Redistribution and employment in frictional labor markets*

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Abstract

This paper analyzes optimal taxation when search frictions generate unemployment and labor supply responds along both the intensive (hours) and extensive (participation) margin. I characterize optimal tax rules in terms of the observable income distribution and sufficient statistics. Search frictions have an ambiguous effect on optimal taxes. High marginal tax rates reduce wage pressure, which lowers unemployment. High average taxes have the opposite effect. The first (second) of these effects calls for higher (lower) tax rates.

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1 Introduction

The question how progressive the income tax should be is among the most important ones in public economics. Formulated in its modern form, the first successful attempt in answering this question was provided by Mirrlees (1971). He recognized that, due to the unobservable nature of individual earnings capacity, the government faces an adverse selection problem when it uses income taxes to achieve its distributional goals. In his model, individuals differ in terms of their (unobservable) ability and supply labor on the intensive margin to a perfectly competitive labor market. The income tax distorts the income-leisure choice, and the associated efficiency costs should optimally be weighed against the distributional benefits from income taxation.

The seminal analysis of Mirrlees (1971) has subsequently been extended in many directions, but the assumption of a perfectly competitive labor market has gone largely uncontested. This assumption, however, is arguably more than an innocuous abstraction. From a theoretical perspective, the consequences of tax progressivity are very different in models with competitive labor markets than in models with frictions. The former point to the detrimental effects of tax progressivity on the incentives to supply labor, whereas the latter predict that higher tax progressivity actually boosts employment, typically through a wage-moderating effect.[1] If, for instance, wages are determined through bargaining, an increase in the marginal tax rate induces workers or unions to bargain less aggressively, which lowers wages and increases employment. Lehmann et al. (2016) find strong empirical support for this prediction in a panel of OECD-countries, which led them to conclude that "...policy-makers should not only focus on the detrimental effects of tax progressivity on in-work effort, but also consider the employment-enhancing effects" (p.455). In this paper, I aim to answer the question how income taxes should be designed if both these forces are present.

To do so, I analyze a model with a frictional labor market in which homogeneous firms compete for workers with different abilities. Workers derive utility from consumption and disutility from providing effort. Firms post vacancies, which specify how much output an employee is expected to produce and the income he or she receives as compensation. Output, in turn, is a function of effort and ability. After observing the posted vacancies, workers optimally choose where to apply. Importantly, because workers cannot coordinate their application strategies, they face a trade-off not only between income and leisure but also between income and the probability of finding a job. Under standard assumptions on preferences and the matching technology, there exists a separating equilibrium in which workers with different ability levels apply at different vacancies. Furthermore, since workers cannot insure against the risk of becoming unemployed, the equilibrium is Pareto efficient only if workers are risk-neutral.

There is a government which aims to maximize social welfare. It levies a non-linear tax on labor income and (possibly) on the share of the match surplus which accrues to firms (henceforth a profit tax) to finance a uniform unemployment benefit. The tax instruments affect the entry decisions by firms, the terms of trade in the posted vacancies, and workers' incentives to provide effort. Crucially, the model predicts that an increase in the marginal income tax reduces working

[1] This is a robust finding across different models with equilibrium unemployment. It holds in the context of matching frictions (Pissarides, 1998), union bargaining (Hersoug, 1984 and Koskela and Vilmunen, 1996), efficiency wages (Pisauro, 1991), and minimum wages with rationing on the extensive margin.

hours, but increases employment. Intuitively, a high marginal tax rate reduces the value of an additional unit of gross income, and thereby induces workers to substitute labor income for both (i) more leisure and (ii) a higher probability of finding a job. Alternatively, paying higher wages becomes a less effective tool for firms to attract applicants if marginal taxes are high. Consequently, a rise in tax progressivity moderates wages and boosts employment. The first of these effects is identical to the standard substitution effect associated with income taxation in competitive labor markets, while the second captures the employment-enhancing (or wage-moderating) effect of tax progressivity typically present in models with frictions.

How should taxes be optimally designed if both these forces are present? First, I show how (un)employment responses modify standard optimal tax rules if the government cannot tax profits. To do so, I derive a sufficient-statistics formula for the optimal marginal tax rate at each point in the income distribution. Compared to the presentation in Saez (2001), the presence of frictions modifies the optimal tax expression in two ways. First, an increase in the *marginal* tax rate boosts employment through the wage-moderating effect. In the typical case that an increase in employment improves the government's budget, this effect calls for higher marginal tax rates. Second, a rise in the *average* tax rate reduces workers' net income and thereby induces firms to post higher wages, which reduces employment. If the reduction in employment negatively affects the government's budget, this second effect calls for lower marginal tax rates so as to prevent average tax rates from rising quickly.

Second, I show that if the government can observe match-specific output (and hence, levy a non-linear profit tax), the expression for the optimal income tax is exactly the same as in an economy without frictions. Put differently, the optimal tax formula from Mirrlees (1971) and Diamond (1998) is not affected by (un)employment responses. Intuitively, in a competitive search environment, both income and profit taxes are equally distortive for the workers' application and firms' entry decisions. However, as in an economy without frictions, only the income tax (and not the profit tax) distorts the income-leisure choice. Consequently, it is optimal to weigh the distributional benefits from income taxation only against distortions in the income-leisure choice (as in Mirrlees, 1971). Profit taxes are then used exclusively to alleviate distortions from income taxes on application decisions and vacancy creation.

1.1 Contributions to the literature

This paper relates to several branches in the literature. First, it contributes to the literature on optimal Mirrleesian taxation by extending the Mirrlees (1971) framework with search frictions. To the best of my knowledge, the only papers to have done so are Boone and Bovenberg (2004, 2013) and, recently, Sleet and Yazici (2017). In the first of these, workers engage in costly search activities, but not every worker manages to find a job. Because tax policy cannot condition on (unobservable) search effort, the optimal income tax balances the distortions on the search margin with those on in-work effort. Sleet and Yazici (2017) analyze optimal income taxation in a Burdett and Mortensen (1998) type frictional labor market with on and off the job search. Search frictions give rise to income and profit dispersion among *ex ante* identical workers and firms, and these effects modify optimal tax formulas. Contrary to my model, both frameworks do not feature an entry decision by firms, nor an application decision by workers.

Consequently, the job-finding (and hence, employment) rate is unaffected by tax policy. By contrast, the interaction between tax policy and (un)employment lies at the heart of the current paper. In particular, I show if and how optimal tax formulas are modified to take into account (un)employment responses.

Second, this paper is closely related to a recent literature on optimal income taxation with search frictions, but which abstracts from the (Mirrleesian) distortion from income taxation on labor supply. Hungerbühler et al. (2006) and subsequent extensions analyze a model in which search is segmented by skill, working hours are fixed and wages are determined through bargaining.[2] An increase in tax progressivity reduces wages and boosts employment. Optimal policy balances the distributional benefits from income taxes against distortions in the bargaining outcome. Contrary to their paper, my model also features an income-leisure choice *à la* Mirrlees (1971) to capture the distortion from tax progressivity on labor supply. Furthermore, I assume wage posting rather than bargaining.[3] Golosov et al. (2013) also use a framework with wage posting to study optimal income taxation and unemployment insurance. Workers are identical and risk-averse, while firms differ in their productivity. As in my model, the government optimally provides an unemployment benefit because workers face an uninsurable risk of becoming unemployed. The income tax is used to redistribute only between workers who apply for the same type of jobs, of whom only a fraction is successful. By contrast, because in my model workers differ in their productivity, the government also redistributes between workers who apply for different jobs. Finally, like Hungerbühler et al. (2006), Golosov et al. (2013) abstract from labor supply considerations, whereas in my model distortions on in-work effort and labor force participation critically affect the pattern of optimal income taxes.

Third, I contribute to the literature on competitive search (see for a recent overview Wright et al., 2017). In particular, I show that standard efficiency properties (as in, e.g., Moen, 1997) generalize to a setting with worker heterogeneity in productivity and intensive-margin labor supply, provided workers are risk-neutral. As is well understood, competitive search equilibria fail to be efficient if workers are risk-averse and cannot privately insure themselves against the risk of not finding a job (see, e.g., Acemoglu and Shimer, 1999, Jacquet and Tan, 2012, Golosov et al., 2013 and Geromichalos, 2015). In an extension, I show how efficiency is restored if workers have access to a competitive insurance market. Furthermore, I study how the labor-market outcomes are affected by different aspects of the the tax-benefit system. The model yields sharp predictions regarding the impact of taxes and benefits, and captures both the standard distortion from income taxation on incentives to supply labor as well as the wage-moderating effect of tax progressivity typically present in models with frictions. Finally, I characterize optimal tax policy and the optimal provision of unemployment benefits under the assumption that individuals' earnings capacity and application strategies are not observable.

[2]In particular, their framework has been extended to include a participation margin (Lehmann et al., 2011), proportional (rather than Nash) bargaining (Jacquet et al., 2014) and a search effort decision (Schaal and Taschereau-Dumouchel, 2014). In addition, the model has been used to study optimum minimum wage policies (Hungerbühler and Lehmann, 2009).

[3]This difference is more than semantics. By assuming wage posting, I do not need to make a rather *ad hoc* assumption on the worker's bargaining power (cf. Hosios, 1990) to obtain an efficient outcome in the absence of taxation and a motive for redistribution. Also, I do not need to impose *ex ante* that search is segmented by skill. Instead, in my model workers with different productivity levels endogenously choose to apply for different jobs.

2 Environment and equilibrium

The economy is populated by a unit mass of individuals, which are also referred to as workers. Each individual is endowed with one (divisible) unit of time and receives a draw from a productivity distribution $F(n)$ with density $f(n)$ and support $[n_0, n_1]$. An individual's productivity (or ability) is private information and equals the number of consumption units he or she produces per unit of time if matched with a firm. Hence, if an individual with ability n works ℓ hours, he or she produces $y = n\ell$ consumption units. Individuals derive utility from consumption c and disutility from labor effort ℓ according to a separable utility function

$$u(c) - v(\ell), \tag{1}$$

where $u(\cdot)$ is increasing, weakly concave and $v(\cdot)$ is increasing and strictly convex. For technical convenience, I normalize $v(0) = 0$ and assume $u(0)$ is bounded from below.

On the firms' side, there is a large pool of potential entrants. Upon entry, a firm can post vacancies at unit cost $k > 0$. A vacancy (y, z) specifies (i) how much output y an employee is expected to produce, and (ii) the income z he or she receives as compensation. If a firm is matched with a worker, it earns a gross profit margin $y - z$. Any profits (or losses) flow back to the individuals through a mutual fund.[4] Note that, because a vacancy specifies income and output, it is irrelevant from the firm's perspective whether the vacancy is filled by a high-productivity employee who works few hours, or by a less productive worker who has to work more hours to produce the same amount of output.[5]

Firms and workers interact through frictional labor markets. In particular, I assume all workers can apply at most once and cannot coordinate their application strategies. As a result, some workers and firms may remain unmatched. How many matches are generated depends on the matching technology, which is assumed to feature constant returns to scale. Under this assumption, the job-finding probability or equivalently the employment rate p (the number of matches divided by the number of job-seekers) depends only on labor-market tightness θ, defined as the ratio of vacancies to job-seekers. I denote the inverse of the relationship by $\theta = \theta(p)$, where $\theta(\cdot)$ is increasing, convex, has a non-decreasing elasticity and satisfies $\theta(0) = 0$.[6] These assumptions guarantee a decreasing relationship between the job-finding rate p and the probability that a vacancy is filled, as given by $p/\theta(p)$. Hence, the more difficult it is for workers to find a job, the easier is it for firms to get their vacancies filled, and vice versa.

There is a government which levies a labor income tax $T(z)$ and provides a (uniform) benefit b to all non-employed individuals. In addition, I allow for the possibility that the profit margin $y - z$ is subject to a tax $\tau(\cdot)$. Because firms incur vacancy creation costs, the profit tax is not a tax on pure economic profits. Rather, it is a tax levied on the firm's share of the match surplus. To implement this tax, the government therefore needs to observe match-specific output y. Clearly this is a very strong informational requirement, especially if a firm hires more than one

[4]Since free entry ensures aggregate profits are zero in equilibrium, the distribution of ownership is irrelevant.

[5]This would be different if firms post vacancies which specify income and working hours. If the firm cannot observe productivity, this gives rise to an adverse selection problem: see Guerrieri et al. (2010). Stantcheva (2014) analyzes optimal income taxation in such a framework, though in her model search is not frictional.

[6]These assumptions are fairly standard in the literature and hold for most commonly employed matching functions (e.g., urn-ball matching, telephone-line matching, Cobb-Douglas matching).

worker. I therefore also analyze the case where profits cannot be taxed.

Firms continue to post vacancies until profits are zero in expectation:

$$k = \frac{p}{\theta(p)}(y - z - \tau(y - z)). \tag{2}$$

The cost of creating a vacancy (on the left-hand side) equals the probability the vacancy is filled, multiplied by the net profit margin (on the right-hand side). Provided the marginal profit tax does not exceed one, the above condition constitutes a decreasing relationship between the profit margin and the probability that a vacancy is filled. From the workers' perspective, there thus exists a trade-off between the 'attractiveness' of a vacancy (which decreases in the profit margin $y - z$) and the likelihood of finding a job (which increases in the profit margin $y - z$).

Workers maximize their expected utility by optimally choosing where to apply. Each worker incurs a (utility) cost of searching, which I denote by $\delta \geq 0$. If an individual of type n decides to search, he or she solves:

$$U(n) = \max_{z,y,p} \left\{ p(u(z - T(z)) - v(y/n)) + (1-p)u(b) \quad \text{s.t.} \quad (2) \right\} \tag{3}$$

Here, I used the normalization $v(0) = 0$ and substituted out for consumption $c = z - T(z)$ and labor $\ell = y/n$. Using (3), an individual of type n participates if

$$U(n) - \delta \geq u(b). \tag{4}$$

I denote the participation threshold by $n^* \geq n_0$, which is assumed to satisfy $n^* < n_1$. Hence, there are always some individuals who find it profitable to search.

