) + \psi_j N \int_{\Phi_j} \frac{\partial p_j / \partial(T_i - T_u)}{\partial E_j / \partial(T_i - T_u)} \left(\frac{u(w_j - T_j - \varphi_j) - u(-T_u)}{\lambda} \right) \mathrm{d}G(\varphi) \right] = 0.$$

The union wedge with an occupational choice is defined as follows:

$$\tau_j^o = \psi_j N \int_{\Phi_j} \frac{\partial p_j / \partial(T_i - T_u)}{\partial E_j / \partial(T_i - T_u)} \left(\frac{u(w_j - T_j - \varphi_j) - u(-T_u)}{\lambda w_j} \right) \mathrm{d}G(\varphi). \tag{96}$$

Using this notation, and the definitions of the labor shares (ω_i and ω_i) and wage and employment elasticities (κ_{ji} and η_{ji}), we obtain the final result from Proposition 5.

C.2 Desirability of unions

To study the desirability of labor unions, start from the Lagrangian

$$\mathcal{L} = N\sum_i \psi_i \int_{\Phi_i} \Big[u(-T_u) + p_i(\boldsymbol{\varphi}, T_1 - T_u, \cdots, T_I - T_u)(u(w_i - (T_i - T_u) - T_u) - u(-T_u))\mathrm{d}G(\varphi)\Big]$$
$$+ \psi_f u(F(\cdot) - \sum_i w_i E_i - T_f) + \lambda\Big[\sum_i E_i T_i + T_f - R\Big]. \tag{97}$$

Equilibrium wages and employment rates depend on the participation taxes $T_i - T_u$ and union power ρ_i in all sectors. As before, we analyze a reform where union power in sector i is increased: $\mathrm{d}\rho_i > 0$. This reform puts upward pressure on the wage w_i sector i and downward pressure on employment E_i. To off-set the impact on the equilibrium wage, the reform is combined by reduction in the income tax in sector i: $\mathrm{d}T_i < 0$. This reduction, in turn, is financed by an increase in the profit tax: $\mathrm{d}T_f > 0$. The combined welfare effect is

$$\frac{\mathrm{d}\mathcal{W}}{\lambda} = E_i(1 - b_i)\mathrm{d}T_i + (1 - b_f)\mathrm{d}T_f, \tag{98}$$

which is very similar to the equation (85) except there is no welfare loss due to an inefficient allocation of jobs over workers (i.e., there is no rationing wedge). Because the reform is budget-neutral, we have $E_i\mathrm{d}T_i = -\mathrm{d}T_f$. Moreover, the social welfare weight of firm-owners equals one if the tax system is optimized: $b_f = 1$. The increase in union power ρ_i combined with a reduction in the income tax T_i financed by a higher profit tax T_f increases the net incomes of workers in sector i. The prospects of a higher net wage could induce some individuals to switch from other sectors j (possibly non-employment) to sector i. However, this is only the case for workers who are *ex ante* indifferent between choosing occupation i and their second-best alternative. Under our assumption of efficient rationing, the employment probability of these individuals is zero: $p_i(\boldsymbol{\varphi}, T_1 - T_u, \cdots, T_I - T_u) = 0$ on the boundary of Φ_i. Hence, there is no welfare effect associated with such changes. According to equation (98), a higher union power then raises social welfare if and only if $b_i > 1$.

D Bargaining over multiple wages

D.1 Labor-market equilibrium

We assume that there is one union with a utilitarian objective and denote union power by $\delta \in [0, 1]$. The union bargains with the firm-owners over the wages all sectors i. Hence, the union affects the entire wage distribution. Under Nash-bargaining, the solution for wages and employment in all sectors i follow from solving the following maximization problem:

$$\max_{\{w_i,E_i\}_{i\in\mathcal{I}}} \Omega \;=\; \delta \log\left(\sum_i N_i \int_{\underline{\varphi}}^{G_i^{-1}(E_i)} (u(w_i - T_i - \varphi) - u(-T_u))\mathrm{d}G_i(\varphi)\right)$$
$$+ (1-\delta)\log\left(u(F(K, E_1N_1, \cdots, E_IN_I) - \sum_i w_iN_iE_i - T_f) - u(F(K, 0, \cdots, 0) - T_f)\right)$$
$$\text{s.t. } w_i - F_i(K, E_1N_1, \cdots, E_IN_I) = 0, \quad \forall i,$$
$$G_i(w_i - T_i + T_u) - E_i \geq 0, \quad \forall i. \tag{99}$$

