

# Tax curvature<sup>\*</sup>

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## Abstract

In a Mirrleesian environment, a monopsonist sets hourly wages and individuals choose how many hours to work. Labor market outcomes do not only depend on the level and slope of the income tax function, but also on its curvature. A more concave tax schedule raises the elasticity of labor supply, which boosts wages. Consequently, optimal marginal tax rates for low-skilled workers are declining in income. I derive an optimal tax formula in terms of sufficient statistics that accounts for the impact of tax curvature on labor market outcomes.

*JEL classification: H21, J38, J42*

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# 1 Introduction

Firms exercise significant market power when setting wages. How much market power they can exert depends critically on the elasticity of labor supply (Robinson, 1933, Manning, 2003). This statistic is not policy-invariant. I show that this insight has an implication for tax policy. In particular, if wages are set by a monopsonist and individuals choose how many hours to work, the government can use the *curvature* of the tax function to boost wages of low-skilled workers by raising their elasticity of labor supply. This is achieved by setting marginal tax rates that are declining in income. Consequently, the optimal tax schedule at the bottom of the income distribution is concave.

To reach this conclusion, I study a Mirrleesian environment where individuals differ in their ability, which is not observed by the government. Individuals take their hourly wage and the tax schedule as given and optimally choose how many hours to work. Wages are not determined competitively as in Mirrlees (1971), but instead set by a monopsonist that observes ability and maximizes profits taking into account how wages affect individual labor supply. The government has a preference for redistribution and levies a nonlinear tax on labor earnings and a confiscatory tax on profits.

A novel feature is that labor market outcomes do not only depend on the level and slope of the tax function, but also on its curvature, i.e., the the second derivative. The latter determines how responsive individuals are to wage changes. To illustrate, suppose the tax function is convex and marginal tax rates increase steeply with income. Individuals then have weak incentives to work longer hours following an increase in the hourly wage. A low elasticity of labor supply, in turn, implies that a profit-maximizing monopsonist sets low wages. Consequently, a local increase in the *second* derivative of the tax function reduces the hourly wage, hours worked and labor earnings.

A government that is interested in redistribution can exploit this feature to boost wages of low-skilled workers. I characterize the second-best allocation and show that the tax schedule that implements it is concave at the bottom of the income distribution. Declining marginal tax rates at low earnings raise the labor supply elasticity of low-skilled workers, which increases their wages. The government sets the curvature of the tax function in such a way that the monopsonist does not extract any rents from hiring the least-skilled workers: if the tax system is optimized, the monopsonist pays the least-skilled workers a wage equal to their productivity. The optimal tax system thus off-sets any distortions from market power at the bottom of the income distribution.

The finding that optimal marginal tax rates for low-skilled workers are declining in income is a local result. I also derive an optimal tax formula that holds at each point in the income distribution. To that end, I study the welfare effects of increasing the *second* derivative of the tax function just below a particular income level, and decreasing it right above. This reform induces a local increase in the marginal tax rate, cf. Saez (2001) and Golosov et al. (2014). The optimal tax formula is obtained by setting the sum of the welfare-relevant effects from this reform equal to zero. Compared to existing results from the literature, the additional ‘sufficient statistics’ that characterize optimal tax policy are the impact of tax curvature on labor earnings and hourly wages, and the impact of the level and slope of the tax function on hourly wages and profits.

A number of recent papers study optimal redistributive taxation in an environment where firms have market power, either in the market for goods (Kaplow, 2019, Boar and Midrigan, 2020, Kushnir and Zubrickas, 2020, Eeckhout et al., 2021, Gürer, 2021) or, as in the current paper, the market for labor (Hariton and Piaser, 2007, Cahuc and Laroque, 2014, da Costa and Maestri, 2019, Hummel, 2021).

Cahuc and Laroque (2014) study the desirability of minimum wages alongside an optimal income tax if firms observe ability and labor supply responds on the extensive (participation) margin. By contrast, Hariton and Piaser (2007) and da Costa and Maestri (2019) characterize optimal tax policy in an environment where firms do not observe ability and workers supply labor on the intensive (hours, effort) margin. Hummel (2021) analyzes the implications of monopsony power for optimal income taxation and welfare if firms observe ability and labor supply responds on both the intensive and extensive margin. As in Hariton and Piaser (2007) and da Costa and Maestri (2019), firms offer workers contracts that specify labor effort and earnings. The key difference with these studies is that in the current paper, the monopsonist only sets hourly wages as individuals choose how many hours to work. An important implication is that, unlike in the aforementioned studies, labor market outcomes depend on the curvature of the tax function, which is the main focus of this paper.

It is well known that measures of labor supply or earnings responses to wage or tax changes, such as the elasticity of taxable income (ETI), depend on the curvature of the tax function. See, among others, Saez (2001) and Jacquet and Lehmann (2021) for a discussion of this issue. A key difference is that in my model, the curvature of the tax function affects labor market outcomes directly and not only the behavioral responses to wage or tax changes. Slemrod and Kopczuk (2002) derive an optimal elasticity of taxable income in an environment where the government can affect this elasticity using administrative instruments. By contrast, in my model the government uses the curvature of the tax function to affect the elasticity of labor supply.

