Taxation and sorting

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Quick summary
How does taxation affect sorting patterns in the labor market? I study this question in a competitive search environment with non-linear taxes on labor income. Taxation hinders positive assortative matching if the tax system is progressive and the matching elasticity is decreasing.

Background
A classic question in economics is: who trades with whom? This question is relevant in the context of two-sided heterogeneity (say, heterogeneous buyers and sellers). Classic examples are the marriage market and the labor market. I focus on the second. In a famous paper, Becker (1973) shows that in a competitive environment one obtains positive assortative matching (PAM, meaning high types trade with each other) whenever the production function – which maps types into output – is supermodular. Under supermodularity, there are complementarities in production. The most productive worker is most valuable in the most productive firm and as a result, the latter can always outbid its competitors for that particular worker's services.

The analysis of Becker (1973) assumes the process by which buyers and sellers meet is frictionless. However, with two-sided heterogeneity, it is natural to think that frictions are important: it takes time and resources to figure out what the right match is. Shimer and Smith (2000) analyze sorting patterns in an environment with frictions. They show that for equilibrium to exhibit PAM, complementarities in production need to be stronger than in the absence of frictions. In particular, log-supermodularity is required. The latter implies supermodularity, but not the other way around. The reason why stronger complementarities are required is because frictions give rise to a trade-off between prices on the one hand and the probability or speed of trading on the other. If you want a good deal, you have to accept a lower probability of trading or search longer. Highly productive workers and firms, in turn, have the highest opportunity costs of not trading. Both parties want to make sure to be matched. But this can only be achieved if highly productive firms and workers 'shy away' from each other. Frictions thus provide a force for NAM, which can be overcome if complementarities in production are sufficiently strong.

The model from Shimer and Smith (2000) assumes random search, meaning that neither buyers nor sellers can influence who they run into. Eeckhout and Kircher (2010) analyze under which conditions the equilibrium is positively (or negatively) assorted whenever market participants can direct their search. In their model, search is competitive. Prices are posted in advance and play an allocative role similar to the role of prices in competitive equilibrium. They find that PAM obtains under a condition on the complementarities 'in between' supermodularity and log-supermodularity (in particular, the relevant condition is n-root supermodularity, where n reflects the elasticity of substitution in the search technology).

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The question I take up in this note is how sorting patterns in the labor market are affected by income taxes. Income taxes significantly alter the costs and benefits of searching for better jobs or for better workers and may therefore affect sorting patterns. To answer this question, I extend the model from Eeckhout and Kircher (2010) with non-linear income taxes. The tax function I consider is one with a constant rate of progressivity (CRP). According to Heathcote et al. (2017), this tax function offers a remarkably good representation of the current U.S. tax and transfer schedule. I derive a condition on the complementarities in production under which PAM obtains. This condition is stronger than the one derived in Eeckhout and Kircher (2010) if two reasonable assumptions are met: (i) the tax system is progressive, (ii) the matching elasticity is decreasing.

Model

The model I analyze is an extension of Eeckhout and Kircher (2010). I follow the presentation from Chade et al. (2017), in particular section 4.4.2. Workers ('sellers') are denoted by y and firms ('buyers') by x. The mass of firms and workers is fixed. If a worker of type y matches with a firm of type x, production is $f(x,y)$. The function $f(\cdot)$ is twice continuously differentiable and increasing in both its arguments. Firms post a wage w, and workers optimally choose where to apply. Workers can apply once and cannot coordinate their application strategies. If there are constant returns to scale in matching, the probability that a worker is matched $m(\theta)$ depends only on the buyer-seller ratio θ, which may vary across 'sub-markets'. The function $m(\cdot)$ is increasing, concave, and satisfies $m(0) = 0$. Furthermore, the matching elasticity $\eta(\theta) = m'(\theta)\theta/m(\theta) \in [0,1]$ is assumed to be non-increasing. As firms and workers meet in pairs, the probability that a firm is matched is given by $q(\theta) = m(\theta)/\theta$.