The payoffs of both parties are taken in deviation from the payoff associated with the disagreement outcome. The Lagrangian is:

$$\mathcal{L} =\delta \log\left(\sum_i N_i \int_{\underline{\varphi}}^{G_i^{-1}(E_i)} (u(w_i - T_i - \varphi) - u(-T_u))\mathrm{d}G_i(\varphi)\right)$$
$$+(1-\delta)\log\left(u(F(K, E_1N_1, \cdots, E_IN_I) - \sum_i w_iN_iE_i - T_f) - u(F(K, 0, \cdots, 0) - T_f)\right)$$
$$+\sum_i \vartheta_i(w_i - F_i(K, E_1N_1, \cdots, E_IN_I)) + \sum_i \mu_i(G_i(w_i - T_i + T_u) - E_i). \tag{100}$$

The first-order conditions are:

$$w_i : \frac{\delta}{\sum_j N_jE_j(\overline{u_j} - u_u)} N_iE_i\overline{u_i'} - \frac{1-\delta}{u_f - \underline{u}_f} N_iE_i\overline{u_f'} + \vartheta_i + \mu_iG_i' = 0, \tag{101}$$
$$E_i : \frac{\delta}{\sum_j N_jE_j(\overline{u_j} - u_u)} N_i(\hat{u}_i - u_u) - N_i\sum_j \vartheta_jF_{ji} - \mu_i = 0, \tag{102}$$
$$\vartheta_i : w_i - F_i = 0, \tag{103}$$
$$\mu_i : G_i - E_i = 0. \tag{104}$$

where $\underline{u}_f \equiv u(F(K, 0, \cdots, 0) - T_f)$. These conditions characterize labor-market equilibrium, which has the following properties.

First, if the union has zero bargaining power ($\delta = 0$), the equilibrium coincides with the competitive outcome (i.e., $G_i = E_i$ and $w_i = F_i$ for all i). To see why, substitute $\delta = 0$ in the first-order conditions for w_i and E_i in equations (101) and (102). Next, use (101) to substitute for ϑ_i in equation (102) and rearrange:

$$\mu_i\underbrace{(N_iG_i'F_{ii} - 1)}_{<0} + \; N_i\sum_{j\neq i}\mu_j\underbrace{G_j'F_{ji}}_{\geq 0} = N_i\underbrace{\frac{u_f'}{u_f - \underline{u}_f}}_{>0}\underbrace{\sum_j N_jE_jF_{ji}}_{=-F_{Ki}K<0}. \tag{105}$$

The inequalities follow from the assumptions of co-operant factors of production and constant returns to scale. Non-increasing marginal productivity and co-operant factors of production imply $F_{ii} \leq 0 \leq F_{ji}$, whereas constant returns to scale implies $\sum_j N_jE_jF_{ji} = -F_{Ki}K \leq 0$.[26]

[26]This follows from differentiating $F(\cdot) = F_K(\cdot)K + \sum_j N_jE_jF_j(\cdot)$ with respect to E_ℓ.

Suppose that there is a sector in which $G_i > E_i$, i.e., the wage is above the market-clearing level. Then, from the Kuhn-Tucker conditions, it must be that $\mu_i = 0$. Because of the non-negativity of all multipliers, however, equation (105) cannot be satisfied unless all labor types would be perfect substitutes, i.e., $F_{ii} = F_{ij} = F_{Ki} = 0$ for all i, j. This is a contradiction. Therefore, $G_i = E_i$ for all i if $\delta = 0$.