Before turning to characterize the solution to the above maximization problem, two remarks are in place. First, the dual formulation to (3) is obtained by letting firms choose which vacancies to post in order to maximize profits, subject to a utility requirement. The utility requirement, in turn, is pinned down by the zero-profit condition. This *market utility* representation is more common in the literature (see Wright et al., 2017 and numerous references therein), but the current (and equivalent) exposition is more convenient when turning to the problem of optimal taxation. Second, for future purposes it is useful to think of (3) as a standard consumer problem. The consumer (in this case, the worker) derives utility from three commodities: consumption, leisure and the probability of finding a job. The zero-profit condition (2) then acts as a budget constraint and determines which combination of these commodities the consumer can afford.

I assume the tax system is such that for the workers who decide to participate, the first-order conditions associated with the maximization problem (3) are both necessary and sufficient.[7] Combining the first-order conditions for income z and output y yields:

$$\frac{v'(y/n)}{nu'(z - T(z))} = 1 - T'(z). \tag{5}$$

This condition is identical to the standard labor-supply equation in a competitive equilibrium without vacancy creation costs. In this case, free entry implies $z = y$. In the optimum, the

[7] In the absence of taxation, the assumptions on $u(\cdot)$, $v(\cdot)$ and $\theta(\cdot)$ guarantee this is indeed the case.

marginal rate of substitution between output and consumption (on the left-hand side) equals the marginal rate of transformation between income and consumption (on the right-hand side). Intuitively, competition between firms forces them to post vacancies in which, at the margin, the productivity of labor is equal to the households' willingness to substitute between income and leisure. As in an economy without frictions, the marginal income tax affects the relative price between consumption and leisure and thereby generates distortions in labor supply.

The second optimality condition is obtained by combining the first-order conditions for income z and job-finding p:

$$\frac{\Delta(z, y/n)}{\chi(p)} = \frac{1 - T'(z)}{1 - \tau'(y - z)}. \tag{6}$$

Here, $\Delta(z, y/n) \equiv (u(z - T(z)) - v(y/n) - u(b))/u'(z - T(z))$ denotes the utility gain a worker of type n experiences when he or she becomes employed, expressed in consumption units. Furthermore, $\chi(p) \equiv (\theta'(p) - \theta(p)/p)k$ measures the difference between the marginal and average costs of getting a vacancy filled. It is best thought of as a congestion externality. Firms, when deciding to post a vacancy, do not take into account that it becomes more difficult for other firms to fill their vacancies. On the other hand, firms also do not internalize the utility gain a worker experiences when a match is formed. The first (second) effect causes excessive (too little) vacancy creation. Condition (6) then states that in the absence of taxation, the effects cancel out and vacancy creation is undistorted. This is why directed search models are often said to 'internalize' the Hosios (1990) efficiency condition. However, because taxes distort the relative price between income and job-finding, this is no longer true once the government levies income and profit taxes. I will discuss the impact of the different tax instruments in much greater detail in the next section.

Combined, the two first-order conditions (5)-(6) and the zero-profit condition (2) constitute a system of three equations in three unknowns. Denote the solution by $z(n)$, $y(n)$ and $p(n)$ respectively. Now, consider the following definition:

Definition 1. *A **competitive search equilibrium** consists of a participation threshold n^*, levels of income $z(n)$, output $y(n)$ and employment rates $p(n)$ for all $n \geq n^*$, such that, for a given system of taxes and benefits $(T(\cdot), \tau(\cdot), b)$:*

(i) $n^* \geq n_0$ and $U(n^*) - \delta \geq u(b)$ with complementary slackness,

(ii) firms make zero profits in expectation: (2),

(iii) individuals optimize: (5), (6).

(iv) The tax-benefit system is such that the government's budget balances:

$$\int_{n^*}^{n_1} p(n)\Big(T(z(n)) + \tau(y(n) - z(n)) + b\Big) f(n)\mathrm{d}n = b. \tag{7}$$

Conditions (i)-(iii) restate that workers optimize and firms make zero profits in expectation. Condition (iv), in turn, states that benefit payments to all individuals (on the right-hand side) are equal to the total payments collected from all matches, including the foregone payments in unemployment benefits (on the left-hand side).

The following figures graphically illustrate the equilibrium in the absence of taxes and benefits. Figure 1 shows the trade-off between income z and output y individuals with different

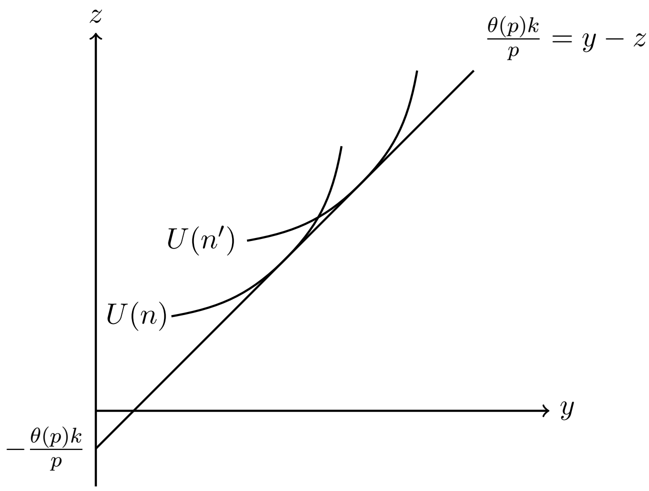

Figure 1: Trade-off between income and leisure

abilities face, for a given employment probability p. Here, the productivity levels satisfy $n' > n$. The zero-profit condition is given by the straight line and acts as the budget constraint. In the optimum, individuals choose to apply at the vacancy where their indifference curve is tangent to the zero-profit condition. Since it is less costly for individuals with a higher ability to produce output, the indifference curve of type n' is flatter than that of type n. Hence, for a given job-finding rate, an individual with a higher productivity applies for a vacancy which specifies both higher income and higher output.

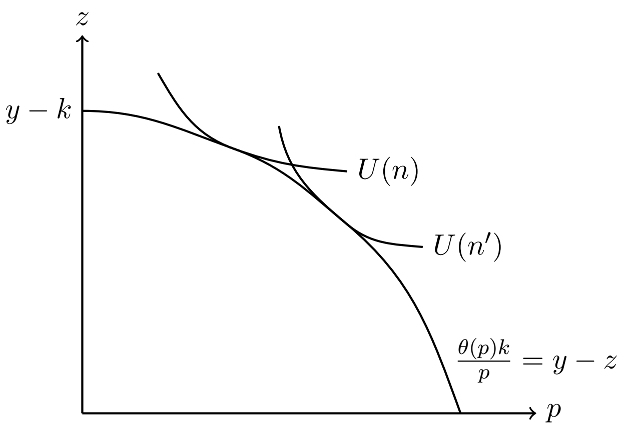

Figure 2: Trade-off between income and job-finding

Figure 2, in turn, illustrates the trade-off between income z and the probably of finding a match (or equivalently, the employment rate) p, holding output y fixed. The zero-profit condition is now represented by the curve bending outward from the origin. Again, individuals choose the point of tangency between their indifference curve and the zero-profit condition. Because individual n' experiences a larger utility gain from finding a job, his or her indifference curve is steeper. Consequently, for a given output, an individual with a higher ability applies for a job which specifies a lower income, where he or she faces a higher probability of being matched with a firm.

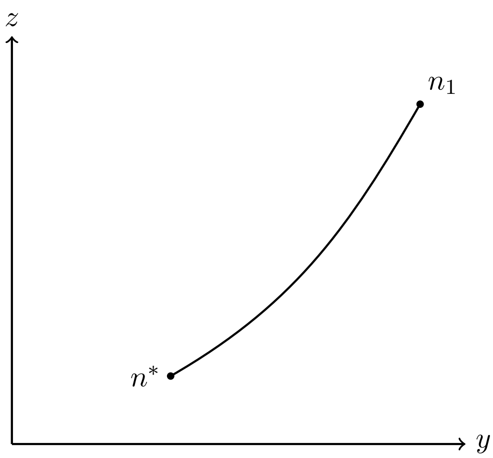

Figure 3: Equilibrium vacancies

Finally, Figure 3 shows which vacancies are offered in equilibrium. This presentation differs from the one in Figure 1 because it accounts for the endogeneity of the employment rate. In Appendix A.2, it is shown that income, output and the employment rate are all increasing in n. Intuitively, high-ability individuals find it less costly (in utility terms) to produce a certain amount of output. Consequently, output increases in productivity. By standard concavity arguments, individuals find it optimal to use the additional output to 'buy' both more income (i.e., consumption), and a higher job-finding probability. Individuals thus sort themselves into different sub-markets according to their productivity, as illustrated in Figure 3. Hence, even though all participants are active in a single labor market, individuals with different abilities find it optimal to apply at different vacancies.

3 Constrained efficiency and comparative statics

Is the competitive search equilibrium in the absence of taxes and benefits (constrained) efficient? Put differently, starting from the laissez-faire outcome and given the presence of frictions, is it possible to construct a feasible Pareto improvement? The answer to these questions is given in the next Proposition.

Proposition 1. *Suppose there are no taxes and benefits:* $T(\cdot) = \tau(\cdot) = b = 0$*. The competitive search equilibrium is efficient if and only if individuals are risk-neutral:* $u''(\cdot) = 0$.

Proof. See Appendix A.1 □

Proposition 1 shows that standard efficiency properties from competitive search models (see, e.g., Moen, 1997) generalize to a setting with worker heterogeneity in productivity and intensive-margin labor supply, but only if individuals are risk-neutral. The terms of trade in the posted vacancies thus play an allocative role similar to the role of prices in an economy without frictions. The reason why the equilibrium is inefficient when individuals are risk-averse is straightforward:

idiosyncratic unemployment risk is – by assumption – not privately insurable.[8] Efficiency then only prevails if individuals do not value insurance, which is the case if they are risk-neutral.

The result from Proposition 1 has an important implication for the optimal design of taxes and benefits: if individuals are risk-averse, the tax-benefit system not only serves to redistribute income from high-ability to low-ability individuals, but also to (partly) correct for the missing insurance market. To strictly separate these issues, I allow for the possibility that individuals are risk-neutral and add a perfectly competitive insurance market in one of the extensions (which restores even if individuals are risk-averse).

How are the equilibrium outcomes affected by the tax-benefit system? To answer this question, I consider perturbations of the income tax $T(\cdot)$, the profit tax $\tau(\cdot)$ and the unemployment benefit b. As opposed to the unemployment benefit, changing the income and profit tax requires perturbing a function rather than a parameter. To that end, I introduce the following functions:

$$T^*(z, \kappa_T) = T(z) + \kappa_T R_T(z) \tag{8}$$

$$\tau^*(y - z, \kappa_\tau) = \tau(y - z) + \kappa_\tau R_\tau(y - z) \tag{9}$$

and replace the income and profit tax in the optimization problem of workers (3) by $T^*(\cdot)$ and $\tau^*(\cdot)$, respectively. The functions $R_T(\cdot)$ and $R_\tau(\cdot)$ specify the direction in which the income and profit tax are perturbed, and the parameters κ_T and κ_τ govern the magnitude of the perturbation. Using these definitions, the impact of a tax reform can be studied by differentiating the equilibrium outcomes with respect to the reform parameters κ_T and κ_τ, evaluated at the reform of interest (see, e.g., Gerritsen, 2016 and Jacquet and Lehmann, 2017). For instance, the impact of changing the intercept of the income tax can be analyzed by setting $R_T(z) = 1$. Similarly, the impact of increasing the marginal tax rate while keeping the average tax rate at income level $z(n)$ unaffected can be analyzed by setting $R_T(z) = z - z(n)$.

I analyze how changing the tax-benefit system affects equilibrium income $z(n)$, output $y(n)$ and the employment rate $p(n)$ of individuals with ability n who decide to participate. In addition to changing the unemployment benefit, I consider a change in both (i) the slope and (ii) the intercept of the income and profit tax. The first of these are compensated reforms as they only affect relative prices, and thereby generate substitution effects. Reforms of the second type do not affect relative prices and generate income effects. Furthermore, I analyze how the equilibrium outcomes vary with ability n. The results are given in the next proposition.

Proposition 2. *Starting from the laissez-faire equilibrium, Table 1 summarizes how the tax-benefit system and productivity affect the equilibrium outcomes. Here, T' and τ' denote the slope of the income and profit tax, whereas T and τ denote the level. Furthermore, output is unaffected (i.e. the $=$ applies) if and only if individuals are risk-neutral: $u''(\cdot) = 0$.*

Proof. See Appendix A.2 □

An increase in the marginal income tax reduces income and output, and raises employment.

[8]The absence of an insurance market can be micro-founded by assuming application strategies are not observable. In that case, it is impossible for workers who apply for the same type of jobs to write contracts insuring them against the risk of applying unsuccessfully, and firms cannot commit to make payments to applicants who are not hired (as in Jacquet and Tan, 2012).

Table 1: Comparative statics

	T'	T	τ'	τ	b	n
income (z)	-	+	+	-	+	+
output (y)	-	+/=	-/=	+/=	-/=	+
employment (p)	+	-	-	-	-	+

Note: A $+/=/-$ indicates that the column variable has a positive/zero/negative impact on the row variable.