The wage responses to tax changes that modify optimal tax formulas are reminiscent of the literature that studies optimal income taxation with general equilibrium effects on wages. See, for example, Stiglitz (1982), Rothschild and Scheuer (2013) and Sachs et al. (2020). In these studies, labor markets are perfectly competitive and wages respond to tax changes because labor types are imperfect substitutes in production. By contrast, in my model labor types are perfect substitutes in production and wages respond to tax changes because firms have labor market power.

The remainder of this paper is organized as follows. Section 2 presents the model and analyzes the impact of tax curvature on labor market outcomes. Section 3 studies the implications for optimal marginal tax rates at the bottom. Section 4 derives an optimal tax formula that holds at each point in the income distribution. Finally, Section 5 concludes.

## 2 Model

There is a continuum of individuals who differ in their ability  $n \in [n_0, n_1]$  with  $n_1 > n_0 > 0$ . Ability measures how much output an individual produces per hour worked. The cumulative distribution of ability is denoted by  $F(n)$  with density  $f(n)$ . Individuals supply labor on the intensive margin to a single monopsonist. The monopsonist observes ability and sets the hourly wage at each ability level in order to maximize profits. The government does not observe individual ability but only their labor earnings. It has a preference for redistribution and levies a nonlinear tax on labor earnings. To focus exclusively on labor income taxation, I assume profits are taxed at a confiscatory rate or, equivalently, that firm ownership is equally distributed. The timing is as follows.

1. The government chooses the tax schedule  $T(\cdot)$  on labor income  $z(n) = w(n)l(n)$ .
2. The monopsonist sets the hourly wage  $w(n)$  at each ability level  $n \in [n_0, n_1]$ .

3. Individuals choose how many hours  $l(n)$  to work.

Before characterizing the equilibrium, a few remarks are in place. First, the assumption of a single monopsonist that perfectly observes ability is of course extreme. However, it captures that (i) as in Mirrlees (1971), firms have an informational advantage compared to the government and (ii) the labor supply curve a firm faces is less than perfectly elastic (Manning, 2003). Second, despite that the firm is a monopsonist, it can only offer (linear) compensation contracts that specify an hourly wage, as individuals choose their own working hours.<sup>1</sup> If instead firms could offer a combination of earnings and working hours, they would typically generate higher profits. See Hariton and Piaser (2007), da Costa and Maestri (2019) and Hummel (2021) for an analysis of this case. While it seems plausible that firms can also influence working hours through other means than the hourly wage, I conjecture that the main effects and results derived below also hold in richer, more realistic environments, provided workers have some say over their working hours and provided firms with labor market power have an informational advantage compared to the government about their workers' abilities.

Working backward, an individual with ability  $n$  takes the hourly wage  $w(n)$  and the tax schedule  $T(\cdot)$  as given. Preferences over consumption  $c$  and labor effort  $l$  are described by a separable utility function  $U(c, l) = u(c) - \phi(l)$ , with  $u'(\cdot), \phi'(\cdot), \phi''(\cdot) > 0$  and  $u''(\cdot) \leq 0$ . The individual chooses how many hours to work in order to maximize utility:

$$v(n) = \max_l \left\{ u(w(n)l - T(w(n)l)) - \phi(l) \right\}. \quad (1)$$

The first-order necessary condition determines the optimal choice of labor effort  $l(n)$ :

$$u'(w(n)l(n) - T(w(n)l(n)))w(n)(1 - T'(w(n)l(n))) = \phi'(l(n)). \quad (2)$$

At the optimum, the marginal benefits of working an extra hour (on the left-hand side) are equal to the marginal costs (on the right-hand side). In what follows, I assume the tax function is such that the second-order condition for utility maximization is satisfied, so that equation (2) pins down the number of hours worked as a function of the hourly wage.

The monopsonist sets the wage  $w(n)$  at each ability level in order to maximize profits. It takes the tax schedule  $T(\cdot)$  as given, but has to take into account that wages affect hours worked, cf. equation (2). The monopsonist thus maximizes aggregate profits  $\Pi = \int_{n_0}^{n_1} \pi(n) f(n) dn$ , where the profits from employing a worker with ability  $n$  are

$$\pi(n) = \max_{w,l} \left\{ (n - w)l \quad \text{s.t.} \quad u'(wl - T(wl))w(1 - T'(wl)) = \phi'(l) \right\}. \quad (3)$$

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<sup>1</sup>A potential micro-foundation for these contracts is that firms might be restricted to pay a worker the same wage for every hour worked. For example, in the Netherlands, individuals can formally request to change the working hours stipulated in their employment contract. Employers can only deny this request if they have good cause (e.g., financial reasons). Also, in some sectors it is common that (weekly) hours worked are not constant. In both cases, labor income typically scales with the number of hours worked, which implies a constant hourly wage. The (legal) requirement that firms must pay a worker the same wage for every hour worked may appear inconsistent with the traditional assumption from Mirrlees (1971) that the government cannot observe wages. However, as explained by Lee and Saez (2012), the informational inconsistency is overcome if one assumes that hourly wages are observable after a careful audit following a whistle blowing claim.