Second, if the union has sufficiently high bargaining power δ, there is at least one sector i for which the wage exceeds the market-clearing level, i.e., there exists a sector i such that $G_i > E_i$. To see why, suppose $\delta = 1$. In this case, the union is a monopoly union, and sets wages in order to maximize the expected utility of all workers, subject to the labor-demand equations $w_i = F_i(K, E_1 N_1, \cdots, E_I N_I)$. Consequently, the union objective can be written as:

$$\Lambda = \sum_i N_i \int_{\underline{\varphi}}^{G_i^{-1}(E_i)} (u(F_i(K, E_1 N_1, \cdots, E_I N_I) - T_i - \varphi) - u(-T_u))\mathrm{d}G_i(\varphi). \tag{106}$$

Now, suppose that, starting from the competitive equilibrium where $G(F_i - T_i - T_u) = E_i$ for all i, the union considers reducing the employment rate in the sector ℓ where the marginal utility of workers' consumption is highest (i.e., $\overline{u'_\ell} > \overline{u'_j}$ for all $j \neq \ell$). This reduction in employment increases the wage of the workers with the highest marginal utility of consumption and reduce the wages for all other workers. The impact of a reduction in employment in sector ℓ on the union's objective is:

$$\mathrm{d}\Lambda = N_\ell \sum_j N_j E_j \overline{u'_j} F_{j\ell} \times \mathrm{d}E_\ell = N_\ell \left(N_\ell E_\ell F_{\ell\ell} \overline{u'_\ell} + \sum_{j \neq \ell} N_j E_j F_{j\ell} \overline{u'_j} \right) \mathrm{d}E_\ell. \tag{107}$$

This expression can be thought of as summing a weighted average of marginal utilities, with weights $N_j E_j F_{j\ell}$. The first term in brackets is negative (because $F_{\ell\ell} < 0$), whereas the second term in brackets is positive (because $F_{j\ell} \geq 0$ for all $j \neq \ell$). The first term unambiguously dominates the second term. This is because the weights sum to less than zero (constant returns to scale implies $\sum_j N_j E_j F_{j\ell} = -F_{Ki} K \leq 0$) and the only negative component (i.e., $N_\ell E_\ell F_{\ell\ell}$) is multiplied by the largest marginal utility (i.e., $\overline{u'_\ell} > \overline{u'_j}$ for all $j \neq \ell$). Consequently, the union objective unambiguously increases if – starting from the competitive equilibrium – the rate of employment for workers in the sector with the lowest wage is reduced (i.e., $\mathrm{d}E_\ell < 0$). Hence, a monopoly union ($\delta = 1$) always demands a wage above the market-clearing level in at least one sector.

D.2 Optimal taxation

In the absence of income effects and under the assumption that firm-owners are risk-neutral, the first-order conditions in equations (101) and (104) characterize equilibrium wages and employment rates as a function the participation tax rates: $w_i = w_i(T_1 - T_u, .., T_1 - T_u)$ and $E_i = E_i(T_1 - T_u, .., T_1 - T_u)$.[27] These reduced-form equations can be used to derive the optimal tax formulas. This case is identical to the one with multiple unions, which is analyzed

[27]Risk-neutrality of firm-owners ensures that equilibrium wages and employment rates do not depend on the profit tax.

in the main text. The optimal tax formulas (written in terms of elasticities) therefore remain unaffected.

D.3 Desirability of unions

To study the desirability of a national union, we analyze the welfare effects of a joint marginal increase in union power δ combined with a tax reform, such that all labor-market outcomes are unaffected. If the tax system is optimized, any change in social welfare must then necessarily be the result of the change in union power.

Which tax reform offsets any impact of the increase in union power on equilibrium wages and employment? First, the tax reform cannot include a change in the participation tax for workers whose wage is at the market-clearing level. To see why, consider the labor-market equilibrium condition in a sector i where the wage is at the market-clearing level:

$$G_i(F_i(\cdot) - (T_i - T_u)) = E_i. \tag{108}$$

A change in the participation tax in this sector needs to be accompanied by a change in either $F_i(\cdot)$ or E_i. For this to be the case, employment in at least one sector i needs to adjust. However, the tax change is intended keep employment in all sectors unaffected. Hence, in sectors where $G_i = E_i$ it must be the case that $\mathrm{d}(T_i - T_u) = 0$. The tax reform thus changes income taxes in all sectors j where the wage is set above above the martket-clearing level, i.e., where $G_i > E_i$. The marginal tax reform should then satisfy:

$$\forall i \in k(\delta): \quad \sum_{j \in k(\delta)} \frac{\partial w_i(T_1 - T_u, \cdots, T_I - T_u, \delta)}{\partial T_j} \mathrm{d}T_j^* + \frac{\partial w_i(T_1 - T_u, \cdots, T_I - T_u, \delta)}{\partial \delta} \mathrm{d}\delta = 0. \tag{109}$$

