Optimal taxation with market power

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Consider the Mirrlees model. Allow for imperfect substitutability between labor types, so that wages are endogenous. Ignore spill-over effects. Pure profits are taxed away. I solved the standard Mirrlees-problem and the problem where wage setting is unionized (and rationing on the intensive margin). The problem I did not yet manage to solve was the optimal-tax problem with a monopsony (unless it could engage in first-degree price discrimination). Main reason was that the second derivative of the tax function shows up in the labor-market equilibrium condition (in particular, HH choose labor supply to maximize utility taking wages as given, firms choose wages and labor hours subject to the labor-supply equation). I think I have found a way to formulate the problem, though I have not yet solved it. Assume preferences are quasi-linear: $u(c,l) = c - v(l)$ with $v(\cdot)$ increasing and strictly convex. Denote income by z_n, taxes by T_n and labor supply by l_n. Social welfare:

$$\mathcal{W} = \int_{\mathcal{N}} \Psi(z_n - T_n - v(l_n))\mathrm{d}F_n$$

Concavity in $\Psi(\cdot)$ generates a motive for redistribution. Otherwise there might still be an efficiency motive. Government budget constraint:

$$0 = \int_{\mathcal{N}} (T_n + a_n g(l_n f_n)/f_n - z_n)\mathrm{d}F_n$$

Note: $g(x) = x$ and $a_n = n$ in Mirrlees problem. Labor-market equilibrium conditions:

$$\begin{aligned}
\dot{T}_n &= p_n \\
\dot{z}_n &= q_n \\
p_n &= (1 - l_n v'/z_n)q_n \\
\dot{p}_n &= p_n r_n/q_n + \left(\frac{l_n v'}{z_n^2} - \frac{v' + v'' l_n}{a_n g' z_n}\right) q_n^2 \\
\dot{q}_n &= r_n
\end{aligned}$$

State variables: T_n, z_n, p_n, q_n, controls: l_n, r_n. This looks like a standard optimal control problem with a path constraint. Variables p_n, q_n and r_n are introduced only to formulate this problem as a standard optimal control problem. The third condition is the labor-supply equation. The fourth equation is something of a mark-up equation.

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Much easier problem formulation: define two state variables u_n and $x_n = a_n l_n - z_n$. By the Envelope theorem (assuming linear production – which is not needed):

$$\dot{u}_n = v'(l_n)\frac{x_n}{a_n l_n - x_n}\dot{l}_n$$
$$\dot{x}_n = \dot{a}_n l_n$$

Welfare:

$$\mathcal{W} = \int_{\mathcal{N}} \Psi(u_n)\mathrm{d}F_n$$

Resource constraint:

$$0 = \int_{\mathcal{N}} (a_n l_n - v(l_n) - u_n)\mathrm{d}F_n$$

Introduce a variable $b_n = \dot{l}_n$ and this is a standard optimal control problem, which is must easier. State variables: u_n, l_n, x_n and control: b_n.

Relatively straightforward to derive end-point results (not done yet). Interesting, without a distributional motive ($\Psi(\cdot)$ linear) and iso-elastic labor supply, the tax function $T(z) = T(0) - z/\varepsilon$ restores first-best. Similar as with unions, where you also need a negative marginal tax. Intuition: output is restricted, so subsidies needed to restore efficiency in both cases.

Main difficulty: deriving an optimal tax formula for the interior, though I get quite far. In the FOC for labor supply there is a wedge, the multiplier on the IC and its derivative (for which I can easily obtain expressions), but also the multiplier on the restriction $\dot{x}_n = \dot{a}_n l_n$. I can try to obtain an expression for this using integration by parts, but that doesn't make life easier. Something strange might be going on: the multiplier is zero at the end-points and in the interior the *change* is proportional to the multiplier on the IC, and the change in labor supply. Seems odd.