Again, I assume the tax schedule is such that the second-order conditions are satisfied.<sup>2</sup> Combining the first-order necessary conditions with respect to the wage and hours worked gives, after rearranging,

$$\frac{w(n)}{n} = \frac{e_{lw}(n)}{1 + e_{lw}(n)}. \quad (4)$$

At the optimum, the markdown of wages relative to productivity depends on the (firm-level) elasticity  $e_{lw}(n)$  of hours worked with respect to the hourly wage, which varies across the skill distribution. Ignoring function arguments to save on notation, the latter can be found by implicitly differentiating the first-order condition (2):

$$e_{lw} = \frac{dl}{dw} \frac{w}{l} = \frac{w}{l} \frac{u''wl(1 - T')^2 + u'(1 - T') - u'wlT''}{-u''w^2(1 - T')^2 + \phi'' + u'w^2T''}. \quad (5)$$

Because there is a single monopsonist, the *firm-level* elasticity of labor supply is equal to the aggregate, or *market-level* elasticity of labor supply: both are given by  $e_{lw}$ .<sup>3</sup> As stated before, the assumption of a single monopsonist is of course extreme, but captures that the labor supply curve a firm faces is less than perfectly elastic (Manning, 2003).

The elasticity of labor supply (5) depends on properties of the tax function, in particular the level  $T$  (which enters  $u'$  and  $u''$ ), the slope  $T'$  and the *curvature*  $T''$ . This leads to the following result.

**Proposition 1.** *A local increase in the curvature of the tax function  $T(\cdot)$  at earnings  $z(n)$  leads to a lower equilibrium wage  $w(n)$ , hours worked  $l(n)$  and labor earnings  $z(n)$ .*

*Proof.* See Appendix A. □

According to Proposition 1, a local increase in the *second* derivative of the tax function  $T(\cdot)$  at earnings  $z(n)$  reduces the hourly wage, hours worked and labor earnings of individuals with ability  $n$ . The reason is that an increase in the curvature of the tax function makes individual labor supply less responsive to a change in the hourly wage. To illustrate, suppose the tax schedule is convex and marginal tax rates are steeply increasing in income. In that case, individuals have weak incentives to work longer hours following an increase in the hourly wage. Put differently, the elasticity of labor supply  $e_{lw}$  is low. A low elasticity of labor supply makes it attractive for the monopsonist to pay low wages as well, cf. equation (4). Consequently, a local increase in the curvature of the tax function reduces the hourly wage, hours worked and labor earnings. The following example illustrates this.

**Example 1.** *Suppose the individual utility function is  $U(c, l) = c - \frac{l^{1+1/\varepsilon}}{1+1/\varepsilon}$  and the tax schedule has a constant rate of progressivity  $p \in (0, 1)$ :  $T(z) = z - \frac{1-\tau}{1-p}z^{1-p}$ . Equilibrium labor supply follows from equation (2):  $l(n) = (1 - \tau)^{\frac{\varepsilon}{1+p\varepsilon}} w(n)^{\frac{(1-p)\varepsilon}{1+p\varepsilon}}$ . Hence, the elasticity of labor supply is  $e_{lw} = \frac{(1-p)\varepsilon}{1+p\varepsilon}$  and the equilibrium wage is, cf. equation (4):*

$$w(n) = (1 - p) \frac{\varepsilon}{1 + \varepsilon} n. \quad (6)$$

<sup>2</sup>As is common in the literature, these conditions can be verified *ex post* after having solved numerically for the optimal tax function. Numerical examples where this is the case are available upon request.

<sup>3</sup>By contrast, if labor markets are perfectly competitive, the market-level elasticity of labor supply is given by equation (5), whereas the firm-level elasticity of labor supply is infinite.

A higher rate of tax progressivity  $p \in (0, 1)$  means that marginal tax rates are more quickly increasing in income. This makes it less attractive for individuals to work longer hours if their wage increases: the elasticity of labor supply  $e_{lw}$  is decreasing in  $p$ . An increase in the rate of tax progressivity thus amplifies the negative impact of market power on wages by making individuals less responsive to wage changes.

The effect described in Proposition 1 and illustrated with the example above differs from the wage-moderating effect of tax progressivity. The latter states that a local increase in the *marginal* tax rate lowers the equilibrium wage. This is a robust prediction in models where labor markets are imperfectly competitive: it holds in the context of union bargaining (Hersoug, 1984), search frictions (Pissarides, 1985) and efficiency wages (Pisauro, 1991). The main difference with Proposition 1 is that the latter concerns the impact of the second (as opposed to the first) derivative of the tax function. In the aforementioned studies, labor market outcomes only depend on the level and slope of the tax function. Consequently, a local change in the curvature of the tax function has no impact. By contrast, a change in the curvature does affect labor market outcomes if, as in my model, a monopsonist sets hourly wages and individuals choose how many hours to work.