Intuitively, a higher marginal tax raises the relative price of consumption and induces individuals to substitute away from consumption (i.e., apply for jobs which specify a lower income) towards more leisure and a higher job-finding rate. The first of these effects is identical to the standard substitution effect in labor supply. The second effect, in turn, is reminiscent of the wage-moderating effect of tax progressivity typically present in models with equilibrium unemployment. Turning to the second reform, an increase in the intercept of the income tax ceteris paribus reduces consumption. To attenuate this drop, individuals apply for higher-income jobs which reduces their chances of being matched (hence, employment decreases). Furthermore, if the marginal utility of consumption is strictly decreasing, the reduction in net income generates an income effect in labor supply, so that output increases.

Turning to the impact of profit taxes, first note that – unlike the marginal income tax – the marginal profit tax only affects the trade-off between income and the probability of being matched and not between income and leisure (compare equations (5)-(6)). If the marginal profit tax is high, a rise in earnings hardly affect firms' net profits hence, their incentives to post vacancies. If the marginal profit tax rises, firms therefore post higher wages, which depresses employment. High *levels* of profit taxes, on the other hand, directly reduce incentives to post vacancies, which lowers wages and depresses employment. In both cases, there may be an effect on output through income effects in labor supply.

Unemployment benefits provide (partial) insurance against the risk of not finding a job. Therefore, a rise in the benefit level induces individuals to apply for higher-income jobs. As a result, income rises and employment falls. The increase in income might furthermore generate a reduction in output if there are income effects in labor supply. Finally – as mentioned before – income, output and job-finding all increase in productivity. Intuitively, more productive individuals use some of the additional output they produce to 'buy' both more income (i.e., consumption) and a probability of being matched.

Crucially, the model captures both the standard effects of taxation in competitive markets, as well as the impact on (un)employment typically present in models with frictions. Both types of responses turn out to play a crucial role in the characterization of the optimal tax-benefit system. More specifically, to determine the distortionary costs of income taxes, two effects are of particular interest: the impact of the marginal income tax on income z and the job-finding rate p. Denote the corresponding elasticities by:

$$\varepsilon_{zT'} \equiv -\frac{\mathrm{d}z}{\mathrm{d}T'}\frac{1-T'}{z} > 0, \quad \varepsilon_{pT'} \equiv \frac{\mathrm{d}p}{\mathrm{d}T'}\frac{1-T'}{p} > 0, \tag{10}$$

where the signs follow from Proposition 2. The first of these – commonly referred to as the elasticity of taxable income (ETI) – captures both the standard intensive-margin response *and* the wage-moderating effect of tax progressivity. This second effect has implications for (un)employment as well, as captured by the second elasticity. Now, consider the following.

Corollary 1. *In the absence of taxes and benefits, if individuals are risk-neutral and the matching elasticity is constant, the elasticity of taxable income (ETI) equals:*

$$\varepsilon_{zT'} = \hat{\varepsilon}_{zT'} + \sigma(1-\mu). \tag{11}$$

Here, $\hat{\varepsilon}_{zT'} \equiv v'(y/n)n/(v''(y/n)z)$ *is the ETI in competitive labor markets without vacancy creation costs (in which case* $z = y$*),* $\sigma \equiv \theta(p)k/(pz)$ *denotes the expected costs of getting a vacancy filled as a fraction of earnings, and* $\mu \equiv \theta(p)/(p\theta'(p))$ *is the matching elasticity, which measures the percentage increase in the job-finding rate if labor-market tightness increases by one percent. Furthermore, the elasticity of the job-finding rate is:*

$$\varepsilon_{pT'} = \mu. \tag{12}$$

Proof. See Appendix A.2.1 □

Corollary 1 illustrates that, under some additional (though fairly standard) assumptions on preferences and the matching technology, the ETI can be split up in two components: the 'standard' intensive-margin response and a wage-moderation effect. In the absence of vacancy creation costs, $\sigma = 0$ and the second effect is absent. If, on the other hand, there are search frictions, taxable income responds more strongly because firms post lower wages. This effect is stronger the more severe are frictions (i.e., the higher σ) and the less responsive is the job-finding rate to an increase in the number of vacancies (i.e., the lower μ). Conversely, an increase in tax progressivity generates a stronger increase in the employment rate if the latter is more responsive to an increase in labor-market tightness, as illustrated by (12).

The result from Corollary 1 can also be used to think about the quantitative importance of the wage-moderating effect of tax progressivity, and its implications for employment. As an illustration, suppose the ETI equals 0.25 (Saez et al., 2012), the matching elasticity is 0.5 (Petrongolo and Pissarides, 2001), and the costs of getting a vacancy filled equal 10% of total earnings. With these numbers, about one-fifth of the total response in taxable income can be attributed to frictions. If the unemployment rate is 0.05, a matching elasticity of 0.5 furthermore implies that the rate of unemployment is reduced by almost 10% if the retention rate (one minus the marginal tax rate) increases by 1%. These numbers, however, should be interpreted with great caution: they can be very different under more realistic assumptions on preferences, the matching technology and the existing tax-benefit system.

4 Optimal taxation (i): restricted problem

I now turn to address the question how tax policy should optimally be designed. Doing so requires a specification of the welfare criterion. For this purpose, I define

$$\mathcal{W} \equiv \int_{n_0}^{n^*} \Psi(u(b))f(n)\mathrm{d}n + \int_{n^*}^{n_1} \Psi(U(n)-\delta)f(n)\mathrm{d}n, \tag{13}$$

where $u(b)$ denotes the utility of the non-participants and $U(n)$ is as defined in (3). The function $\Psi(\cdot)$ is assumed to be increasing and weakly concave. Strict concavity in either $u(\cdot)$ or $\Psi(\cdot)$ generates a motive for redistribution, which is absent only if individuals are risk-neutral and the government is utilitarian: $u''(\cdot) = \Psi''(\cdot) = 0$.

In this section, I make the critical assumption that individual match surplus y is not observable. Consequently, the profit margin cannot be taxed: $\tau(\cdot) = 0$. The government's budget constraint (7) then reads:

$$\int_{n^*}^{n_1} p(n)(T(z(n)) + b)f(n)\mathrm{d}n = b. \tag{14}$$

The government optimally chooses $T(\cdot)$ and b to maximize social welfare (13) subject to the budget constraint (14), taking into account the behavioral responses to the tax-benefit system (see Proposition 2). I characterize the solution to this problem using the tax-perturbation approach.[9] In particular, I consider two perturbations of the tax function $T(z)$ and leave a more intuitive characterization of the optimal benefit level b to be derived the next section. The first reform raises the tax liability of all employed individuals by an amount $\mathrm{d}T(0) > 0$. This reform is best thought of as an increase in the intercept $T(0)$ of the tax function. The second reform, in turn, raises the marginal tax rate on a small interval $[z, z+\omega]$ by an amount $\mathrm{d}T'(z) > 0$. Optimal tax formulas are then derived from the notion that, if the tax system is optimal, a small change in the tax system should have no impact on social welfare.

Following Saez (2001), I express optimal tax formulas in terms of the (observable) income rather than the (unobservable) ability distribution. I denote the former by $H(\cdot)$, and the corresponding density by $h(\cdot)$. The income and ability distribution are related through $H(z(n)) = 1 - \int_n^{n_1} p(m)f(m)\mathrm{d}m$. In words, the fraction of the population with income below $z(n)$ equals one minus the fraction of individuals whose ability exceeds n and who succeed in finding a job (hence the multiplication with the employment rate $p(\cdot)$). I denote the highest income level by $z_1 = z(n_1)$ and the lowest by $z^* = z(n^*)$. Concerning the participation threshold n^*, I assume δ is sufficiently high (or equivalently, n_0 sufficiently low) so that full participation is not socially optimal: $n^* > n_0$. This assumption is relaxed in the next section.

Starting with the first reform, what are the welfare effects associated with increasing the tax intercept? First, this reform transfers resources from all working individuals to the government budget. The direct welfare effect is:

$$\eta \mathrm{d}T(0) \times \int_{z^*}^{z_1} (1-g(z))h(z)\mathrm{d}z. \tag{15}$$

[9] For a recent and detailed discussion of this approach, see Gerritsen (2016). How the approach is related to the mechanism-design approach used in the next section is explained in detail in Jacquet and Lehmann (2017).

Here, η is the multiplier on the government budget constraint and measures the (shadow) value of an additional unit of public resources. In addition, $g(z) = \Psi'(\bar{U}(z))u'(z - T(z))/\eta$ is the welfare weight the government attaches to individuals who earns income z, and whose expected utility net of search costs is $\bar{U}(z)$.[10]

A higher tax liability also generates income effects in earnings z, job finding p and (possibly) output y (see Proposition 2). By the Envelope theorem, these responses have no direct effect on individuals' expected utility. However, changes in earnings and employment do affect the government's budget. Denote the corresponding income effects by $\zeta_z \equiv \mathrm{d}z/\mathrm{d}T > 0$ and $\zeta_p \equiv -(\mathrm{d}p/\mathrm{d}T)/p > 0$. Using these definitions, the impact on welfare is:

$$\eta \mathrm{d}T(0) \times \int_{z^*}^{z_1} (\zeta_z T'(z) - \zeta_p (T(z) + b))h(z)\mathrm{d}z. \tag{16}$$

Intuitively, a change in earnings has a budgetary effect equal to the marginal tax rate $T'(z)$, whereas a change in employment affects the budget by an amount $T(z) + b$.

Furthermore, with an interior participation threshold n^*, a change in the intercept of the tax function affects the participation decision of individuals who are marginally indifferent between participation and non-participation. Again, the participation response has no direct impact on the expected utility of the individuals whose decisions are affected. There is, however, an impact on the government budget. The associated welfare effect is:

$$-\eta \mathrm{d}T(0) \times \pi(T(z^*) + b). \tag{17}$$

Here, $\pi \equiv h(z^*)\mathrm{d}z^*/\mathrm{d}T > 0$ captures the change in participation resulting from a change in the tax liability.[11] The latter is multiplied by the participation tax $T(z^*) + b$, which measures the change in government revenue if a (marginally indifferent) individual decides to participate and is successful in finding a job.

Turning to the second reform, a change in the marginal tax rate generates substitution effects in earnings, job finding and output (see Proposition 2). The measure of individuals whose marginal tax rate is increased is given by $\omega h(z)$. Again, by the Envelope theorem these behavioral responses have no first-order effect on individuals' expected utility. The budgetary effects generate a welfare effect equal to:

$$\eta\omega \mathrm{d}T'(z) \times zh(z) \left(-\varepsilon_{zT'} \frac{T'(z)}{1 - T'(z)} + \varepsilon_{pT'} \frac{(T(z) + b)/z}{1 - T'(z)} \right). \tag{18}$$

As before, changes in earnings are multiplied by $T'(z)$ and changes in employment by $T(z) + b$ to capture the impact on the government budget.

The reform also increases the tax liability of individuals who earn an income above $z + \omega$.

[10]Formally, $\bar{U}(z) \equiv U(z^{-1}(z)) - \delta$, where $z^{-1}(\cdot)$ is the inverse of $z(n) = z$.

[11]The response equals $\mathrm{d}z^*/\mathrm{d}T = \mathrm{d}z/\mathrm{d}n \times \mathrm{d}n^*/\mathrm{d}T$, where $\mathrm{d}n^*/\mathrm{d}T$ can be determined by calculating $\mathrm{d}n^*/\mathrm{d}\kappa_T$ from the threshold constraint $p(u(z - T(z) - \kappa_T R_T(z) - v(y/n^*) - u(b)) = \delta$ using the implicit function theorem, evaluated at the reform $R_T(z) = 1$.

For ω small, the associated welfare effect is:

$$\eta\omega \mathrm{d}T'(z) \times \int_{z}^{z_1} (1 - g(\tilde{z}) + \zeta_z T'(\tilde{z}) - \zeta_p (T(\tilde{z}) + b))h(\tilde{z})\mathrm{d}\tilde{z}. \tag{19}$$

Equation (19) captures both the mechanical as well as the revenue effects resulting from income effects: see (15) and (16). For this purpose, it is useful to define

$$\alpha(z) \equiv g(z) - \zeta_z T'(z) + \zeta_p (T(z) + b) \tag{20}$$

as the welfare weight adjusted for income effects. Intuitively, $\alpha(z)$ measures the total welfare effect of giving an additional unit of income to an individual who earns income z. Now, consider the following result.

Proposition 3. *Suppose there are no profit taxes and suppose there is non-participation in the social optimum: $\tau(\cdot) = 0$ and $n^* > n_0$. If the tax-benefit is optimal, the participation tax satisfies:*

$$\int_{z^*}^{z_1} (1 - \alpha(z))h(z)\mathrm{d}z = \pi(T(z^*) + b). \tag{21}$$

In addition, for all $z \in [z^*, z_1]$ the following condition must hold:

$$zh(z)\left(\varepsilon_{zT'}\frac{T'(z)}{1 - T'(z)} - \varepsilon_{pT'}\frac{(T(z) + b)/z}{1 - T'(z)}\right) = \int_{z}^{z_1} (1 - \alpha(\tilde{z}))h(\tilde{z})\mathrm{d}\tilde{z}. \tag{22}$$

Proof. Add equations (15), (16) and (17) and set the resulting expression equal to zero. Using the definition (20), this leads to (21). Similarly, add equations (18) and (19) and set the resulting expression equal to zero. This immediately implies (22). □

Both conditions in Proposition 3 balance distributional benefits of taxation against the distortionary costs. First, consider the expression for the optimal participation tax (21). Raising the intercept of the tax function redistributes resources from all working individuals (who earn at least z^*) to the government budget, both directly and indirectly through income effects. The left-hand side captures the benefits of doing so. The right-hand side captures the distortionary costs. An increase in the intercept of the tax function reduces participation, which has a budgetary effect proportional to the participation tax $T(z^*) + b$. Naturally, the optimal participation tax is higher the higher are the distributional benefits, and the lower is the participation response π (see also Diamond, 1980).