Here, $k(\delta) \equiv \{i : G_i > E_i\}$ is the set of sectors where the wage is raised above the market-clearing level. As before, assume that the government adjusts the profit tax to keep the budget balanced. Since the combined increase in union power δ and the tax reform $\mathrm{d}T_j^*$ for all j leaves all labor-market outcomes unaffected, there is only a transfer of resources from firm-owners to the workers whose wage is higher than the market-clearing level (i.e., for whom $G_i > E_i$). The welfare effect is thus equal to:

$$\frac{\mathrm{d}\mathcal{W}}{\lambda} = \sum_{i \in k(\delta)} N_i E_i (1 - b_i) \mathrm{d}T_i^*, \tag{110}$$

where λ is the multiplier on the government budget constraint. Divide the latter by $\sum_i N_i > 0$. The remaining term is positive if and only if

$$\sum_{i \in k\delta} \omega_i (1 - b_i) \mathrm{d}T_i^* > 0. \tag{111}$$

E Efficient bargaining

E.1 Derivation elasticities

Partial equilibrium in labor market i is obtained by combining the contract curve from equation (26) and the rent-sharing rule from equation (27):

$$\overline{u'(w_i(1-t_i)-T_u-\varphi)}(w_i-F_i(E_i)) = u(w_i(1-t_i)-T_u-G_i^{-1}(E_i))-u(-T_u), \tag{112}$$

$$w_i = (1-\upsilon_i)F_i(E_i)+\upsilon_i\phi_i(E_i). \tag{113}$$

Unlike before, here we directly express our results in terms of participation tax rates t_i, as opposed to levels $T_i - T_u$. This has no implications for the main insights. In the absence of income effects, these equations define $E_i = E_i(t_i)$ and $w_i = w_i(t_i)$. As before, the absence of income effects implies a change in T_u does not affect equilibrium wages and employment if the participation tax rate t_i remains constant. Hence, the derivative of equation (112) with respect T_u, while keeping t_i constant, is zero:

$$-\overline{u''(w_i(1-t_i)-T_u-\varphi)}(w_i-F_i(E_i)) = -u'(w_i(1-t_i)-T_u-G_i^{-1}(E_i))+u'(-T_u). \tag{114}$$

To derive the elasticities of employment and wages with respect to the participation tax rate, we first linearize the rent-sharing rule:

$$\frac{\mathrm{d}w_i}{w_i} = -\left((1-m_i)\frac{(1-\upsilon_i)}{\varepsilon_i}+m_i\right)\frac{\mathrm{d}E_i}{E_i}, \tag{115}$$

where $m_i \equiv (w_i - F_i)/w_i = 1 - F_i/w_i$ is the implicit subsidy on labor demand, as a fraction of the wage. If union power is zero, $\upsilon_i = 0$, $m_i = 0$, and equation (115) reduces to the linearized labor-demand equation.

Second, linearizing the contract curve yields:

$$\frac{\mathrm{d}\overline{u_i'}}{\overline{u_i'}} + \frac{\mathrm{d}(w_i-F_i)}{w_i-F_i} = \frac{\mathrm{d}(\hat{u}_i-u_u)}{\hat{u}_i-u_u}. \tag{116}$$

Using equation (114), the linearized sub-parts are given by:

$$\frac{\mathrm{d}\overline{u_i'}}{\overline{u_i'}} = \frac{\overline{u_i''}w_i(1-t_i)}{\overline{u_i'}}\left(\frac{\mathrm{d}w_i}{w_i}-\frac{\mathrm{d}t_i}{1-t_i}\right)+\frac{(\hat{u}_i'-\overline{u_i'})}{\overline{u_i'}}\frac{\mathrm{d}E_i}{E_i}, \tag{117}$$

$$\frac{\mathrm{d}(w_i-F_i)}{w_i-F_i} = \frac{1}{m_i}\left(\frac{\mathrm{d}w_i}{w_i}+\frac{(1-m_i)}{\varepsilon_i}\frac{\mathrm{d}E_i}{E_i}\right), \tag{118}$$

$$\frac{\mathrm{d}(\hat{u}_i-u_u)}{\hat{u}_i-u_u} = \frac{\hat{u}_i'w_i(1-t_i)}{(\hat{u}_i-u_u)}\left(\frac{\mathrm{d}w_i}{w_i}-\frac{\mathrm{d}t_i}{1-t_i}\right)-\frac{\hat{u}_i'E_i}{G_i'(\hat{\varphi}_i)(\hat{u}_i-u_u)}\frac{\mathrm{d}E_i}{E_i}. \tag{119}$$