Before turning to the optimal tax problem, it is useful to highlight the following result.

**Lemma 1.** *At an interior solution characterized by the first-order conditions (2) and (4), hourly wages  $w(n)$ , hours worked  $l(n)$  and labor earnings  $z(n) = w(n)l(n)$  are increasing in ability  $n$ .*

*Proof.* See Appendix B. □

In words, for a given nonlinear tax schedule  $T(\cdot)$ , individuals with a higher ability  $n$  earn a higher hourly wage  $w(n)$  and work longer hours  $l(n)$ . These observations imply that labor earnings  $z(n)$  are increasing in ability  $n$  as well.

The government chooses the tax schedule  $T(\cdot)$  on labor earnings to maximize social welfare

$$\mathcal{W} = \int_{n_0}^{n_1} \Psi(v(n))f(n)dn. \quad (7)$$

The function  $\Psi(\cdot)$  is an increasing, weakly concave transformation of individual utilities. Together with the concavity in the individual utility function  $u(\cdot)$ , it determines the government's preferences for redistribution from high-skilled to low-skilled workers. The government chooses the tax schedule  $T(\cdot)$  to maximize social welfare (7), taking into account how a change in the tax function affects labor market outcomes and subject to the budget constraint

$$\int_{n_0}^{n_1} T(z(n))f(n)dn + \int_{n_0}^{n_1} \pi(n)f(n)dn = G, \quad (8)$$

where  $G$  is some exogenous spending. This formulation makes clear that profits are taxed at a confiscatory rate. Equivalently, one could assume that profits are not taxed but that firm ownership is uniformly distributed among all workers. To focus exclusively on labor income taxation, the model thus abstracts from capital income inequality.

The literature offers two methods to solve an optimal tax problem of this kind: the mechanism design approach (see, e.g., Mirrlees, 1971) and the tax perturbation approach (see, e.g., Saez, 2001 and Golosov et al., 2014). See Jacquet and Lehmann (2021) for a formal discussion and comparison of both

methods. In what follows, I use the mechanism design approach to derive a property of marginal tax rates at the bottom of the income distribution (Section 3) and the tax perturbation approach to derive an optimal tax formula in terms of sufficient statistics that holds at each point in the income distribution (Section 4).

### 3 Declining marginal tax rates

Solving the optimal tax problem using the mechanism design approach requires finding the allocation  $(v(n), \pi(n), l(n))$  for each  $n \in [n_0, n_1]$  that maximizes welfare (7) subject to resource and incentive constraints. The resource constraint is obtained by inverting the relationship  $v(n) = u(z(n) - T(z(n))) - \phi(l(n))$  with respect to  $T(z(n))$  and using the property  $z(n) = nl(n) - \pi(n)$ . Substituting these in the government budget constraint (8) gives

$$\int_{n_0}^{n_1} [nl(n) - u^{-1}(v(n) + \phi(l(n)))] f(n) dn = G. \quad (9)$$

The incentive constraints, in turn, describe the optimizing behavior of the monopsonist and individuals. To derive the first of these, differentiate the expression for profits (3) with respect to ability  $n$  and apply the envelope theorem:<sup>4</sup>

$$\pi'(n) = l(n). \quad (10)$$

To derive the second incentive constraint, differentiate the expression for individual utility (1) and again apply the envelope theorem:

$$\begin{aligned} v'(n) &= u'(w(n)l(n) - T(w(n)l(n)))l(n)(1 - T'(w(n)l(n)))w'(n) \\ &= \phi'(l(n))l(n) \frac{w'(n)}{w(n)} = \phi'(l(n)) \left( \frac{\pi(n)}{nl(n) - \pi(n)} \right) b(n), \quad \text{where } b(n) = l'(n). \end{aligned} \quad (11)$$

The first step uses equation (2) and the second step uses the relationship  $\pi(n) = (n - w(n))l(n)$ .<sup>5</sup>

As will be made clear below, in order to find the allocation that maximizes social welfare subject to resource and incentive constraints, it is important to take the constraint  $\pi(n_0) \geq 0$  explicitly into account. Equation (10) then implies firms make non-negative profits from hiring each worker. Lastly, to make sure that the allocation can be implemented using a non-linear tax  $T(z(n))$ , labor earnings must satisfy the monotonicity constraint  $z'(n) \geq 0$ . Differentiating  $z(n) = nl(n) - \pi(n)$  and imposing equation (10), the latter requires  $b(n) = l'(n) \geq 0$ . According to Lemma 1, this condition is satisfied at any interior solution described by the labor market equilibrium conditions (2) and (4).<sup>6</sup>

The problem of finding the optimal tax schedule  $T(\cdot)$  can now be written as an optimal control

<sup>4</sup>To see this, write the Lagrangian of the firm's problem as

$$\Lambda(n) = (n - w)l + \lambda [u'(wl - T(wl))w(1 - T'(wl)) - \phi'(l)],$$

where  $\lambda$  is the multiplier on the constraint (2). By the envelope theorem,  $\pi'(n) = \Lambda'(n) = l(n)$ .