Production:

$$Y = \left[\int_{\mathcal{N}} a(n) L(n)^{\frac{\sigma-1}{\sigma}} \mathrm{d}n \right]^{\frac{\sigma}{\sigma-1}}$$

Here, $L(n) = l(n)f(n)$ is the aggregate labor input of type n and $a(n)$ is a productivity shifter. The parameter $\sigma \geq 0$ is the constant elasticity of substitution. $\sigma = 0$ corresponds to Leontief production, $\sigma = 1$ to Cobb-Douglas production and $\sigma = \infty$ to perfect substitutability. The standard Mirrlees formulation is obtained when setting $a(n) = n$ and $\sigma = \infty$. Important to note: function exhibits CRS. Hence, when wages are equal to marginal products, profits are zero. Important incidence implication: wage increases of one group equal wage losses for other workers (see Lee and Saez, 2012). Also, σ equals the 'own-wage elasticity of labor demand' (i.e., the percentage decrease in employment if the wage increases by one percent, holding aggregate output fixed), which – importantly – need not be bigger than one. The types are ordered such that $a(n)f(n)^{\frac{\sigma-1}{\sigma}}$ is increasing in n, w.l.o.g.

Monopsony problem. Set $a_n = n$ and maintain the assumption of perfect substitutability between labor types (necessary to keep the problem tractable). In addition, there is another constraint to consider: $x_{\underline{n}} \geq 0$. The problem now reads:

$$
\begin{aligned}
\max_{[u_n, l_n, x_n, b_n]_{\mathcal{N}}} \quad & \mathcal{W} = \int_{\mathcal{N}} \Psi(u_n) \mathrm{d}F_n \\
\text{s.t.} \quad & \int_{\mathcal{N}} (nl_n - v(l_n) - u_n) \mathrm{d}F_n = 0 \\
\forall n: \quad & \dot{u}_n = v'(l_n) \frac{x_n}{nl_n - x_n} b_n \\
\forall n: \quad & \dot{x}_n = l_n \\
\forall n: \quad & \dot{l}_n = b_n \\
& x_{\underline{n}} \geq 0
\end{aligned}
$$

Here, u_n, l_n and x_n are the control variables and b_n is the state variable. In the FB: $n = v'(l_n)$ and u_n equalized (provided a motive for redistribution). It can be verified that the final constraint is binding. This implies that the wage equals productivity at the bottom. The government prevents the firm from extracting any rents from the lowest type. It can do so by ensuring the lowest type is infinitely responsive to changes in the wage. This can be done by 'cleverly' setting the first and second derivative of the tax function. There is also a result at the top: see the slides from the presentation in the research afternoon (20/4/2018). Obtaining an expression for optimal taxes (or wedges) in the interior is very challenging. Or, more precisely: it is very challenging to meaningfully interpret the optimality conditions. In this sense, the best I can get out of this is something like this:

$$
(\eta - n) v' f_n - \dot{\lambda}_n v' \frac{x_n}{nl_n - x_n} - \lambda_n v' \frac{l_n}{nl_n - x_n} + \int_n^{\bar{n}} \lambda_m v' \frac{ml_m}{(ml_m - x_m)^2} \dot{l}_m \mathrm{d}m = 0
$$

The last two terms can be shown to equal (using integration by parts):

$$
\int_n^{\bar{n}} \frac{1}{ml_m - x_m} \dot{\lambda}_m v' l_m \mathrm{d}m + \left(1 + \frac{1}{\varepsilon}\right) \int_n^{\bar{n}} \frac{1}{ml_m - x_m} \lambda_m v' \dot{l}_m \mathrm{d}m
$$

assuming a constant Frisch elasticity. Further simplifying the first or final term using integration by parts does not simplify the expression (in fact, I get exactly the same expression). Also 'repeated substitution' does not seem to work, unless this would imply (which I do not think is correct) the final term of the last line equals zero. This I can check numerically. Integration by substitution also does not really simplify matters. I am a little bit surprised that it is so difficult to obtain an optimal tax expression. The problem seems tractable.

The Lagrangian reads:

$$
\begin{aligned}
\mathcal{L} = & \int_{\underline{n}}^{\bar{n}} \left[(\Psi(u_n) + \eta(nl_n - u_n - v(l_n))\, f_n + \lambda_n v'(l_n) \frac{x_n}{nl_n - x_n} b_n + \dot{\lambda}_n u_n + \mu_n l_n + \dot{\mu}_n x_n + \nu_n b_n + \dot{\nu}_n l_n \right] \mathrm{d}n \\
& + \lambda_{\underline{n}} u_{\underline{n}} - \lambda_{\bar{n}} u_{\bar{n}} + \mu_{\underline{n}} x_{\underline{n}} - \mu_{\bar{n}} x_{\bar{n}} + \nu_{\underline{n}} l_{\underline{n}} - \nu_{\bar{n}} l_{\bar{n}} + \xi x_{\underline{n}}
\end{aligned}
$$