With full participation and in the absence of an unemployment benefit (either for exogenous reasons, or because individuals are risk-neutral and do not value unemployment insurance), condition (21) is replaced by:

$$\int_{z_0}^{z_1} (1 - \alpha(z))h(z)\mathrm{d}z = 0. \tag{23}$$

Here, the lowest income is denoted by $z_0 = z(n_0)$. This result states that the government continues to provide a lump-sum transfer (here, equal to $-T(0)$) until the average welfare weight of the recipients (in this case, the individuals who find a job) is equal to one. If, on the other hand, the government optimally provides (partial) unemployment insurance, the average welfare weight of the working falls short of one. In this case, taxing workers uniformly generates distributional benefits because it allows the government to finance an unemployment benefit.

Expression (22) equates the distributional benefits of raising the marginal tax rate at income level z against the distortionary costs of doing so. The tax reform transfers resources from individuals who earn at least z to the government budget (again, directly and indirectly through income effects). The right-hand side gives the associated impact on welfare. The left-hand side, in turn, captures the efficiency costs of distorting relative prices. By reducing the value of an additional unit of income, a high marginal tax rate induces individuals to substitute away from consumption (i.e., apply for lower-income jobs) towards more leisure and a higher job-finding rate. The left-hand side captures the total impact on government revenue associated with these substitution effects.

To gain some further intuition for the pattern of optimal marginal taxes, rewrite (22):

$$\frac{T'(z)}{1-T'(z)} = \frac{1}{\varepsilon_{zT'}} \frac{\int_z^{z_1} (1-\hat{\alpha}(\tilde{z}) - \zeta_p(T(\tilde{z})+b))h(\tilde{z})\mathrm{d}\tilde{z}}{1-H(z)} \frac{1-H(z)}{zh(z)} + \frac{\varepsilon_{pT'}}{\varepsilon_{zT'}} \frac{(T(z)+b)/z}{1-T'(z)}. \tag{24}$$

Here, $\hat{\alpha}(z)$ refers to the welfare weight which only accounts for income effects in earnings z: $\hat{\alpha}(z) \equiv g(z) - \zeta_z T'(z)$. This presentation clearly illustrates how frictions modify the optimal tax formula. Compared to the standard formula without frictions (see, e.g., Saez, 2001 and, more recently, Gerritsen, 2016 and Jacquet and Lehmann, 2017), two additional effects show up. First, a higher marginal tax boosts employment through the wage-moderating effect ($\varepsilon_{pT'} > 0$). In the typical case that a change in employment positively affects the government's budget (i.e., if $T(z) + b > 0$), this effect calls for *higher* marginal tax rates. On the other hand, a higher average tax reduces employment ($\zeta_p > 0$). This effect modifies the optimal tax formula in a very similar fashion as a participation response: see Jacquet et al. (2013). To prevent average tax rates from rising quickly, this second effect calls for *lower* marginal tax rates. Without frictions, $p = 1$ and consequently $\varepsilon_{pT'} = \zeta_p = 0$. In this case, the optimal tax formula (22) reduces to the one which holds under perfectly competitive labor markets.

An immediate implication of (22) is that the optimal marginal tax rate is generally not zero at the top of the income distribution in frictional labor markets, as is the case in competitive labor markets (see, e.g., Sadka, 1976 and Stiglitz, 1982). Evaluating (22) at income level $z = z_1$ yields:

$$\frac{T'(z_1)}{(T(z_1)+b)/z_1} = \frac{\varepsilon_{pT'}}{\varepsilon_{zT'}}. \tag{25}$$

At the top of the income distribution, the ratio between the 'extensive' (employment) and 'intensive' (earnings) elasticity equals the inverse ratio of the associated budgetary effects. There is a clear intuition behind condition (25): because a change in the marginal tax rate does not affect individuals' expected utility, the top *marginal* tax rate $T'(z_1)$ is set to maximize total revenue collected from top-income earners. The revenue, in turn, is proportional to $p(n_1)(T(z(n_1)) + b)$, where both z and p are endogenous to the tax system. Consequently, the zero top rate prevails only if there is no wage-moderating effect of tax progressivity at the top ($\varepsilon_{pT'} = 0$), or in the atypical case that a change in employment at the top of the income distribution has no budgetary effects ($T(z_1)+b = 0$). This result bears close resemblance to Hungerbühler et al. (2006). They show that, in a model with search frictions and wage bargaining (but no intensive-margin responses), the top rate is optimally positive because it serves to generate revenue through the wage-moderation effect.

5 Optimal taxation (ii): general problem

This section analyzes optimal taxation under the assumption that the government can observe match-specific output y. Consequently, the profit tax $\tau(\cdot)$ is no longer restricted to be zero. I characterize the solution to this problem using the mechanism-design approach (introduced in Mirrlees, 1971), rather than the perturbation approach employed in the previous section.[12] Instead of optimizing over tax functions, in the mechanism-design formulation the government directly chooses an allocation, which is formally defined as follows.

Definition 2. *An* ***allocation*** $\left(n^*, c_u, [p(n), c(n), y(n)]_{n^*}^{n_1}\right)$ *specifies a participation threshold* $n^* \geq n_0$*, a level of consumption* c_u *for the unemployed and, for each individual who participates, an employment probability* $p(n)$ *and a level of consumption* $c(n)$ *and output* $y(n)$ *if the individual is employed.*

Once an allocation is specified, social welfare follows from (13) upon substituting $b = c_u$ and $U(n) = p(n)(u(c(n)) - v(y(n)/n)) + (1 - p(n))u(c_u)$. What are the constraints the government faces when designing an allocation? First, the allocation must be resource feasible:

$$\int_{n^*}^{n_1} \Big[p(n)(y(n) - c(n)) - (1 - p(n))c_u - \theta(p(n))k\Big] f(n)\mathrm{d}n - \int_{n_0}^{n^*} c_u f(n)\mathrm{d}n = 0. \tag{26}$$

In words, aggregate consumption equals aggregate output net of vacancy creation costs. Second, there is a voluntary participation constraint

$$U(n^*) - \delta \geq u(c_u), \qquad n^* \geq n_0, \tag{27}$$

which holds with complementary slackness. Finally, since ability is private information, the allocation must be incentive compatible: it should be in the best interest of each participating individual to pick the bundle $(p(n), c(n), y(n))$ designed for his or her type. This boils down to requiring, for all $n \geq n^*$:

$$n = \arg\max_{n'} \Big\{p(n')(u(c(n')) - v(y(n')/n)) + (1 - p(n'))u(c_u)\Big\}. \tag{28}$$

In the remainder, I assume the first-order condition associated with the maximization problem

[12] As it turns out, the mechanism-design approach generates results which can readily be interpreted and linked to the existing literature. On the other hand, the perturbation approach becomes more complicated with more tax instruments and behavioral responses to account for.

(28) is both necessary and sufficient. Under this assumption, (28) can be replaced by:

$$U'(n) = p(n)\frac{v'(y(n))y(n)}{n^2}. \tag{29}$$

This condition is obtained by differentiating $U(n) = p(n)(u(c(n)) - v(y(n)/n)) + (1-p(n))u(c_u)$ with respect to n under the Envelope condition (28).

It is routine to verify that the competitive search equilibrium satisfies resource feasibility (26), voluntary participation (27) and incentive compatibility (29).[13] This observation, however, does not imply that the allocation which maximizes social welfare subject to these constraints can also be decentralized using the tax-benefit system $(T(\cdot), \tau(\cdot), b)$. This is because the incentive-compatibility constraint (29) only prevents individuals from choosing a bundle intended for another type, whereas individuals can also choose a bundle not intended for *any* type when confronted with the tax-benefit system $(T(\cdot), \tau(\cdot), b)$. Such "joint deviations" can be prevented by assuming the government optimizes over a non-separable tax function $\mathcal{T}(z, y-z)$ (rather than over separable functions $T(z)$ and $\tau(y-z)$), which assigns a large penalty if an individual picks a bundle not intended for any type. Here, I suffice by stating that, while implementability is not guaranteed theoretically, I will check *ex post* in the simulations whether the identified allocation can be implemented with the separable tax-benefit system analyzed in the current paper.

In Appendix A.3, I formulate the government's optimization problem and show how the corresponding first-order conditions and labor-market equilibrium conditions can be used to derive the following results.

Proposition 4. *If the tax-benefit system $(T(\cdot), \tau(\cdot), b)$ is optimal, the following condition must hold:*

$$\int_{n_0}^{n_1} \Psi'(\tilde{U}(n))f(n)\mathrm{d}n = \eta\left(\int_{n_0}^{n^*}\frac{1}{u'(c_u)}f(n)\mathrm{d}n + \int_{n^*}^{n_1}\left[\frac{p(n)}{u'(c(n))} + \frac{1-p(n)}{u'(c_u)}\right]f(n)\mathrm{d}n\right), \tag{30}$$

where $\tilde{U}(n) \equiv \max\{u(c_u), U(n)-\delta\}$ and η is the multiplier on the aggregate resource constraint. The optimal income tax satisfies, for all $n \geq n^*$:

$$\frac{T'(z(n))}{1-T'(z(n))} = \left(1+\frac{1}{\varepsilon(n)}\right)\frac{u'(c(n))\int_n^{n_1}\frac{1}{u'(c(m))}\left[1-\tilde{g}(m)\right]f(m)\mathrm{d}m}{1-F(n)}\frac{1-F(n)}{nf(n)}. \tag{31}$$

Here, $\varepsilon(n) \equiv v'(\ell(n))/(v''(\ell(n))\ell(n))$ is the Frisch elasticity of labor supply and $\tilde{g}(n) \equiv \Psi'(\tilde{U}(n))$ $u'(c(n))/\eta$ denotes the welfare weight the government attaches to an employed individual with ability n. The optimal profit tax is given by, again for all $n \geq n^*$:

$$\tau(y(n)-z(n)) = \chi(p(n)) - \Delta(z(n), y(n)/n) - (T(z(n))+b) + \frac{\varepsilon(n)}{1+\varepsilon(n)}y(n)T'(z(n)). \tag{32}$$

Proof. See Appendix A.3. □

[13] In particular, condition (i) from Definition 1 is identical to (27) upon substituting $b = c_u$. The resource constraint (26) is obtained by substituting $b = c_u$ and $T(z(n) + \tau(y(n)-z(n)) = y(n) - c(n) - \theta(p(n))k/p(n)$ in (7). Incentive compatibility (29) follows from differentiating (3) with respect to n.

To gain some intuition for these results, first consider (30). This equation is best thought of as determining the optimal unemployment benefit b. It equates the benefits of giving all individuals an additional unit of 'guaranteed utility' (on the left-hand side) to the resource costs necessary to generate this increase (on the right-hand side). To obtain a unit increase in utility, consumption in the state of unemployment has to be raised by $1/u'(c_u)$ units, which accrues both to the non-participants (first term), and the participants who did not succeed in finding a job (captured in the second term). Similarly, raising the utility of an employed individual with ability n requires an increase in consumption of $1/u'(c(n))$ units. The right-hand side integrates the resource costs over all individuals and expresses the result in welfare units (through multiplication with η).

The condition for the optimal income tax (31) is almost identical to the one derived in Diamond (1998), who – like Mirrlees (1971) – studies optimal income taxation in competitive labor markets.[14] Remarkably, (un)employment considerations do *not* modify the optimal tax formula. This sharply contrasts the finding from the previous section, where it was argued that standard optimal tax rules are adjusted to account for income and substitution effects in job finding. To understand why this is not the case if the government can levy a non-linear profit tax, reconsider the first-order conditions associated with the individual optimization problem (3):

$$\frac{v'(y(n)/n)}{nu'(z(n)-T(z(n)))} = 1 - T'(z(n)), \quad \frac{\Delta(z(n), y(n)/n)}{\chi(p(n))} = \frac{1-T'(z(n))}{1-\tau'(y(n)-z(n))}. \tag{33}$$

From (33), it can be seen that the income tax distorts the relative price between income and leisure (first term) *and* the relative price between income and job finding (second term). By contrast, the profit tax only distorts the relative price between income and job finding, and this distortion is symmetric to the one generated by income taxes. Consequently, optimal policy balances the distributional benefits from income taxes only against distortions in the income-leisure choice (as in Mirrlees, 1971 and Diamond, 1998). Profit taxes are then used to alleviate distortions in (un)employment, which occur because income taxes affect application decisions and vacancy creation.

The optimal profit tax is given by equation (32). A first noteworthy observation is that this condition does not depend explicitly on social preferences. The profit tax can hence be thought of as a purely corrective tax, which the government does not employ if income taxes and benefits are optimally zero (in this case (32) holds because (33) implies $\Delta(\cdot) = \chi(\cdot)$ in the absence of taxes). The latter is optimal only if there is no redistributive motive: $u''(\cdot) = \Psi''(\cdot) = 0$. With a preference for redistribution, the optimal profit tax generally differs from zero. The first two terms indicate that the optimal profit tax increases in the amount of 'excess vacancy creation'. Intuitively, vacancy creation should be discouraged (i.e., profit taxes should be raised) if the congestion externality $\chi(\cdot)$ is large compared to the utility gain of finding a job $\Delta(\cdot)$, ceteris paribus. These two terms cancel in the laissez-faire equilibrium (recall: the Hosios (1990) is endogenously satisfied in the competitive search equilibrium), but this is generally not true

[14]The only difference in presentation compared to Diamond (1998) is that I do not impose a constant marginal utility of consumption.

if the government raises (distortionary) taxes. Income taxes and unemployment benefits also directly affect the optimal profit tax: see the final two terms in (32). In particular, the profit tax should optimally be lowered (i.e., labor demand should be stimulated) if an increase in employment generates a large fiscal externality $T(z) + b$. Vacancy creation should thus be encouraged if benefits and average taxes are high. By contrast, the optimal profit tax decreases in the marginal income tax $T'(z)$. This is because high marginal taxes push employment rates above their efficient values through the wage-moderation effect. If this effect is strong, profit taxes should be high to discourage vacancy creation.