<sup>5</sup>To see this, differentiate both sides with respect to ability to find  $\pi'(n) = l(n) + (n - w(n))l'(n) - w'(n)l(n)$ . Equation (10) then implies  $w'(n)l(n) = (n - w(n))l'(n)$ .

<sup>6</sup>As stated before, these conditions are only necessary but not sufficient. Sufficiency can be checked *ex post* after having numerically solved for the optimal tax system. Examples where this is the case are available upon request.

problem, with state variables  $(v(n), \pi(n), l(n))$  and control variable  $b(n)$ . Appendix D formulates the problem and derives the first-order necessary conditions. This leads to the following result.

**Proposition 2.** *At a solution with positive labor effort, optimal marginal tax rates for the least-skilled workers are declining in income. Put differently, the optimal tax schedule is concave at the bottom of the income distribution:  $T''(z(n_0)) < 0$ .*

*Proof.* See Appendix C. □

The main insight from Proposition 2 is that the government can use the curvature of the tax function to boost wages of low-skilled workers. This is achieved by setting a tax schedule that is concave at the bottom of the income distribution. A more concave tax schedule, i.e., a decrease in the second derivative of the tax function, positively affects wages: see Proposition 1. As explained before, declining marginal tax rates make it attractive for individuals to work longer hours following an increase in the hourly wage. A high elasticity of labor supply, in turn, induces the monopsonist to pay high wages as well, cf. equation (4). A government that is interested in redistribution can exploit this feature to raise the wages of low-skilled workers, and finds it optimal to do so.

At the optimal tax system, the monopsonist does not extract any rents from hiring the least productive workers. Hence, despite that there is a single monopsonist, these workers get paid a wage equal to their productivity. The optimal tax system thus off-sets any negative impact from market power on the wages of the least-skilled workers. To achieve this, the government sets the curvature of the tax function at the bottom of the income distribution in such a way that labor supply of the least-skilled workers becomes infinitely elastic:  $e_{lw}(n_0) \rightarrow \infty$ . Naturally, doing so requires that marginal tax rates at the bottom are declining in income:  $T''(z(n_0)) < 0$ .

The finding that optimal marginal tax rates are declining in income is a local result: it holds at the bottom of the income distribution. Unfortunately, using the mechanism design approach to derive optimal tax rules that hold at each point in the income distribution and that can be meaningfully interpreted turns out to be particularly challenging. The next section attempts to derive such a result using an alternative approach to solving the optimal tax problem.

## 4 Optimal tax formula

The optimal tax problem can also be solved using the tax perturbation approach. See Saez (2001) and Golosov et al. (2014), among many others. The idea behind this approach is to study a perturbation of the nonlinear tax schedule  $T(\cdot)$ . Such a reform induces welfare-relevant effects. Optimal tax formulas can then be derived from the requirement that if the tax schedule is optimized, the welfare-relevant effects of the reform sum to zero.

Before studying a particular tax reform, it is useful to restate the labor market equilibrium conditions. In modified form, equations (2) and (4) can be written as

$$u'(z - T(z) - \alpha)z(1 - T'(z) - \beta) = \phi'(l)l, \quad (12)$$

$$\begin{aligned} n \left[ u''(z - T(z) - \alpha)z(1 - T'(z) - \beta)^2 + \frac{\phi'(l)l}{z} - u'(z - T(z) - \alpha)z(T''(z) + \gamma) \right] \\ = \phi'(l) + \phi''(l)l. \end{aligned} \quad (13)$$

The first of these is obtained from multiplying equation (2) by  $l$  and the second from combining equations (4)–(5), using the property  $z = w \times l$ . These equations pin down equilibrium labor effort  $l$  and earnings  $z$  as a function of ability  $n$  and the reform parameters  $\alpha$ ,  $\beta$  and  $\gamma$ . These reform parameters can be used to study the impact of a local increase in the level, slope and curvature of the tax function, respectively. To illustrate, the effect of the tax curvature on earnings and labor effort is given by  $dz/d\gamma$  and  $dl/d\gamma$ . The behavioral responses on equilibrium earnings and labor effort can be obtained by implicitly differentiating equations (12)–(13), evaluated at  $\alpha = \beta = \gamma = 0$ .<sup>7</sup> The effects on profits and wages, in turn, follow from the relationships  $\pi = nl - z$  and  $w = z/l$ .