Solving for the relative changes in employment and wages yields:

$$\frac{\mathrm{d}E_i}{E_i}=\frac{-u_u' w_i(1-t_i)}{\frac{\hat{u}_i' E_i}{G_i'(\hat{\varphi}_i)}+u_u' w_i(1-t_i)\left(\frac{(1-m_i)(1-\upsilon_i)}{\varepsilon_i}+m_i\right)+(\hat{u}_i-u_u)\left(\frac{(1-m_i)}{m_i}\frac{(1-\upsilon_i)}{\varepsilon_i}-1+\frac{(\hat{u}_i'-\overline{u_i'})}{\overline{u_i'}}\right)}\frac{\mathrm{d}t_i}{1-t_i}, \tag{120}$$

$$\frac{\mathrm{d}w_i}{w_i}=\frac{u_u' w_i(1-t_i)\left(\frac{(1-m_i)(1-\upsilon_i)}{\varepsilon_i}+m_i\right)}{\frac{\hat{u}_i' E_i}{G_i'(\hat{\varphi}_i)}+u_u' w_i(1-t_i)\left(\frac{(1-m_i)(1-\upsilon_i)}{\varepsilon_i}+m_i\right)+(\hat{u}_i-u_u)\left(\frac{(1-m_i)}{m_i}\frac{(1-\upsilon_i)}{\varepsilon_i}-1+\frac{(\hat{u}_i'-\overline{u_i'})}{\overline{u_i'}}\right)}\frac{\mathrm{d}t_i}{1-t_i}. \tag{121}$$

The elasticities are now as given in Proposition 9.

E.2 Optimal taxation

Start with the Lagrangian for the maximization of social welfare if the government has utilitarian preferences:[28]

$$\begin{aligned}\mathcal{L}=&\sum_i N_i\left(\int_{\underline{\varphi}}^{G_i^{-1}(E_i)}u(w_i(1-t_i)-T_u-\varphi)\mathrm{d}G_i(\varphi)+\int_{G_i^{-1}(E_i)}^{\overline{\varphi}}u(-T_u)\mathrm{d}G_i(\varphi)\right)\\&+u(F(\cdot)-\sum_i w_iN_iE_i-T_f)+\lambda\left(\sum_i N_i(T_u+E_it_iw_i)+T_f-R\right).\end{aligned} \tag{122}$$

Differentiating with respect to T_u, T_f, and t_i yields:

$$T_u:-\sum_i N_iE_i\overline{u_i'}-\sum_i N_i(1-E_i)u_u'+\lambda\sum_i N_i=0, \tag{123}$$

$$T_f:-u_f'+\lambda=0, \tag{124}$$

$$\begin{aligned}&t_i:-N_iE_iw_i(\overline{u_i'}-\lambda)+\frac{\partial E_i}{\partial t_i}\left(N_i(\hat{u}_i-u_u)+u_f'N_i(F_i-w_i)+\lambda N_it_iw_i\right)\\&+\frac{\partial w_i}{\partial t_i}\left(N_iE_i\overline{u_i'}(1-t_i)-N_iE_iu_f'+\lambda N_iE_it_i\right)=0.\end{aligned} \tag{125}$$

The first two expressions from Proposition 9 are obtained by dividing equation (123) by $\lambda\sum_i N_i$ and equation (124) by λ, and imposing the definitions of the social welfare weights $b_i\equiv\overline{u'(c_i)}/\lambda$, $b_u\equiv u'(c_u)/\lambda$ and the employment shares $\omega_i\equiv N_iE_i/\sum_j N_j$ and $\omega_u\equiv\sum_i N_i(1-E_i)/\sum_j N_j$. The second result can be found by dividing equation (124) by λ and using $b_f\equiv u'(c_f)/\lambda$. The expression for the optimal participation tax rate t_i is obtained by substituting $u_f'=\lambda$ in equation (125) and dividing the expression by $N_iE_i\lambda w_i$. After imposing the definitions of the union wedge $\tau_i\equiv\frac{u(\hat{c}_i)-u(c_u)}{\lambda w_i}$, the mark-up $m_i=\frac{w_i-F_i}{w_i}$ and the elasticities κ_{ii} and η_{ii} as defined in equations (32)–(33), we arrive at the final expression stated in Proposition 9.