5.1 A test for efficiency

As argued above, the condition for the optimal profit tax (32) does not depend on the redistributive preferences of the government. Consequently, it can also be thought of as an efficiency condition which relates different aspects of the tax-benefit system. To see why, combine the result with the household's first-order condition (6). After some rearranging,

$$\frac{T(z(n)) + \tau(y(n) - z(n)) + b}{y(n)} + \frac{\tau'(y(n) - z(n)) - T'(z(n))}{1 - T'(z(n))} \frac{\chi(p(n))}{y(n)} = \frac{\varepsilon(n)}{1 + \varepsilon(n)} T'(z(n)). \quad (34)$$

Clearly, the above condition only depends on characteristics of the tax-benefit system. In this sense, the result bears resemblance to Koehne and Sachs (2017), who derive an efficiency condition which relates the rate of tax deductibility for work-related goods to the marginal tax rate on labor income. Like (34), their condition holds irrespective of the government's taste for redistribution and can therefore be used to test for the efficiency of the tax-benefit system. Equation (34) holds trivially if the government abstains from levying taxes to provide an unemployment benefit, which is optimal only if individuals are risk-neutral ($u''(\cdot) = 0$) and if the government is utilitarian ($\Psi''(\cdot) = 0$).[15] In the typical case that individuals are risk-averse and/or the government cares about redistribution, this is no longer true.

Bringing condition (34) to the data is challenging for at least two reasons. First, there is no clear empirical counterpart of $\tau(\cdot)$. While it is true that in reality (accounting) profits are subject to a tax, the latter is levied at the firm (and not at the match) level. Furthermore, the costs firms incur to fill their vacancies are typically tax-deductible, irrespective of whether the vacancy is filled or not. This corresponds to restricting $\tau(\cdot) = 0$ (the problem analyzed before), because free entry ensures the profit margin equals the *expected* vacancy creation costs. As a result, the tax base vanishes if a firm posts many vacancies.

Second, also the congestion externality $\chi(p(n))$ lacks a clear empirical counterpart. To circumvent this problem, it is useful to replace the 'sharp' efficiency condition (34) by the following:

$$T'(z(n)) > \tau'(y(n) - z(n)) \quad \Leftrightarrow \quad \frac{T(z(n)) + \tau(y(n) - z(n)) + b}{y(n)} > \frac{\varepsilon(n)}{1 + \varepsilon(n)} T'(z(n)), \quad (35)$$

which follows from imposing $\chi(p(n)) > 0$ in (34). This result has a clear intuition: it is optimal to distort the job-finding choice (6) upward (i.e., to set a marginal income tax which exceeds

[15] In this case, there are no redistributive preferences and the laissez-faire outcome is efficient (Proposition 1).

the marginal profit tax) if and only if the fiscal externality from raising employment is high compared to the wage-moderating effect of tax progressivity. The latter is proportional to the marginal income tax, which generates an upward distortion in employment. This condition is easier to bring to the data than (34), but still suffers from the fact that there is no clear empirical counterpart of the tax on the profit margin $\tau(\cdot)$.

5.2 End-point results

Proposition 4 has an immediate implication for the optimal top rate: if the government can observe individual match surplus (and hence, tax profits), the optimal marginal income tax for the highest-income earners equals zero. Intuitively, at each point in the income distribution the marginal tax rate serves to redistribute income from individuals above to individuals below that income level. The distributional benefits are then optimally weighed against distortions in labor supply. At the top of the income distribution, however, there is nobody to redistribute from. The marginal tax rate only generates distortions and therefore optimally equals zero. This is a well-known result (see Sadka, 1976 and Stiglitz, 1982 among many others) which generalizes to the current setting, but only if the government can levy a non-linear profit tax. Without a profit tax, the top rate serves to generate revenue through the wage-moderation effect: see (25). This role is taken over by the marginal profit tax if the government can observe individual match surplus. In particular, the top rate on profits is then given by:[16]

$$\frac{\tau'(y(n_1)-z(n_1))}{1-\tau'(y(n_1)-z(n_1))} = -\frac{1}{\chi(p(n_1))}\Big[T(z(n_1))+\tau(y(n_1)-z(n_1))+b\Big]. \tag{36}$$

In the typical case that an increase in employment of the individuals with the highest ability positively affects the government budget (i.e., if the term in brackets on the right-hand side is positive), it is optimal to set a negative marginal profit tax in order to boost vacancy creation and stimulate employment (see Proposition 2). Naturally, the optimal subsidy is lower the larger is the congestion externality, as captured by the denominator in (36).

The typical finding that the optimal marginal tax rate also equals zero at the bottom of the income distribution (see, e.g., Seade, 1977) does *not* hold in frictional labor markets, even if there is full participation. In particular, the optimal tax rate at the bottom equals:[17]

$$\begin{aligned}\frac{T'(z(n^*))}{1-T'(z(n^*))} = \left(1+\frac{1}{\varepsilon(n^*)}\right)\frac{u'(c(n^*))}{n^*f(n^*)}\Bigg[\int_{n_0}^{n^*}\left(\frac{\Psi'(u(c_u))}{\eta}-\frac{1}{u'(c_u)}\right)f(n)\mathrm{d}n \\ +\int_{n^*}^{n_1}(1-p(n))\left(\frac{1}{u'(c(n))}-\frac{1}{u'(c_u)}\right)f(n)\mathrm{d}n\Bigg].\end{aligned} \tag{37}$$

If the government cares for redistribution (i.e., if $\Psi(\cdot)$ is strictly concave), the right-hand side is strictly positive unless everybody participates and individuals are risk-neutral (in which case the right-hand side equals zero). The reason is intuitive: with less than full participation and involuntary unemployment, setting a positive marginal tax rate at the bottom generates distributional gains, because it allows the government to finance an unemployment benefit. The

[16] To derive this result, substitute $T'(z(n_1)) = 0$ in (32) and (33). Combining the results gives (36).

[17] To see why, evaluate (31) at $n = n^*$ and combine the result with (30).

latter is used to redistribute towards the non-participants and to provide (partial) insurance against the risk of not finding a job. The associated distributional benefits and insurance gains are captured by the first and second integral on the right-hand side of (37), respectively. This condition clearly illustrates that the tax-benefit system not only serves to redistribute income, but also to (partly) correct for the missing insurance market. The next section strictly separates these issues.

6 Extension: private unemployment insurance

Up to this point, I have assumed the (idiosyncratic) risk of becoming unemployed is not privately insurable. A direct implication is that the laissez-faire equilibrium is inefficient if individuals are risk-averse: see Proposition 1. The tax-benefit system therefore not only serves to redistribute income between high-ability and low-ability individuals, but also to (partly) correct for the missing insurance market. To strictly separate these issues, I now extend the model to include a perfectly competitive insurance market.

As before, the government levies a non-linear tax on earnings z and (possibly) the profit margin $y-z$ to finance a uniform benefit level b. The latter is paid to both non-participants and to participants who are involuntary unemployed. In addition to the (state-provided) benefit, individuals can buy additional coverage. Denote the latter by q, so that consumption in the state of unemployment equals $b+q$. If an individual applies for a job where he or she becomes employed with probability p, the actuarially fair premium equals $(1-p)q/p$, to be paid if the individual applies successfully.[18] The maximization problem of an individual who decides to participate then becomes:

$$U(n) = \max_{z,y,p,q} \left\{ p(u(z - T(z) - (1-p)q/p) - v(y/n)) + (1-p)u(b+q) \ \text{ s.t. } (2) \right\}. \tag{38}$$

The participation constraint is again given by $U(n) - \delta \geq u(b)$. Solving the above problem leads to the counterparts of (5) and (6):

$$\frac{v'(y/n)}{nu'(z - T(z) - (1-p)q/p)} = 1 - T'(z), \quad \frac{z - T(z) - b - \frac{v(y/n)}{u'(b+q)}}{\chi(p)} = \frac{1 - T'(z)}{1 - \tau'(y-z)} \tag{39}$$

The difference between the second condition and (6) is that the 'consumption difference' $z - T(z) - b$ replaces the 'utility difference' $(u(z-T(z)) - u(b))/u'(z-T(z))$. Intuitively, individuals use the insurance market to fully insure their consumption risk, so that there is no variation in the marginal utility of consumption. This is formally demonstrated by considering the first-order condition with respect to q:

$$u'(z - T(z) - (1-p)q/p) = u'(b+q). \tag{40}$$

The optimal coverage thus equalizes consumption in both states: $q = p(z - T(z) - b)$. Because there are utility costs associated with working (i.e., $v(\cdot) > 0$), a direct implication is that utility

[18]To see why, note that an insurer makes zero profits if a fraction p of all participants in an insurance contract pay $(1-p)q/p$ to finance a payment q paid to a fraction $1-p$ of the participants.

in the state of unemployment exceeds utility in the state of employment. Put differently, once an individual has secured his or her consumption through the insurance contract, he or she actually has an incentive *not* to find a job. To prevent the insurance market from breaking down, it is therefore required that application strategies are contractible.[19]

Not surprisingly, with a perfectly competitive insurance market efficiency is restored in the absence of taxation.[20] A direct implication is that the tax-benefit system only serves to redistribute income between high-ability and low-ability individuals, and not to correct for the missing insurance market. The impact of the tax-benefit system on the labor-market outcomes, in turn, can be analyzed by combining the first-order conditions (39) and (40) with the zero-profit condition (2). These equations implicitly characterize the optimal coverage, income, output and employment rate for all individuals who decide to participate, as a function of the tax-benefit system. The comparative statics can be determined in analogous fashion as in Proposition 2. Doing so, however, does not generate any substantive additional insights and for this reason I directly turn to the problem of optimal taxation.

First, consider the restricted problem where the profit margin cannot be taxed: $\tau(\cdot) = 0$. Optimal tax formulas can again be derived by analyzing perturbations of the income tax. As it turns out, the results from Proposition 3 continue to hold with a perfectly competitive insurance market. Put differently, the optimal tax formulas from Proposition 3 generalize when expressed in terms of the income distribution and sufficient statistics. This, however, does not mean that the optimal tax-benefit system is the same as in the absence of an insurance market. The reason is that both the income distribution and the behavioral responses are endogenous to tax policy.

Turning to the general problem (where the profit tax is not restricted to be zero), I again characterize the solution using the mechanism design approach. The formulation of the problem is almost identical as before. The objective is given by (13) and the participation and incentive compatibility constraint are given by (27) and (29), respectively. The presence of an insurance market only affects the definition of expected utility and the resource constraint. Because individuals fully insure their consumption risk, the former reads:

$$U(n) = u(c(n)) - p(n)v(y(n)/n) \tag{41}$$

for all $n \geq n^*$. In addition, the aggregate resource constraint (26) is replaced by:

$$\int_{n^*}^{n_1} (p(n)y(n) - c(n) - \theta(p(n))k) f(n)\mathrm{d}n - \int_{n_0}^{n^*} c_u f(n)\mathrm{d}n = 0. \tag{42}$$

In Appendix A.4, I solve the government's optimization problem to derive the following results.

Proposition 5. *If the tax-benefit system $(T(\cdot), \tau(\cdot), b)$ is optimal, the following condition must*

[19] Alternatively, insurance may come through firms as in Jacquet and Tan (2012). In their model, individuals are risk-averse and firms attract applicants by making payments also to individuals who apply unsuccessfully. Such wage-vacancy contracts may restore efficiency, but only if firms can credibly commit to making payments to unsuccessful applicants, and if these payments can be conditioned on the number of applicants.

[20] Efficiency can be established along similar lines as in the second part of Appendix A.1. The only difference is that the result continues to hold under risk-aversion.

hold:

$$\int_{n_0}^{n_1} \Psi'(\tilde{U}(n)) f(n) \mathrm{d}n = \eta \left(\int_{n_0}^{n^*} \frac{1}{u'(c_u)} f(n) \mathrm{d}n + \int_{n^*}^{n_1} \frac{1}{u'(c(n))} f(n) \mathrm{d}n \right). \tag{43}$$

The optimal income tax satisfies, for all $n \geq n^*$:

$$\frac{T'(z(n))}{1 - T'(z(n))} = \left(1 + \frac{1}{\varepsilon(n)}\right) \frac{u'(c(n)) \int_n^{n_1} \frac{1}{u'(c(m))} \left[1 - \tilde{g}(m)\right] f(m) \mathrm{d}m}{1 - F(n)} \frac{1 - F(n)}{n f(n)}. \tag{44}$$

The optimal profit tax is given by, again for all $n \geq n^*$:

$$\tau(y(n) - z(n)) = \chi(p(n)) - \tilde{\Delta}(n) - (T(z(n)) + b) + \frac{\varepsilon(n)}{1 + \varepsilon(n)} y(n) T'(z(n)), \tag{45}$$

where $\tilde{\Delta}(n) \equiv q(n)/p(n) - v(y(n)/n)/u'(c(n))$ is the (private) utility gain of raising employment, expressed in consumption units.