An important advantage of the tax perturbation approach is that it allows for a derivation of optimal tax formulas in terms of sufficient statistics (Chetty, 2009). In the current setting, these are: i) the income distribution, ii) behavioral responses and iii) welfare weights. Starting with the first, let  $H(z)$  denote the cumulative distribution of earnings, with corresponding density  $h(z)$ . Monotonicity of labor earnings  $z'(n) \geq 0$  implies that the distributions of earnings and ability are related through  $H(z(n)) = F(n)$  and hence,  $h(z(n))z'(n) = f(n)$ . The behavioral responses, in turn, capture how an increase in the level, slope or curvature of the tax function affect labor market outcomes. With a slight abuse of notation, I denote by  $y_x$  the impact of a local increase in  $x \in \{T, T', T''\}$  on outcome  $y \in \{z, l, w, \pi\}$ . As explained before, these behavioral responses can be obtained by implicitly differentiating equations (12)–(13) with respect to  $\alpha$ ,  $\beta$  and  $\gamma$  respectively, and using the relationships  $\pi = nl - z$  and  $w = z/l$ . Lastly, the welfare weight of an individual with earnings  $z$  is

$$g(z) = \frac{1}{\eta} \Psi'(v(\hat{n}(z))) u'(z - T(z)), \quad (14)$$

where  $\eta$  is the multiplier on the government budget constraint and  $\hat{n}(z)$  denotes the ability level that corresponds to earnings  $z$ . In words, the welfare weight  $g(z)$  measures by how much social welfare increases if an individual with earnings  $z$  receives an additional unit of after-tax income. These weights summarize in a reduced-form way the government's preferences for redistribution. Because both  $\Psi(\cdot)$  and  $u(\cdot)$  are concave, the welfare weights are declining in income  $z$ .

Figure 1 graphically illustrates the reform that is used to derive an optimal tax formula. The black, dotted line shows the original tax schedule. For simplicity, it is drawn as a straight line. The red, solid line shows the perturbed tax schedule after the reform is implemented. Below earnings  $z'$ , the two tax functions are the same. In the small interval  $[z', z' + \zeta]$ , the government increases the curvature (i.e., the second derivative) of the tax function by  $dT''$ . This is shown by the convex part of the solid line. Following the increase in the tax curvature, the marginal tax rates above earnings  $z' + \zeta$  increase by an amount equal to  $dT' = dT''\zeta$ . As a result, the perturbed tax schedule is steeper than the original tax schedule: see Figure 1. In the interval  $[z' + \delta, z' + \delta + \zeta]$  with  $\delta \gg \zeta$ , the government reverses the increase in the curvature by lowering the second derivative of the tax function by  $dT''$ . This is shown by the concave part of the solid line. Following this reversal, the marginal tax rates of the perturbed and original tax schedule are the same at earnings above  $z' + \delta + \zeta$ : the dotted and solid line are parallel. However, the reform does increase the tax burden for individuals with earnings above this level by an

<sup>7</sup>When implicitly differentiating equations (12)–(13), it is important to take the dependency of  $T(z)$ ,  $T'(z)$  and  $T''(z)$  on earnings  $z$  into account. The behavioral responses then capture the changes along the actual tax schedule, taking into account its higher-order derivatives. See Jacquet et al. (2013) for a discussion of this issue.