The first-order conditions are:

$$
\begin{aligned}
u_n: &\quad (\Psi' - \eta) f_n + \dot{\lambda}_n = 0 \\
x_n: &\quad \lambda_n v' \frac{n l_n}{(n l_n - x_n)^2} b_n + \dot{\mu}_n = 0 \\
l_n: &\quad \eta(n - v') f_n + \lambda_n \left[v'' \frac{x_n}{n l_n - x_n} - v' n \frac{x_n}{(n l_n - x_n)^2} \right] b_n + \mu_n + \dot{\nu}_n = 0 \\
b_n: &\quad \lambda_n v' \frac{x_n}{n l_n - x_n} + \nu_n = 0 \\
\lambda_n: &\quad \dot{u}_n = v' \frac{x_n}{n l_n - x_n} b_n \\
\mu_n: &\quad \dot{x}_n = l_n \\
\nu_n: &\quad \dot{l}_n = b_n \\
\eta: &\quad \int_{\mathcal{N}} (n l_n - u_n - v) f_n \mathrm{d}n = 0 \\
u_{\bar{n}}: &\quad \lambda_{\bar{n}} = 0 \\
u_{\underline{n}}: &\quad \lambda_{\underline{n}} = 0 \\
x_{\bar{n}}: &\quad \mu_{\bar{n}} = 0 \\
x_{\underline{n}}: &\quad \mu_{\underline{n}} + \xi = 0 \\
l_{\bar{n}}: &\quad \nu_{\bar{n}} = 0 \\
l_{\underline{n}}: &\quad \nu_{\underline{n}} = 0 \\
\xi: &\quad \xi x_{\underline{n}} = 0, \quad \xi \geq 0, \quad x_{\underline{n}} \geq 0
\end{aligned}
$$

The first of these combined with the transversality conditions yields:

$$\int_{\mathcal{N}} \left(\frac{\Psi'}{\eta} - 1 \right) f_n \mathrm{d}n = 0$$

Furthermore, it is straightforward to verify that $\lambda_n < 0$ almost everywhere and that $\dot{\lambda}_n > 0$ iff $g_n < 1$, where $g_n = \Psi'/\eta$ denotes the welfare weight. Furthermore, differentiating the first-order condition with respect to b_n and integrating the first-order condition with respect to x_n combined with the transversality condition on $x_{\bar{n}}$ yields – after substituting in the first-order condition for l_n:

$$0 = \eta(n - v') f_n - \dot{\lambda}_n v' \frac{x_n}{n l_n - x_n} - \lambda_n v' \frac{l_n}{n l_n - x_n} + \int_n^{\bar{n}} \lambda_m v' \frac{m l_m}{(m l_m - x_m)^2} \dot{l}_m \mathrm{d}m$$

Also, it can be verified that $x_{\underline{n}} = 0$, which implies $\underline{n} = w_{\underline{n}}$. Other properties: first best can be implemented if Ψ is linear. In that case, $n = v'$ for all n. In the iso-elastic case, this can be implemented by setting $T'(z_n) = -1/\varepsilon$, where ε is the Frisch elasticity of labor supply. At the top, we find:

$$\frac{\bar{n}}{v'} - 1 = (1 - g_{\bar{n}}) \left[\frac{\bar{n}}{w_{\bar{n}}} - 1 \right]$$

Very interesting: if $g_{\bar{n}} = 0$, then this result implies:

$$v'(l_{\bar{n}}) = w_{\bar{n}}$$

which is equivalent to saying $T' = 0$ at the top. This does not mean labor supply is undistorted. In fact, labor supply is too low at the top (note: wage is below MPL). Furthermore, if $g_{\bar{n}} > 0$, then the marginal income tax is negative at the top: $T' < 0$. At the bottom, you can make sure the firm does not extract any rents by making individuals extremely responsive to tax changes. This you can achieve by setting the curvature.