Proof. See Appendix A.4. □

The results are very similar to those presented in Proposition 4, with two notable differences. First, the right-hand side of (43) no longer accounts for differences in the marginal utility of consumption in the different employment states. Intuitively, the difference is absent if individuals can insure against the risk of not finding a job. This has an interesting implication for the pattern of optimal marginal income taxes at the bottom of the income distribution. Evaluating (44) at n^* and combining with (43) yields:

$$\frac{T'(z(n^*))}{1 - T'(z(n^*))} = \left(1 + \frac{1}{\varepsilon(n^*)}\right) \frac{u'(c(n^*))}{n^* f(n^*)} \int_{n_0}^{n^*} \left(\frac{\Psi'(u(c_u))}{\eta} - \frac{1}{u'(c_u)} \right) f(n) \mathrm{d}n. \tag{46}$$

Compared to (37), there is no longer an insurance motive for setting a positive marginal tax rate for the lowest-income earners (i.e., the second term in (37) is absent). Consequently, if all individuals participate (i.e., $n^* = n_0$), the optimal marginal tax rate equals zero at the bottom of the income distribution (as in Seade, 1977). This is not true if unemployment risk is not privately insurable. In this case, setting a positive marginal tax rate at the bottom of the income distribution generates insurance benefits as it allows the government to finance an unemployment benefit.

Second, in the expression for the optimal profit tax the utility gain of finding a job $\Delta(\cdot)$ from (32) is replaced by the similar term

$$\tilde{\Delta}(n) \equiv \frac{q(n)}{p(n)} - \frac{v(y(n)/n)}{u'(c(n))}. \tag{47}$$

This term can be thought of as a measure of the 'private' utility gain from finding a job, and therefore serves a very similar role as before. Moving from unemployment to employment raises private consumption by an amount equal to $q(n)/p(n) = z(n) - T(z(n)) - b$, which is the difference in (gross) consumption levels between an employed and an unemployed individual in the absence of an insurance market. It thus represents the additional income individuals can use to buy consumption. Subtracted from this income gain is the utility costs of producing output, expressed in consumption units. As before, the expression for the optimal profit tax (45) states that profit taxes should be lowered (i.e., vacancy creation should be stimulated) if (i) excess vacancy creation $\chi(\cdot) - \tilde{\Delta}(\cdot)$ is low, (ii) the fiscal externality of raising employment $T(\cdot) + b$ is high, (iii) employment is not driven far above its efficient level as a result of wage moderation (i.e., $T'(\cdot)$ is low).

7 Conclusion

How progressive should the income tax be if marginal tax rates discourage in-work effort, but boost employment through a wage-moderation effect? I analyze this question in a model where search is frictional and firms compete for workers with different abilities. Firms post vacancies which specify how much output an employee is expected to produce an the income he or she receives as compensation. Workers derive utility from consumption and leisure and optimally choose where to apply. Because search is frictional, some workers and firms remain unmatched. Workers therefore not only face a trade-off between income and leisure but also between income and the probability of finding a job. Under standard assumptions on preferences and the matching technology, there exists a separating equilibrium in which workers with different abilities apply at different vacancies. Furthermore, in the absence of a distributive or insurance motive, the laissez-faire equilibrium generates a socially efficient allocation of resources (as in, e.g., Moen, 1997).

In order to redistribute income and partially insure workers against the risk of becoming unemployed, the government levies a non-linear tax on labor income and (possibly) profits to finance a uniform unemployment benefit. I analyze how the labor-market outcomes are affected by different aspects of the tax-benefit system. Crucially, the model predicts that high *marginal* income taxes discourage in-work effort (as in competitive labor markets), but boost employment through a wage-moderation effect. Intuitively, promising higher earnings is a less effective tool to attract workers if marginal tax rates are high. Consequently, an increase in the marginal tax rate reduces wages and boosts employment. High *average* taxes, on the other hand, raise posted wages and thereby reduce employment.

Concerning the question how tax policy should optimally be designed in such an environment, I first show that if the government cannot tax profits, (un)employment responses modify standard optimal tax rules (as in, e.g., Saez, 2001) in two ways. First, in the typical case that an increase in employment improves the government's budget, the wage-moderating effect of tax progressivity calls for *higher* marginal tax rates. High marginal taxes, however, mechanically raise average tax rates further up in the income distribution, which reduces employment further up in the ability distribution. This second effect therefore provides a force for *lower* marginal tax rates. How search frictions affect the optimal degree of tax progressivity thus ultimately remains an empirical question, which I aim to address in future research.

Second, I show that if the government can tax profits, the expression for the optimal income tax from Mirrlees (1971) and Diamond (1998) generalizes. Thus, in contrast to the previous finding, (un)employment responses do *not* modify optimal tax rules if the government can levy a non-linear profit tax. Intuitively, the income-leisure choice is distorted only by the income tax, whereas the workers' application and firms' entry decisions are distorted (symmetrically) by both income and profit taxes. Optimal income taxes therefore balance the distributional gains of higher tax progressivity only against the distortions in labor supply (as in Mirrlees, 1971). Profit taxes are then used exclusively to alleviate distortions from income taxes on application decisions and vacancy creation. Whether such a profit tax can actually be implemented in practice is questionable, as it relies on the assumption the government can observe match-specific output.

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A Proofs

A.1 Proof proposition 1

The structure of the proof is as follows. First, I show that if individuals are risk-averse, there exists a Pareto-improving and resource feasible perturbation of the equilibrium allocation. Then, I show that no such perturbation exists if individuals are risk-neutral.

In the absence of taxes and benefits, an individual of type n who decides to participate, solves

$$U(n) \equiv \max_{c,y,p} \left\{ p(u(c) - v(y/n)) + (1-p)u(0) \quad \text{s.t.} \quad \theta(p)k = p(y-c) \right\}, \tag{48}$$

where consumption c replaces income z. Denote the solution to the above optimization problem by $(c(n), y(n), p(n))$. Hence, a fraction $p(n)$ of the individuals with ability n becomes employed, produces $y(n)$ and consumes $c(n)$, whereas a fraction $1 - p(n)$ remains unemployed and does not consume at all. Now, consider a perturbation where the consumption of individuals with ability n in the state of unemployment is marginally raised by $\mathrm{d}c_u > 0$ and consumption in the state of employment is reduced by $\mathrm{d}c_u(1 - p(n))/p(n)$. Such a perturbation is resource feasible as it does not affect aggregate consumption of individuals with productivity n. The impact on expected utility is:

$$\mathrm{d}U(n) = -(1 - p(n))(u'(c(n)) - u'(0))\mathrm{d}c_u. \tag{49}$$

The latter is strictly positive whenever $u(\cdot)$ is strictly concave. Since the above perturbation raises the expected utility of type n individuals without decreasing the expected utility of any other type, the laissez-faire equilibrium is not Pareto efficient if individuals are risk-averse.

To show the result in the opposite direction, suppose individuals are risk-neutral and suppose there is a planner which aims to maximize the sum of expected utilities subject only to the aggregate resource constraint. Its objective is:

$$\mathcal{W} = \int_{n_0}^{n^*} c_u(n) f(n) \mathrm{d}n + \int_{n^*}^{n_1} \Big[p(n)(c(n) - v(y(n)/n)) - (1 - p(n)) c_u(n) - \delta \Big] f(n) \mathrm{d}n. \quad (50)$$

Here, I imposed risk-neutrality ($u(c) = c$) and $n^* \geq n_0$ denotes the participation threshold, which is assumed to satisfy $n^* < n_1$. In addition, $c_u(n)$ denotes the consumption if an individual of type n is not employed. The aggregate resource constraint reads:

$$\int_{n^*}^{n_1} \Big[p(n)(y(n) - c(n)) - (1 - p(n)) c_u(n) - \theta(p(n))k \Big] f(n) \mathrm{d}n - \int_{n_0}^{n^*} c_u(n) f(n) \mathrm{d}n = 0. \quad (51)$$

The government chooses the allocation which maximizes social welfare (50) subject to the resource constraint (51) and the restriction $n^* \geq n_0$. The Lagrangian reads:

$$\begin{aligned} \mathcal{L} = & \int_{n_0}^{n^*} c_u(n) f(n) \mathrm{d}n + \int_{n^*}^{n_1} \Big[p(n)(c(n) - v(y(n)/n)) + (1 - p(n)) c_u(n) - \delta \Big] f(n) \mathrm{d}n \\ & + \eta \left[\int_{n^*}^{n_1} \Big[p(n)(y(n) - c(n)) - (1 - p(n)) c_u(n) - \theta(p(n))k \Big] f(n) \mathrm{d}n - \int_{n_0}^{n^*} c_u(n) f(n) \mathrm{d}n \right] \\ & + \mu(n^* - n_0). \end{aligned} \quad (52)$$

The first-order conditions are given by:

$$c_u(n): \begin{cases} (1 - \eta) f(n) = 0 & n < n^* \\ (1 - p(n))(1 - \eta) f(n) = 0 & n \geq n^* \end{cases} \quad (53)$$

$$c(n): \quad p(n)(1 - \eta) = 0 \quad (54)$$

$$y(n): \quad p(n)(\eta - v'(y(n))/n) f(n) = 0 \quad (55)$$

$$p(n): \quad (c(n) - v(y(n)/n) - c_u(n) + \eta(y(n) - c(n) + c_u(n) - \theta'(p(n))k)) f(n) = 0 \quad (56)$$

$$\begin{aligned} n^*: \quad & -p(n^*)(c(n^*) - v(y(n^*)/n^*) - c_u(n^*) - \delta/p(n^*) \\ & + \eta(y(n^*) - c(n^*) + c_u(n^*) - \theta(p(n^*))k/p(n^*))) f(n^*) + \mu = 0 \end{aligned} \quad (57)$$

$$\begin{aligned} \eta: \quad & \int_{n^*}^{n_1} (p(n)(y(n) - c(n)) - (1 - p(n)) c_u(n) - \theta(p(n))k) f(n) \mathrm{d}n \\ & - \int_{n_0}^{n^*} c_u(n) f(n) \mathrm{d}n = 0 \end{aligned} \quad (58)$$

$$\mu: \quad \mu(n^* - n_0) = 0, \quad \mu \geq 0, \quad n^* \geq n_0 \quad (59)$$

From (53) and (54), it follows that $\eta = 1$. Substituting in (55)-(56) yields:

$$v'(y(n)/n) - n = 0 \quad (60)$$

$$y(n) - v(y(n)/n) - \theta'(p(n))k = 0 \quad (61)$$

Now, consider as a candidate solution to the above system of first-order conditions the competitive search equilibrium (see Definition 1). For the types who participate, the equilibrium is characterized by the following conditions:[21]

$$\theta(p(n))k = p(n)(y(n) - c(n)) \tag{62}$$

$$v'(y(n)/n) = n \tag{63}$$

$$c(n) - v(y(n)/n) = (\theta'(p(n)) - \theta(p(n))/p(n))k \tag{64}$$

In addition, in the laissez-faire equilibrium $c_u(n) = 0$ for all n. The participation constraint is pinned down by (see again Definition 1):

$$U(n^*) \geq \delta, \qquad n^* \geq n_0 \tag{65}$$

with complementary slackness. To see why this candidate solution solves the government maximization problem, first multiply (62) by $f(n)$ and integrate over all individuals. Together with $c_u(n) = 0$ for all n, this implies the resource constraint (58) is satisfied. In addition, condition (63) is identical to (60) and (62) and (64) jointly imply (61). To see why (65) also applies, first use (64) to write:

$$U(n^*) = p(n^*)(c(n^*) - v(y(n^*)/n^*)) = p(n^*)(\theta'(p(n^*)) - \theta(p(n^*))/p(n^*))k. \tag{66}$$

Next, substitute $\eta = 1$ and (61) in (57):

$$(\delta - p(n^*)(\theta'(p(n^*)) - \theta(p(n^*))/p(n^*))k)f(n^*) + \mu = 0 \tag{67}$$

There are two cases to consider. First, if the threshold constraint is not binding (i.e., $n^* > n_0$ and $\mu = 0$), it follows that (using (66))

$$\delta = p(n^*)(\theta'(p(n^*)) - \theta(p(n^*))/p(n^*))k = U(n^*), \tag{68}$$

as required by (65). On the other hand, if the threshold constraint binds (i.e., $n^* = n_0$), $\mu \geq 0$ implies

$$U(n^*) = p(n^*)(\theta'(p(n^*)) - \theta(p(n^*))/p(n^*))k \geq \delta, \tag{69}$$

again as required by (65). The allocation implied by the competitive search equilibrium thus maximizes the sum of expected utilities subject to the aggregate resource constraint. Hence, there exists no feasible Pareto improvement if individuals are risk-neutral.