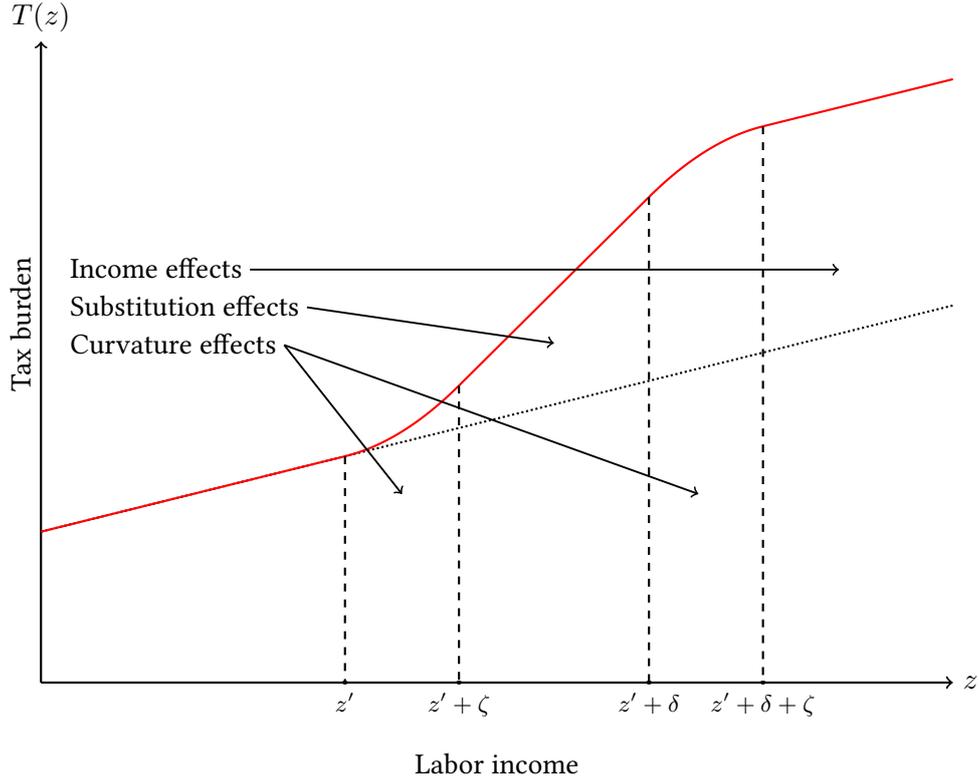


Figure 1: Tax reform and behavioral responses

amount equal to  $dT = dT' \delta = dT'' \zeta \delta$ .<sup>8</sup>

To keep track of the welfare-relevant effects associated with the tax reform, write the Lagrangian of the government's optimization problem

$$\mathcal{L} = \int_{z_0}^{z_1} \left[ \Psi(u(z - T(z)) - \phi(l)) + \eta(T(z) + \pi - G) \right] h(z) dz, \quad (15)$$

where  $z_0 = z(n_0)$  and  $z_1 = z(n_1)$ . Note that labor effort  $l$  and profits  $\pi$  vary along the earnings distribution as well and that earnings, labor effort and profits all depend on the level, slope and curvature of the tax function. The first part of the Lagrangian (15) states the objective, integrated over the income (as opposed to the ability) distribution. The second part captures the government budget constraint, again integrated over the income distribution, with associated multiplier  $\eta$ .

The reform graphically illustrated in Figure 1 generates three types of welfare-relevant effects. In what follows, I discuss each of these in turn and then turn to derive and explain the optimal tax formula. First, there are behavioral responses due to a change in the curvature of the tax function in the small intervals  $[z', z' + \zeta]$  and  $[z' + \delta, z' + \delta + \zeta]$ . In Figure 1, these are labeled 'curvature effects'. Ignoring

<sup>8</sup>While Figure 1 graphically illustrates the reform, it is also possible to give a formal definition. Let  $\rho(z)$  denote the difference between the perturbed and the original tax schedule. This reform function is given by

$$\rho(z) = \begin{cases} 0 & \text{if } z \leq z', \\ \frac{1}{2} dT'' (z - z')^2 & \text{if } z \in (z', z' + \zeta], \\ -\frac{1}{2} dT'' \zeta^2 + dT'' \zeta (z - z') & \text{if } z \in (z' + \zeta, z' + \delta], \\ -\frac{1}{2} dT'' \zeta^2 + dT'' \zeta (z - z') - \frac{1}{2} dT'' (z - (z' + \zeta))^2 & \text{if } z \in (z' + \delta, z' + \delta + \zeta], \\ dT'' \zeta \delta & \text{if } z > z' + \delta + \zeta. \end{cases}$$

function arguments to save on notation, the welfare impact of raising the curvature in the interval  $[z', z' + \zeta]$  by an amount equal to  $dT''$  is

$$\int_{z'}^{z'+\zeta} \left[ \left( \Psi'(u'(1-T')z_{T''} - \phi'l_{T''}) + \eta(T'z_{T''} + \pi_{T''}) \right) h \right] (z) dz \times dT''. \quad (16)$$

Here, the entire function between square brackets is evaluated at earnings  $z$ . To understand this expression, note that a change in the tax curvature affects earnings  $z$ , labor effort  $l$  and profits  $\pi$  for individuals with earnings in the interval  $[z', z' + \zeta]$ . The effects on earnings, labor effort and profits, in turn, affect social welfare and government finances. The impact on the Lagrangian is then obtained by integrating  $\frac{dL}{dT''} \times dT'' \times h$  from  $z'$  until  $z' + \zeta$ , where  $L$  is the term in square brackets below the integral sign of equation (15). Working out  $\frac{dL}{dT''}$  and accounting for the impact of tax curvature on earnings  $z$ , labor effort  $l$  and profits  $\pi$  gives the result from equation (16).

Equation (16) can be simplified in a number of steps. First, use equation (2) to substitute out for  $\phi' = u'w(1-T')$ . Second, the relationship  $z = w \times l$  implies  $z_{T''} = w_{T''}l + wl_{T''}$  and hence,  $w_{T''}l = z_{T''} - wl_{T''}$ . Third, a local increase in the curvature of the tax function has no impact on profits:  $\pi_{T''} = 0$ .<sup>9</sup> Fourth, use the definition of the welfare weight (14). Rearranging gives

$$\eta \times \int_{z'}^{z'+\zeta} \left[ \left( T'z_{T''} + g(1-T')lw_{T''} \right) h \right] (z) dz \times dT''. \quad (17)$$