A worker of type y chooses which firm x and which sub-market (w,θ) to visit. Assuming linear utility, the optimization problem reads:

$$\max_{x,\theta,w} \quad m(\theta)(w - T(w)) \quad \text{s.t.} \quad q(\theta)(f(x,y) - w) = v(x), \tag{1}$$

where $v(x)$ is the payoff of firms and $T(\cdot)$ is the tax function. Since neither firms nor workers have market power, workers take 'market utility' $v(x)$ as given: they need to promise a firm of type x at least $v(x)$. Next, substitute $w = f(x,y) - v(x)/q(\theta)$ from the constraint in the optimization problem and consider the first-order condition with respect to θ:

$$m'(\theta)\left(f(x,y) - \frac{v(x)}{q(\theta)}\right)(1-t_a) = -\frac{q'(\theta)\theta}{q(\theta)}v(x)(1-t_m). \tag{2}$$

Here, $t_a \equiv T(w)/w$ and $t_m \equiv T'(w)$ denote the average and the marginal tax rate, respectively. This condition equates the marginal benefits of visiting a market with a higher buyer-seller ratio to the marginal costs. A higher buyer-seller ratio implies a higher matching probability: $m'(\theta) > 0$. The latter is multiplied by the net wage $w - T(w) = w(1-t_a) = (f(x,y) - v(x)/q(\theta))(1-t_a)$ to obtain the marginal benefits. Turning to the marginal costs, a higher buyer-seller ratio implies that firms have a lower probability of matching. For their utility to remain constant (recall: workers take $v(x)$ as given), workers must accept a lower wage. The right-hand side captures the wage sacrifice workers must make for firm's utility to remain constant if the buyer-seller ratio increases.

From (2), it can readily be seen that the optimal buyer-seller ratio (and hence, the probability that workers find a match) increases in the marginal tax rate and decreases in the average tax rate. If the average tax rate is high, the benefits of finding a job are low. This induces workers to apply for a high-wage job and accept a low probability of finding a match. Conversely, if the marginal tax rate is high, workers care less about a marginally higher wage. Consequently, they will apply for a job with a somewhat lower wage, where they have a higher chance of being matched.

To proceed, divide both sides of (2) by $1-t_a$. In what follows, the coefficient $p = 1-(1-t_m)/(1-t_a) = (t_m-t_a)/(1-t_a)$ is going to play an important role. It is a measure of the difference between the marginal and the average tax rate and can be thought of as a measure of tax progressivity. A tax function with a constant rate of progressivity (CRP) is the following:

$$T(w) = w - \gamma w^{1-p} \tag{3}$$

with $p < 1$ by assumption. If $p > 0$ ($p < 0$), the tax system is said to be progressive (regressive). In this case, both marginal and average tax rates are increasing (decreasing) with income. If $p = 0$, the tax system is proportional. This particular choice of the tax function is not merely convenient from a theoretical perspective. According to Heathcote et al. (2017), it provides a very good approximation of the current U.S. tax and transfer schedule.

Using this tax function and the property $m(\theta) = \theta q(\theta)$, the first-order condition can be rearranged to:

$$m'(\theta)\left(f(x,y) - \frac{v(x)}{q(\theta)}\right) = (1-\eta(\theta))v(x)(1-p), \tag{4}$$

where $\eta(\theta)$ is the matching elasticity, assumed to be non-increasing. The more progressive the tax system (i.e., the higher p), the less workers care about wage increases (i.e., the lower the costs of accepting a lower wage). As a result, tax progressivity leads to wage moderation, which translates into a higher buyer-seller ratio and hence, a lower unemployment rate.

Under which condition do we obtain PAM? This happens if a more productive worker is willing to 'pay' more to get matched with a better firm. Put differently, PAM occurs if it is less costly for a more productive worker to promise a higher utility to a more productive firm. In this case, the more productive workers can outbid less productive workers when trying to match with a more productive firm. Technically, the relevant requirement is that the marginal rate of substitution between x (the firm's type) and v (the firm's reservation utility) is increasing in worker type y: see Chade et al. (2017). To derive the MRS, denote by

$$\psi(x,y,v) = \gamma m(\theta(x,y,v))\left(f(x,y) - \frac{v}{q(\theta(x,y,v))}\right)^{1-p} \tag{5}$$

the utility of worker y if matched with a firm of type x which is promised a utility of v. Here, I substituted out for the tax function using (3) and the optimal queue length $\theta(x,y,v)$ solves (4). The MRS is given by $-\psi_x/\psi_v$, or

$$MRS = q(\theta(x,y,v))f_x(x,y) \tag{6}$$

by the Envelope theorem (recall: θ is maximized). As mentioned, the requirement for PAM is that the MRS is increasing in y. Following the derivations in Chade et al. (2017), this requirement boils down to (after some algebra):

$$\left(\frac{ff_{xy}}{f_xf_y}\right) > \xi(\theta)\left[\frac{1-p(1-\eta(\theta))}{1-p(1-\xi(\theta)\eta(\theta))}\right] \tag{7}$$

This is the counter-part of Lemma 1 in Eeckhout and Kircher (2010). The left-hand side is a measure for the complementarities in production. The right-hand side is related to the matching function and the coefficient of tax progressivity. The term $\xi(\theta) = m'(\theta)(\theta m'(\theta) - m(\theta))/(m''(\theta)m(\theta)\theta)$ is the elasticity of substitution of the aggregate search technology, which plays an important role in the analysis of Eeckhout and Kircher (2010).