What other properties can we show? Assuming $\dot{l}_n \geq 0$, it must be that $n > v'$ at the bottom (second and third term are zero, fourth term is negative). So again, labor supply is distorted downward. What if the government is Rawlsian? Doesn't make things much easier.

Other nice result. Marginal tax is positive at the bottom, but tax function is very concave. Purpose is to make labor supply very elastic, so that firms cannot exploit market power. Hence, marginal tax rates are positive but decreasing at the bottom. The 'decreasing' I understand, but not so much why they are positive.

Perturbation approach

Maybe it is possible to solve the problem using the perturbation approach. The perturbation I have in mind is the same as in the standard problem: increase the marginal tax rate in some interval $[Z, Z+\delta]$ by $\mathrm{d}T'$. For individuals with income above Z, the tax bill increases by $\delta\mathrm{d}T'$. A complication compared to the standard problem is that in the monopsony problem the outcomes are also affected by the curvature of the tax function (i.e., by the second derivative $T''(z)$). This has an important implication: in order to raise the marginal tax rate by $\mathrm{d}T'$, the curvature of the tax function has to be raised in an interval *before* Z, and reduced after $Z+\delta$. Local changes in the curvature would normally not affect labor-market outcomes, but here they do! So how does this work? The idea would be to raise the curvature in the tax function in the interval $[Z-\gamma, Z]$ by $\mathrm{d}T''$. This raises the marginal tax rate by $\mathrm{d}T' = \gamma\mathrm{d}T''$, which is applied on the interval $[Z, Z+\delta]$. To maintain the marginal tax rates for higher income levels, the increase is reversed by reducing the curvature in the interval $[Z+\delta, Z+\delta+\gamma]$ by $\mathrm{d}T''$. For individuals with income above Z (or strictly, above $Z+\delta$), the tax bill is increased by $\delta\mathrm{d}T' = \gamma\delta\mathrm{d}T''$. Then I have to be a bit clever in taking limits with respect to γ and δ. Look at the paper by Kevin for this.

Following the approach by Spiritus, I consider the following policy reform. At some interval $[Z, Z+\mathrm{d}Z]$, I raise the *second* derivative of the tax function by $\mathrm{d}T'' > 0$. Then, at income level $Z+\delta$, with $\delta \gg \mathrm{d}Z$, I reverse this reform (i.e., lower the second derivative of the tax function by $\mathrm{d}T''$ over an interval width $\mathrm{d}Z$).

A first question we have to ask is: how does this reform affect the tax function $T(z)$? First, in the interval $[Z, Z+\delta]$ (more precisely: $[Z+\mathrm{d}Z, Z+\delta-\mathrm{d}Z]$) the marginal tax rate increases by $\mathrm{d}T' = \mathrm{d}T''\mathrm{d}Z$. Consequently, for all income levels above Z (or, more precisely, above $Z+\delta$), the tax bill increases by $\mathrm{d}T = \mathrm{d}T'\delta = \mathrm{d}T''\mathrm{d}Z\delta$. In addition, in two intervals (of width $\mathrm{d}Z$), the second derivative of the tax function is changed by $\mathrm{d}T''$ and $-\mathrm{d}T''$, respectively.

For notation, let me denote by $H(z)$ the income distribution with corresponding density $h(z)$. I use the convention from Spiritus (2018) regarding derivatives and function arguments. What is probably good to emphasize is that I can write $l(z)$ as being the labor supply corresponding to an individual with earnings z. Similarly, I can denote by $n(z)$ the ability of an individual earning income z.

Let's re-state the labor-market equilibrium conditions:

$$z(1-T'(z)) = v'(l)l$$

$$\frac{n}{v'(l)}\left[(1-T'(z)) - zT''(z)\right] = 1 + \frac{1}{\varepsilon}$$

Here, ε is the Frisch elasticity of labor supply (assumed to be independent of l) and quasi-linear preferences are assumed. A first noteworthy observation: there are no income effects. Hence, l and z are not affected by a change in the level of taxes.

The Lagrangian of the government problem is:

$$\mathcal{L} = \int_{\mathcal{Z}} \left[\Psi(z - T(z) - v(l)) + \eta\Big(T(z) + nl - z\Big)\right]\mathrm{d}H(z)$$

where it is to be understood that both l and z respond endogenously to changes in the tax system $T(z)$. In the remainder, I scale everything by η (so that the objective is $\mathcal{L}/\eta$).