[28]It is straightforward to add Pareto weights ψ_i and adjust the definitions of the social welfare weights accordingly. This has no implications for our results.

E.3 Desirability of unions

To determine how a change in union power υ_i affects social welfare, we formulate the Lagrangian by taking the labor-market equilibrium conditions explicitly into account:

$$\begin{aligned}
\mathcal{L} =& \sum_i N_i \left(\int_{\underline{\varphi}}^{G_i^{-1}(E_i)} u(w_i(1-t_i) - T_u - \varphi)\mathrm{d}G_i(\varphi) + \int_{G_i^{-1}(E_i)}^{\overline{\varphi}} u(-T_u)\mathrm{d}G_i(\varphi) \right) \\
&+ u(F(\cdot) - \sum_i w_i N_i E_i - T_f) + \lambda \left(\sum_i N_i(T_u + E_i t_i w_i) + T_f - R \right) \\
&+ \sum_i \vartheta_i N_i (w_i - (1-\upsilon_i)F_i(\cdot) - \upsilon_i \phi_i(\cdot)) \\
&+ \sum_i \mu_i N_i \Bigg(\int_{\underline{\varphi}}^{G_i^{-1}(E_i)} u'(w_i(1-t_i) - T_u - \varphi)\mathrm{d}G_i(\varphi)(F_i(\cdot) - w_i) \\
&+ E_i(u(w_i(1-t_i) - T_u - G_i^{-1}(E_i)) - u(-T_u)) \Bigg). \qquad (126)
\end{aligned}$$

To determine how a change in the union power affects social welfare, differentiate the Lagrangian with respect to υ_i, and apply the envelope theorem:

$$\frac{\partial \mathcal{W}}{\partial \upsilon_i} = \frac{\partial \mathcal{L}}{\partial \upsilon_i} = N_i \vartheta_i (F_i - \phi_i). \qquad (127)$$

Because the production function $F(\cdot)$ is concave in E_i, $w_i - F_i = \upsilon_i(\phi_i(\cdot) - F_i(\cdot)) > 0$ if $\upsilon_i > 0$. Hence, $\frac{\partial \mathcal{L}}{\partial \upsilon_i}$ is positive if and only if $\vartheta_i < 0$. To determine the sign of ϑ_i ,use the first-order conditions of the Lagrangian with respect to t_i, w_i and T_f:

$$\begin{aligned}
t_i :& - w_i N_i E_i(\overline{u_i'} - \lambda) - \mu_i w_i N_i E_i \left(\overline{u_i''}(F_i - w_i) + \hat{u}_i' \right) = 0, \qquad (128) \\
w_i :&(1-t_i)N_i E_i \overline{u_i'} - N_i E_i u_f' + \lambda t_i N_i E_i + \vartheta_i N_i \\
&+ \mu_i (1-t_i) N_i \left(E_i \overline{u_i''}(F_i - w_i) + E_i \hat{u}_i' \right) - \mu_i N_i E_i \overline{u_i'} = 0, \qquad (129) \\
T_f :& - u_f' + \lambda = 0. \qquad (130)
\end{aligned}$$

Combining equations (128) and (129) and substituting equation (130) yields:

$$\vartheta_i = \mu_i E_i \overline{u_i'}. \qquad (131)$$

Substituting for μ_i using equation (128) and simplifying gives:

$$\vartheta_i = E_i \left(\frac{\lambda \overline{u_i'}(1-b_i)}{\overline{u_i''}(F_i - w_i) + \hat{u}_i'} \right). \qquad (132)$$

From equations (127) and (132), it follows that an increase in υ_i increases social welfare if and only if the term on the right-hand side of expression (132) is negative:

$$b_i > 1. \qquad (133)$$