Before turning to interpret this result, first suppose taxes are absent, or proportional. In both cases, $p = 0$ and (7) is identical to the condition derived in Eeckhout and Kircher (2010). Under the (natural) condition that the matching elasticity $\eta(\theta)$ is non-increasing, it can be shown that $\xi(\theta) \in [0, 1]$. If the matching elasticity is constant, $\xi(\theta) = 1$ and log-supermodularity is required for the equilibrium to be positively assorted (as in Shimer and Smith, 2000). If the matching elasticity is decreasing, a weaker condition than log-supermodularity suffices: see Eeckhout and Kircher (2010). Finally, the condition boils down to supermodularity if $\xi(\theta) = 0$, which occurs if $m(\theta) = \theta$. In this case, it is as if frictions disappear. Naturally, the condition then coincides with the condition derived by Becker (1973) (i.e., supermodularity).

Now, suppose there is a progressive tax system ($p > 0$) and suppose that the matching elasticity is decreasing, so that $\xi(\theta) < 1$. In this case, the second term on the right-hand side is larger than one. Consequently, for given $\xi(\theta)$, stronger complementarities in production are required to obtain PAM. In that sense, progressive taxation hinders positive assortative matching. The intuition behind this result is fairly subtle, and can best be understood by combining a number of observations.

Observation 1: If the buyer-seller ratio increases in worker type y (i.e., if more productive workers trade with higher probability), stronger complementarities are required to obtain PAM.

Recall, the most productive workers (firms) have most to lose from not trading. In equilibrium, they will make sure to have a high matching probability. However, if one side of the market trades fast (i.e., with high probability), the other side must trade slow (i.e., with low probability). Absent any complementarities in production, high productive firms and workers thus 'shy away' from each other and the equilibrium is negatively assorted. This explains why complementarities must be sufficiently strong to obtain PAM if there are search frictions (cf. Shimer and Smith, 2000).

Observation 2: If the matching elasticity is decreasing, more productive workers must make larger wage sacrifices if they want to visit a market with a higher buyer-seller ratio.

To obtain an increase in the matching probability, workers must visit a market with a higher buyer-seller ratio. Doing so, however, comes at a cost. An increase in the buyer-seller ratio means firms have a lower chance of finding a match. For firms to accept this, workers must make a wage sacrifice. If the matching elasticity is decreasing, additional buyers become less and less efficient in generating additional matches – in percentage terms. As a result, the wage sacrifice workers must make increases in the buyer-seller ratio. In terms of (4), the right-hand side increases in θ if the matching elasticity is decreasing (and is constant if the matching elasticity is constant). And because more productive workers visit markets with higher buyer-seller ratios, high types must make larger wage sacrifices.

Observation 3: Making a wage sacrifice (i.e., accepting a lower wage) is less costly if the tax system is progressive.

If the tax system is progressive (i.e., the marginal tax rate is high compared to the average tax rate), workers do not care much about a marginal increase in the wage. Conversely, accepting a lower wage is less costly if the tax system is progressive.

If the matching elasticity is decreasing, more productive workers must make larger wage sacrifices if they want to visit a market with a higher buyer-seller ratio (observation 2). This is less costly if the tax system is progressive (observation 3). Combined, these observations imply that the buyer-seller ratio increases *faster* in worker productivity as a result of progressive taxation. By observation 1, stronger complementarities are required to obtain PAM. This is why the requirement (7) is stronger than in Eeckhout and Kircher (2010). In that sense, taxation hinders positive assortative matching if the tax system is progressive and the matching elasticity decreasing. This finding has two (potentially) important implications. First, sorting patters are efficient in the decentralized allocation (Eeckhout and Kircher, 2010). Consequently, if progressive taxation leads to different sorting patterns, this comes at an efficiency cost. Second, if it is the case that tax progressivity hinders PAM, this model could potentially help explain why pre-tax wage inequality is higher in countries with a more progressive tax system (of course, in addition to many other reasons).

References

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