What are all the welfare relevant effects? First, consider the direct effects of changing the second derivative (and reversing this reform). The welfare effect is:

$$\left[\left(\left(\frac{\Psi'}{\eta}-1\right)(1-T')z_{T''}-\left(\frac{\Psi'v'}{\eta}-n\right)l_{T''}\right)h\right](Z)\mathrm{d}T''\mathrm{d}Z$$
$$-\left[\left(\left(\frac{\Psi'}{\eta}-1\right)(1-T')z_{T''}-\left(\frac{\Psi'v'}{\eta}-n\right)l_{T''}\right)h\right](Z+\delta)\mathrm{d}T''\mathrm{d}Z$$

Here, both (Z) and $(Z+\delta)$ indicate that the functions are evaluated at these points.

Second, the change in the marginal tax rates generates welfare-relevant effects in the interval $[Z, Z+\delta]$. These are equal to:

$$\int_{Z}^{Z+\delta}\left[\left(\left(\frac{\Psi'}{\eta}-1\right)(1-T')z_{T'}-\left(\frac{\Psi'v'}{\eta}-n\right)l_{T'}\right)h\right](z)\mathrm{d}z\times\mathrm{d}T''\mathrm{d}Z$$

By the way: note the subscripts. These indicate the derivatives.

Third, and finally, there is a transfer for all individuals with income above Z. The welfare effect:

$$\int_{Z}^{\infty}\left(1-\frac{\Psi'}{\eta}\right)\mathrm{d}H(z)\times\delta\mathrm{d}T''\mathrm{d}Z$$

Now, proceed as follows. Set the sum of all welfare effects equal to zero. The common term $\mathrm{d}T''\mathrm{d}Z$ cancels immediately. Then, divide by δ and take the limit as $\delta\to 0$, keeping $\mathrm{d}Z \ll \delta$. Defining by g_z the welfare weight of an individual with income z, this leads to:

$$\frac{\mathrm{d}}{\mathrm{d}z}\left[\left((g_z-1)(1-T')z_{T''}-(g_zv'-n)l_{T''}\right)h\right](Z)$$
$$=\int_{Z}^{\infty}(1-g_z)\mathrm{d}H(z)+\left[(g_z-1)(1-T')z_{T'}-(g_zv'-n)l_{T'}\right](Z)$$

OK, so this is a complicated expression. You see immediately why the mechanism design problem was so difficult. Plus, as expected, integrating this expression over z also implies integrating over an integral.

Can we simplify these expressions a bit? Maybe possible to combine some of the terms regarding the derivatives of l and z. I can use the first-order conditions for this purpose. This is all fairly straightforward. However, it doesn't significantly simplify the conditions (except that I can substitute out for $l_{T''}$ and $l_{T'}$). Yet another possibility for simplification is if I can use the property $z = wl$ and express some of the terms in wage changes. Again, I do not believe this is of much help.

Additional note: we can apply the Envelope theorem on $nl - z$. Only works for T'', not for T' I think (because $nl - z$ is maximized s.t. $z(1 - T'(z)) = v'(l)l$, where $T'(z)$ shows up explicitly).

This paper explores the implications of labor market power for optimal redistributive taxation. To do so, I extend the Mirrlees (1971) framework with endogenous wages, which are either (i) determined competitively, (ii) set by workers (through unions), or (iii) set by a monopsonistic firm. Three robust findings emerge. First, a non-linear tax on labor income can always restore efficiency. Second, market power restricts output, which calls for lower marginal taxes. Third, the optimal marginal tax is never positive at the top. Welfare under perfect competition is generally higher than with unions, but might be lower than with a monopsony.

What properties can we show using the MD formulation? First, average welfare weight equals one. Second, $w = n$ at the bottom of the distribution. This implies $1 - T' = -\varepsilon z T''$. Assuming $\dot{l}_n \geq 0$, it follows that $T' > 0$ at the bottom. But $T'' < 0$. Hence, positive bottom rate, but decreasing. Alternative presentation: $\varepsilon_{lw} \to \infty$. Tax function is concave at the bottom to prevent firms from extracting surplus. Regarding the top: top rate is negative if $g > 0$ and zero if $g = 0$.