A.2 Proof proposition 2

For an individual with ability n who decides to participate, the equilibrium is characterized by:

$$p(y - z - \tau(y - z) - \kappa_\tau R_\tau(y - z)) - \theta(p)k = 0 \tag{70}$$

[21]To see why, substitute $u(c) = c$ and $T(\cdot) = \tau(\cdot) = b = 0$ in the zero-profit condition (2) and the first-order conditions (5)-(6) and note that $c = z$ in the absence of taxes.

$$u'(z - T(z) - \kappa_T R_T(z))n(1 - T'(z) - \kappa_T R_T'(z)) - v'(y/n) = 0 \tag{71}$$

$$(u(z - T(z) - \kappa_T R_T(z)) - v(y/n) - u(b))n(1 - \tau'(y - z) - \kappa_\tau R_\tau'(y - z))$$
$$- v'(y/n)(\theta'(p) - \theta(p)/p)k = 0 \tag{72}$$

These correspond to the zero-profit condition (2), and the first-order conditions (5)-(6) under the tax-benefit system $(T^*(\cdot), \tau^*(\cdot), b)$. Denote the above system by $\Lambda(\mathbf{x}; \mathbf{t}) = 0$, which implicitly defines the equilibrium outcomes $\mathbf{x} = (z, y, p)'$ as a function of parameters $\mathbf{t} = (\kappa_T, \kappa_\tau, b, n)'$. The comparative statics can be determined via the implicit function theorem:

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}\mathbf{t}} = -\left(\frac{\partial \Lambda(\mathbf{x}; \mathbf{t})}{\partial \mathbf{x}}\right)^{-1} \frac{\partial \Lambda(\mathbf{x}; \mathbf{t})}{\partial \mathbf{t}} = -\Lambda_{\mathbf{x}}^{-1} \Lambda_{\mathbf{t}}. \tag{73}$$

Working out (73) using co-factor expansion yields:

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}\mathbf{t}} = \frac{-1}{|\Lambda_{\mathbf{x}}|} \begin{pmatrix} \Lambda_y^2 \Lambda_p^3 - \Lambda_p^2 \Lambda_y^3 & \Lambda_p^1 \Lambda_y^3 - \Lambda_y^1 \Lambda_p^3 & \Lambda_y^1 \Lambda_p^2 - \Lambda_p^1 \Lambda_y^2 \\ \Lambda_p^2 \Lambda_z^3 - \Lambda_z^2 \Lambda_p^3 & \Lambda_z^1 \Lambda_p^3 - \Lambda_p^1 \Lambda_z^3 & \Lambda_p^1 \Lambda_z^2 - \Lambda_z^1 \Lambda_p^2 \\ \Lambda_z^2 \Lambda_y^3 - \Lambda_y^2 \Lambda_z^3 & \Lambda_y^1 \Lambda_z^3 - \Lambda_z^1 \Lambda_y^3 & \Lambda_z^1 \Lambda_y^2 - \Lambda_y^1 \Lambda_z^2 \end{pmatrix} \begin{pmatrix} \Lambda_{\kappa_T}^1 & \Lambda_{\kappa_\tau}^1 & \Lambda_b^1 & \Lambda_n^1 \\ \Lambda_{\kappa_T}^2 & \Lambda_{\kappa_\tau}^2 & \Lambda_b^2 & \Lambda_n^2 \\ \Lambda_{\kappa_T}^3 & \Lambda_{\kappa_\tau}^3 & \Lambda_b^3 & \Lambda_n^3 \end{pmatrix} \tag{74}$$

The superscripts correspond to the rows in $\Lambda(\mathbf{x}; \mathbf{t})$. The elements of $\Lambda_{\mathbf{x}}$ are (ignoring function arguments for notational convenience):

$$\begin{aligned} &\Lambda_z^1 = -p(1 - \tau' - \kappa_\tau R_\tau'), \quad \Lambda_y^1 = p(1 - \tau' - \kappa_\tau R_\tau'), \quad \Lambda_p^1 = -\chi, \\ &\Lambda_z^2 = u'' n(1 - T' - \kappa_T R_T')^2 - u' n(T'' + \kappa_T R_T''), \quad \Lambda_y^2 = -v''/n, \quad \Lambda_p^2 = 0, \\ &\Lambda_z^3 = v'(1 - \tau' - \kappa_\tau R_\tau') + \Delta u' n(\tau'' + \kappa_\tau R_\tau''), \\ &\Lambda_y^3 = -v'(1 - \tau' - \kappa_\tau R_\tau') - \Delta u' n(\tau'' + \kappa_\tau R_\tau'') - \chi v''/n, \quad \Lambda_p^3 = -v'(\theta'' k - \chi/p) \end{aligned} \tag{75}$$

The elements in $\Lambda_{\mathbf{t}}$ are:

$$\begin{aligned} &\Lambda_{\kappa_T}^1 = 0, \quad \Lambda_{\kappa_\tau}^1 = -pR_\tau, \quad \Lambda_b^1 = 0, \quad \Lambda_n^1 = 0, \\ &\Lambda_{\kappa_T}^2 = -u'' n(1 - T' - \kappa_T R_T') R_T - u' n R_T', \quad \Lambda_{\kappa_\tau}^2 = 0, \quad \Lambda_b^2 = 0, \quad \Lambda_n^2 = v'/n + v'' y/n^2, \\ &\Lambda_{\kappa_T}^3 = -u' n(1 - \tau' - \kappa_\tau R_\tau') R_T, \quad \Lambda_{\kappa_\tau}^3 = -\Delta u' n R_\tau', \quad \Lambda_b^3 = -u_0' n(1 - \tau' - \kappa_\tau R_\tau'), \\ &\Lambda_n^3 = v' \chi/n + v' y(1 - \tau' - \kappa_\tau R_\tau')/n + \chi v'' y/n^2 \end{aligned} \tag{76}$$

The exposition in (75)-(76) is simplified somewhat using the conditions (70)-(72) and the definitions of $\chi(\cdot)$ and $\Delta(\cdot)$. In addition, $u_0' \equiv u'(b)$ denotes the marginal utility of consumption of the unemployed.

The impact of a reform of the income (profit) tax on an equilibrium outcome can be determined by calculating the partial effect of the reform parameter κ_T (κ_τ), evaluated at the reform of interest. For the income tax, I consider the following reforms:

$$R_T(z) = z - z(n) \tag{77}$$

$$R_T(z) = 1 \tag{78}$$

The first of these increases the marginal tax rate while leaving the average tax rate at the equilibrium income level $z(n)$ unaffected. The second, in turn, generates an increase in the tax bill, but does not affect the marginal tax rate. For the profit tax I consider the perturbations:

$$R_\tau(y-z) = y - z - (y(n) - z(n)) \tag{79}$$

$$R_\tau(y-z) = 1 \tag{80}$$

Again, these reforms generate an increase in the marginal tax rate and the tax liability, respectively, holding the average and marginal tax rate constant.

In order to simplify the exposition and to sign the partial effects, I perturb the tax functions starting from the laissez-faire equilibrium: $T(\cdot) = \tau(\cdot) = b = \kappa_T = \kappa_\tau = 0$. The impact of the marginal income tax on the equilibrium outcomes is then obtained by working out the first column of (74) evaluated at the reform (77). With a slight abuse of notation, I denote the results by:

$$\frac{\mathrm{d}z}{\mathrm{d}T'} = \frac{u'n(\chi^2 v''/n + pkv'\theta'')}{u''n(\chi^2 v''/n + pkv'\theta'') - pkv'\theta''v''/n} < 0 \tag{81}$$

$$\frac{\mathrm{d}y}{\mathrm{d}T'} = \frac{u'npkv'\theta''}{u''n(\chi^2 v''/n + pkv'\theta'') - pkv'\theta''v''/n} < 0 \tag{82}$$

$$\frac{\mathrm{d}p}{\mathrm{d}T'} = \frac{-\chi u'npv''/n}{u''n(\chi^2 v''/n + pkv'\theta'') - pkv'\theta''v''/n} > 0 \tag{83}$$

Similarly, for the reform (78):

$$\frac{\mathrm{d}z}{\mathrm{d}T} = \frac{u''n(\chi^2 v''/n + pkv'\theta'') - \chi u'v''}{u''n(\chi^2 v''/n + pkv'\theta'') - pkv'\theta''v''/n} > 0 \tag{84}$$

$$\frac{\mathrm{d}y}{\mathrm{d}T} = \frac{u''n(pkv'\theta'' - \chi u'n)}{u''n(\chi^2 v''/n + pkv'\theta'') - pkv'\theta''v''/n} \geq 0 \tag{85}$$

$$\frac{\mathrm{d}p}{\mathrm{d}T} = \frac{p(u'v'' - u''n(\chi v''/n + u'n))}{u''n(\chi^2 v''/n + pkv'\theta'') - pkv'\theta''v''/n} < 0 \tag{86}$$

Turning to the impact of profit taxes, first consider the reform (79):

$$\frac{\mathrm{d}z}{\mathrm{d}\tau'} = \frac{-\Delta u'\chi v''}{u''n(\chi^2 v''/n + pkv'\theta'') - pkv'\theta''v''/n} > 0 \tag{87}$$

$$\frac{\mathrm{d}y}{\mathrm{d}\tau'} = \frac{-\Delta u'\chi u''n^2}{u''n(\chi^2 v''/n + pkv'\theta'') - pkv'\theta''v''/n} \leq 0 \tag{88}$$

$$\frac{\mathrm{d}p}{\mathrm{d}\tau'} = \frac{\Delta u'np(v''/n - u''n)}{u''n(\chi^2 v''/n + pkv'\theta'') - pkv'\theta''v''/n} < 0 \tag{89}$$

Perturbing the intercept of the profit tax (as in (80)) generates:

$$\frac{\mathrm{d}z}{\mathrm{d}\tau} = \frac{pv'(\theta''k - \chi/p)v''/n}{u''n(\chi^2 v''/n + pkv'\theta'') - pkv'\theta''v''/n} < 0 \tag{90}$$

$$\frac{\mathrm{d}y}{\mathrm{d}\tau} = \frac{pv'(\theta''k - \chi/p)u''n}{u''n(\chi^2 v''/n + pkv'\theta'') - pkv'\theta''v''/n} \geq 0 \tag{91}$$

$$\frac{\mathrm{d}p}{\mathrm{d}\tau} = \frac{p(v'v''/n - u''n(v' + \chi v''/n))}{u''n(\chi^2 v''/n + pkv'\theta'') - pkv'\theta''v''/n} < 0 \tag{92}$$

The impact of the unemployment benefit b is given by:

$$\frac{\mathrm{d}z}{\mathrm{d}b} = \frac{-\chi v'' u_0'}{u''n(\chi^2 v''/n + pkv'\theta'') - pkv'\theta''v''/n} > 0 \tag{93}$$

$$\frac{\mathrm{d}y}{\mathrm{d}b} = \frac{-\chi u'' n^2 u_0'}{u''n(\chi^2 v''/n + pkv'\theta'') - pkv'\theta''v''/n} \leq 0 \tag{94}$$

$$\frac{\mathrm{d}p}{\mathrm{d}b} = \frac{pnu_0'(v''/n - u''n)}{u''n(\chi^2 v''/n + pkv'\theta'') - pkv'\theta''v''/n} < 0 \tag{95}$$

Finally, the impact of productivity n is:

$$\frac{\mathrm{d}z}{\mathrm{d}n} = \frac{-v'(p(\theta''k - \chi/p)v''y/n^2 + pv'\theta''k/n)}{u''n(\chi^2 v''/n + pkv'\theta'') - pkv'\theta''v''/n} > 0 \tag{96}$$

$$\frac{\mathrm{d}y}{\mathrm{d}n} = \frac{-((v'/n + v''y/n^2)(pv'\theta''k - \chi^2 u''n) - \chi v'yu'')}{u''n(\chi^2 v''/n + pkv'\theta'') - pkv'\theta''v''/n} > 0 \tag{97}$$

$$\frac{\mathrm{d}p}{\mathrm{d}n} = \frac{p((v'/n + v''y/n^2)\chi u''n - v'y(v''/n - u''n)/n)}{u''n(\chi^2 v''/n + pkv'\theta'') - pkv'\theta''v''/n} > 0 \tag{98}$$

The signs of all these effects follow from the assumptions on $u(\cdot)$, $v(\cdot)$ and $\theta(\cdot)$ combined with the observation that $u'(\cdot)n = v'(\cdot)$ in the absence of taxes.[22]

A.2.1 Proof Corollary 1

To derive the first result, note that risk-neutrality implies $u''(\cdot) = 0$. Furthermore, a constant matching elasticity implies:

$$\frac{\mathrm{d}}{\mathrm{d}p}\left[\frac{\theta'(p)p}{\theta(p)}\right] = 0 \quad \Rightarrow \quad p\theta''(p) = \frac{p\theta'(p)}{\theta(p)}\left(\theta'(p) - \frac{\theta(p)}{p}\right) = \frac{\chi(p)}{\mu k}, \tag{99}$$

where μ denotes the constant matching elasticity. Working out the elasticity of taxable income using (81), imposing $u''(\cdot) = 0$, $u'(\cdot)n(1 - T'(\cdot)) = v'(\cdot)$ and substituting in (99) gives:

$$\varepsilon_{zT'} \equiv -\frac{\mathrm{d}z}{\mathrm{d}T'}\frac{1 - T'}{z} = \frac{v'(\chi^2 v''/n + pkv'\theta'')}{zpkv'\theta''v''/n} = \frac{v'n}{v''z} + \frac{\chi^2}{zp\theta''k} = \hat{\varepsilon}_{zT'} + \sigma(1 - \mu), \tag{100}$$

where I used the definitions from Corollary 1. Similarly, the elasticity of the job-finding rate can be obtained using (83), again imposing $u''(\cdot) = 0$, $u'(\cdot)n(1 - T'(\cdot)) = v'(\cdot)$ and substituting in (99):

$$\varepsilon_{pT'} \equiv \frac{\mathrm{d}p}{\mathrm{d}T'}\frac{1 - T'}{p} = \frac{(1 - T')\chi u'npv''/n}{p^2kv'\theta''v''/n} = \frac{\chi}{pk\theta''} = \mu. \tag{101}$$

A.3 Proof proposition 4

The government chooses the allocation which maximizes social welfare, subject to the resource constraint (26), voluntary participation (27) and incentive compatibility (29). To write the problem as an optimal control problem, let the government also optimize over $U(n)$ and add

[22] The assumption that the elasticity of $\theta(\cdot)$ is non-decreasing implies $\theta''k - \chi/p > 0$, which is the only term of which the sign might appear ambiguous.

$U(n) = p(n)(u(c(n)) - v(\ell(n))) + (1-p(n))u(c_u)$ as a constraint. The exposition can be simplified somewhat by replacing $y(n) = n\ell(n)$ everywhere. The maximization problem then reads:

$$\max_{c_u, n^*, [U(n), c(n), \ell(n), p(n)]_{n^*}^{n_1}} \quad \mathcal{W} = \int_{n_0}^{n^*} \Psi(u(c_u)) f(n)\mathrm{d}n + \int_{n^*}^{n_1} \Psi(U(n) - \delta) f(n)\mathrm{d}n \tag{102}$$

$$\text{s.t.} \quad \int_{n^*}^{n_1} \Big[p(n)(n\ell(n) - c(n)) - (1-p(n))c_u - \theta(p(n))k \Big] f(n)\mathrm{d}n - \int_{n_0}^{n^*} c_u f(n)\mathrm{d}n = 0$$

$$\forall n \geq n^* : \quad U(n) = p(n)(u(c(n)) - v(\ell(n))) + (1-p(n))u(c_u)$$

$$\forall n \geq n^* : \quad U'(n) = p(n)\frac{v'(\ell(n))\ell(n)}{n}$$

$$U(n^*) \geq u(c_u) + \delta$$

$$n^* \geq n_0,$$

where the last two conditions hold with complementary slackness. To reduce the number of control variables, invert the definition of expected utility $U(n)$ with respect to $c(n)$ and write $c(n) = \hat{c}(U(n), \ell(n), p(n), c_u)$. By the implicit function theorem:

$$\frac{\partial \hat{c}(\cdot)}{\partial U(n)} = \frac{1}{p(n)u'(c(n))}, \quad \frac{\partial \hat{c}(\cdot)}{\partial \ell(n)} = \frac{v'(\ell(n))}{u'(c(n))},$$
$$\frac{\partial \hat{c}(\cdot)}{\partial p(n)} = -\frac{u(c(n)) - v(\ell(n)) - u(c_u)}{p(n)u'(c(n), l(n))}, \quad \frac{\partial \hat{c}(\cdot)}{\partial c_u} = -\frac{1-p(n)}{p(n)}\frac{u'(c_u)}{u'(c(n))}. \tag{103}$$

The Lagrangian associated with the above maximization problem reads:

$$\begin{aligned}
\mathcal{L} = & \int_{n_0}^{n^*} \Psi(u(c_u)) f(n)\mathrm{d}n + \int_{n^*}^{n_1} \Psi(U(n) - \delta) f(n)\mathrm{d}n - \eta \Bigg[\int_{n_0}^{n^*} c_u f(n)\mathrm{d}n \\
& - \int_{n^*}^{n_1} (p(n)(n\ell(n) - \hat{c}(U(n), \ell(n), p(n), c_u)) - (1-p(n))c_u - \theta(p(n))k) f(n)\mathrm{d}n \Bigg] \\
& - \int_{n^*}^{n_1} \left(\lambda(n)p(n)\frac{v'(\ell(n))\ell(n)}{n} + \lambda'(n)U(n) \right) \mathrm{d}n + \lambda(n_1)U(n_1) - \lambda(n^*)U(n^*) \\
& + \mu(U(n^*) - u(c_u) - \delta) + \nu(n^* - n_0)
\end{aligned} \tag{104}$$

Here, $\lambda(n)$ is the multiplier on the incentive compatibility constraint and μ and ν are the multipliers associated with voluntary participation. Ignoring function arguments for notational convenience (except when the function argument is n), the first-order conditions are given by:

$$U(n): \quad \left(\Psi' - \frac{\eta}{u'} \right) f(n) - \lambda'(n) = 0 \tag{105}$$

$$\ell(n): \quad \eta p(n) \left(n - \frac{v'}{u'} \right) f(n) - \lambda(n)p(n)\frac{v' + v''\ell(n)}{n} = 0 \tag{106}$$

$$p(n): \quad \eta \left[n\ell(n) - c(n) + c_u + \hat{\Delta}(n) - \theta' k \right] f(n) - \lambda(n)\frac{v'\ell(n)}{n} = 0 \tag{107}$$

$$c_u: \quad \eta \int_{n^*}^{n_1} (1-p(n)) \left(\frac{u_0'}{u'} - 1 \right) f(n)\mathrm{d}n + \int_{n_0}^{n^*} (\Psi' u_0' - \eta) f(n)\mathrm{d}n - \mu u_0' = 0 \tag{108}$$

$$\begin{aligned}
n^*: \quad & -\Delta\Psi(n^*) f(n^*) + \eta(\theta k - p(n^*)(n^*\ell(n^*) - c(n^*) + c_u)) f(n^*) \\
& + \mu U'(n^*) + \nu = 0
\end{aligned} \tag{109}$$

$$U(n^*): \quad -\lambda(n^*) + \mu = 0 \tag{110}$$

$$U(n_1): \quad \lambda(n_1) = 0 \tag{111}$$

$$\lambda(n): \quad U'(n) = p(n)\frac{v'\ell(n)}{n} \tag{112}$$

$$\eta: \quad \int_{n^*}^{n_1} \Big[p(n)(n\ell(n) - c(n)) - (1-p(n))c_u - \theta(p(n))k\Big] f(n)\mathrm{d}n$$

$$- \int_{n_0}^{n^*} c_u f(n)\mathrm{d}n = 0 \tag{113}$$

$$\mu: \quad \mu(U(n^*) - u(c_u) - \delta) = 0, \quad \mu \geq 0, \quad U(n^*) - u(c_u) - \delta \geq 0 \tag{114}$$

$$\nu: \quad \nu(n^* - n_0) = 0, \quad \nu \geq 0, \quad n^* - n_0 \geq 0 \tag{115}$$

Here, $\hat{\Delta}(n) \equiv (u(c(n)) - v(\ell(n)) - u(c_u))/u'(c(n))$ is the utility gain an individual with ability n experiences if he or she finds a job, $u_0' \equiv u'(c_u)$ denotes the marginal utility of the unemployed and $\Delta\Psi(n) \equiv \Psi(U(n) - \delta) - \Psi(u(c_u))$ is the direct welfare effect if an individual with ability n decides to participate.

To obtain the first result from the proposition, combine (105) with the transversality condition (111):

$$\lambda(n) = \lambda(n_1) - \int_n^{n_1} \lambda'(m)\mathrm{d}m = -\int_n^{n_1} \left(\Psi' - \frac{\eta}{u'}\right) f(m)\mathrm{d}m. \tag{116}$$

Evaluate (116) at $n = n^*$ and substitute $\mu = \lambda(n^*)$ from (110) in (108). Rearranging gives the first result from Proposition 4.

To obtain an expression for the optimal income tax, divide (106) by $p(n)$ and use (116) to substitute out for $\lambda(n)$. Multiply the expression by $-u'(c(n))/v'(\ell(n))$ to find:

$$\eta\left(1 - \frac{nu'}{v'}\right) f(n) = \left(1 + \frac{v''\ell(n)}{v'}\right) \frac{u'}{n} \int_n^{n_1} \left(\Psi' - \frac{\eta}{u'}\right) f(m)\mathrm{d}m. \tag{117}$$

Use the household's first-order condition (5) to simplify the left-hand side, and divide by $\eta f(n)$:

$$\frac{T'(z(n))}{1 - T'(z(n))} = \left(1 + \frac{v''\ell(n)}{v'}\right) \frac{u' \int_n^{n_1} \frac{1}{u'} \left(1 - \frac{\Psi' u'}{\eta}\right) f(m)\mathrm{d}m}{nf(n)}. \tag{118}$$

As a final step, multiply (118) by $(1-F(n))/(1-F(n))$ and define $\varepsilon(n) \equiv v'(\ell(n))/(v''(\ell(n))\ell(n))$ as the Frisch elasticity of labor supply and $\tilde{g}(n) \equiv \Psi'(U(n) - \delta)u'(c(n))/\eta$ as the welfare weight attached to an employed individual with ability n. This leads to the second result from the proposition.

The final result is obtained as follows. First consider (107) and use (106) to substitute out for $\lambda(n)$:

$$\eta f(n)\left[\left(n\ell(n) - c(n) + c_u + \hat{\Delta}(n) - \theta' k\right) - n\ell(n)\frac{\varepsilon(n)}{1+\varepsilon(n)}\left(1 - \frac{v'}{nu'}\right)\right] = 0. \tag{119}$$

Next, substitute $c(n) = z(n) - T(z(n))$, $c_u = b$, $n\ell(n) = y(n)$, $1 - v'(\ell(n))/(nu'(c(n))) = T'(z(n))$ and use the zero-profit condition (2) to substitute out for $y(n) - z(n)$. Rearranging gives (32).

A.4 Proof proposition 5

The proof follows an almost identical structure as in Appendix A.3. The maximization problem reads:

$$\max_{c_u, n^*, [U(n), c(n), \ell(n), p(n)]_{n^*}^{n_1}} \mathcal{W} = \int_{n_0}^{n^*} \Psi(u(c_u)) f(n) \mathrm{d}n + \int_{n^*}^{n_1} \Psi(U(n) - \delta) f(n) \mathrm{d}n \tag{120}$$

$$\begin{aligned}
\text{s.t.} \quad & \int_{n^*}^{n_1} \Big[p(n) n \ell(n) - c(n) - \theta(p(n)) k \Big] f(n) \mathrm{d}n - \int_{n_0}^{n^*} c_u f(n) \mathrm{d}n = 0 \\
& \forall n \geq n^*: \; U(n) = u(c(n)) - p(n) v(\ell(n)) \\
& \forall n \geq n^*: \; U'(n) = p(n) \frac{v'(\ell(n)) \ell(n)}{n} \\
& U(n^*) \geq u(c_u) + \delta \\
& n^* \geq n_0.
\end{aligned}$$

Invert the definition of expected utility $U(n)$ and write $c(n) = \hat{c}(U(n), \ell(n), p(n))$. By the implicit function theorem:

$$\frac{\partial \hat{c}(\cdot)}{\partial U(n)} = \frac{1}{u'(c(n))}, \quad \frac{\partial \hat{c}(\cdot)}{\partial \ell(n)} = \frac{p(n) v'(\ell(n))}{u'(c(n))}, \quad \frac{\partial \hat{c}(\cdot)}{\partial p(n)} = \frac{v(\ell(n))}{u'(c(n), l(n))}. \tag{121}$$

The Lagrangian associated with the above maximization problem reads:

$$\begin{aligned}
\mathcal{L} = & \int_{n_0}^{n^*} \Psi(u(c_u)) f(n) \mathrm{d}n + \int_{n^*}^{n_1} \Psi(U(n) - \delta) f(n) \mathrm{d}n - \eta \Big[\int_{n_0}^{n^*} c_u f(n) \mathrm{d}n \\
& - \int_{n^*}^{n_1} (p(n) n \ell(n) - \hat{c}(U(n), \ell(n), p(n)) - \theta(p(n)) k) f(n) \mathrm{d}n \Big] \\
& - \int_{n^*}^{n_1} \left(\lambda(n) p(n) \frac{v'(\ell(n)) \ell(n)}{n} + \lambda'(n) U(n) \right) \mathrm{d}n + \lambda(n_1) U(n_1) - \lambda(n^*) U(n^*) \\
& + \mu(U(n^*) - u(c_u) - \delta) + \nu(n^* - n_0)
\end{aligned} \tag{122}$$

The first-order conditions are given by:

$$U(n): \quad \left(\Psi' - \frac{\eta}{u'} \right) f(n) - \lambda'(n) = 0 \tag{123}$$

$$\ell(n): \quad \eta p(n) \left(n - \frac{v'}{u'} \right) f(n) - \lambda(n) p(n) \frac{v' + v'' \ell(n)}{n} = 0 \tag{124}$$

$$p(n): \quad \eta \left[n \ell(n) - \frac{v}{u'} - \theta' k \right] f(n) - \lambda(n) \frac{v' \ell(n)}{n} = 0 \tag{125}$$

$$c_u: \quad \int_{n_0}^{n^*} (\Psi' u_0' - \eta) f(n) \mathrm{d}n - \mu u_0' = 0 \tag{126}$$

$$n^*: \quad -\Delta \Psi(n^*) f(n^*) + \eta(c(n^*) + \theta k - p(n^*) n^* \ell(n^*) - c_u) f(n^*) + \mu U'(n^*) + \nu = 0 \tag{127}$$

$$U(n^*): \quad -\lambda(n^*) + \mu = 0 \tag{128}$$

$$U(n_1): \quad \lambda(n_1) = 0 \tag{129}$$

$$\lambda(n): \quad U'(n) = p(n) \frac{v' \ell(n)}{n} \tag{130}$$

$$\eta: \quad \int_{n^*}^{n_1} \Big[p(n)n\ell(n) - c(n) - \theta(p(n))k\Big] f(n)\mathrm{d}n - \int_{n_0}^{n^*} c_u f(n)\mathrm{d}n = 0 \tag{131}$$

$$\mu: \quad \mu(U(n^*) - u(c_u) - \delta) = 0, \qquad \mu \geq 0, \quad U(n^*) - u(c_u) - \delta \geq 0 \tag{132}$$

$$\nu: \quad \nu(n^* - n_0) = 0, \qquad \nu \geq 0, \quad n^* - n_0 \geq 0 \tag{133}$$

The first two results are obtained in identical fashion as in Appendix A.3. The final result is obtained as follows. Consider (125) and use (124) to substitute out for $\lambda(n)$:

$$\eta f(n)\left[\left(n\ell(n) - \frac{v}{u'} - \theta' k\right) - n\ell(n)\frac{\varepsilon(n)}{1+\varepsilon(n)}\left(1 - \frac{v'}{nu'}\right)\right] = 0. \tag{134}$$

Next, replace $n\ell(n) = y(n)$ and use the zero-profit condition to substitute out for $y(n)$. Then, use the result $q(n) = p(n)(z(n) - T(z(n)) - b)$ to substitute out for $z(n)$. Rearranging and imposing the definitions of $\chi(\cdot)$ and $\tilde{\Delta}(\cdot)$ gives the desired result.