By analogous reasoning, the welfare impact of a reduction in the curvature of the tax function in the small interval  $[z' + \delta, z' + \delta + \zeta]$  by an amount equal to  $dT''$  is

$$-\eta \times \int_{z'+\delta}^{z'+\delta+\zeta} \left[ \left( T'z_{T''} + g(1-T')lw_{T''} \right) h \right] (z) dz \times dT''. \quad (18)$$

Second, the tax reform raises the marginal tax rate in the interval  $[z' + \zeta, z' + \delta]$ : see Figure 1 and recall that  $\delta \gg \zeta$ . A higher marginal tax rate generates behavioral responses on earnings  $z$ , labor effort  $l$  and profits  $\pi$ . The impact on welfare associated with these ‘substitution effects’ is

$$\int_{z'+\zeta}^{z'+\delta} \left[ \left( \Psi'(u'(1-T')z_{T'} - \phi'l_{T'}) + \eta(T'z_{T'} + \pi_{T'}) \right) h \right] (z) dz \times \zeta dT''. \quad (19)$$

This expression is obtained by integrating  $\frac{dL}{dT'} \times dT' \times h$  from  $z' + \zeta$  until  $z' + \delta$ , where  $L$  is the term in square brackets below the integral sign of equation (15), as before. The term  $\frac{dL}{dT'}$  accounts for the equilibrium responses of earnings  $z$ , labor effort  $l$  and profits  $\pi$  to a change in the marginal tax rate. To arrive at equation (19), note that the reform increases the marginal tax rate for individuals with earnings in the interval  $[z' + \zeta, z' + \delta]$  by an amount equal to  $dT' = \zeta dT''$ .

The above expression can be simplified in a similar way as before. In particular, use the property  $\phi' = u'w(1-T')$  and the definition of the welfare weight (14). Moreover, the relationship  $z = w \times l$

<sup>9</sup>To see this, note that profits can be written as

$$\pi = \max_{l,z} \left\{ nl - z \quad \text{s.t.} \quad u'(z - T(z) - \alpha)z(1 - T'(z) - \beta) = \phi'(l)l \right\},$$

where I introduced the reform parameters  $\alpha$  and  $\beta$  in the constraint (2). A change in any of these generally has an impact on profits. However, because  $\gamma$  does not show up, a local increase in the curvature does not affect profits.

implies that  $z_{T'} = w_{T'}l + wl_{T'}$ . Substituting this in equation (19) gives

$$\eta \times \int_{z'+\zeta}^{z'+\delta} \left[ \left( T'z_{T'} + \pi_{T'} + g(1 - T')lw_{T'} \right) h \right] (z) dz \times \zeta dT''. \quad (20)$$

Third, the reform increases the tax burden for individuals with earnings above  $z' + \delta + \zeta$  by an amount  $dT = dT'\delta = dT''\zeta\delta$ . This generates two types of welfare-relevant effects. First, the reform mechanically transfers income from individuals with earnings above this level to the government budget. Second, a change in the tax burden generates behavioral responses on earnings  $z$ , labor effort  $l$  and profits  $\pi$ . Figure 1 labels these ‘income effects’. The total impact on welfare can be found by integrating  $\frac{dL}{dT} \times dT \times h$  for earnings above  $z' + \delta + \zeta$ :

$$\int_{z'+\delta+\zeta}^{z_1} \left[ \left( (-\Psi'u' + \eta) + \Psi'(u'(1 - T')z_T - \phi'l_T) + \eta(T'z_T + \pi_T) \right) h \right] (z) dz \times \zeta \delta dT'', \quad (21)$$

where the increase in the tax burden is  $dT = \zeta \delta dT''$ . Note that, when computing  $\frac{dL}{dT}$  from equation (15), there is both a mechanical effect (first term) as well as behavioral responses (other terms) to an increase in the tax burden. To simplify this expression, use the relationship  $\phi' = u'w(1 - T')$ , the expression for the welfare weight (14) and the property  $z = w \times l$ . Differentiating the latter with respect to  $T$  gives  $z_T = w_Tl + wl_T$ . Equation (21) can then be written as

$$\eta \times \int_{z'+\delta+\zeta}^{z_1} \left[ \left( 1 - g + T'z_T + \pi_T + g(1 - T')lw_T \right) h \right] (z) dz \times \zeta \delta dT''. \quad (22)$$

If the tax system is optimized, any reform should have no impact on social welfare. Put differently, the sum of the three welfare-relevant effects associated with the reform graphically illustrated in Figure 1 is equal to zero. This leads to the following result.

**Proposition 3.** *At the optimal tax system, the following condition must hold at each point  $z' \in [z_0, z_1]$  in the income distribution:*

$$\begin{aligned} 0 = & -\frac{d}{dz} \left[ \left( T'(z')z_{T''}(z') + g(z')(1 - T'(z'))\hat{l}(z')w_{T''}(z') \right) h(z') \right] \\ & + \left( T'(z')z_{T'}(z') + \pi_{T'}(z') + g(z')(1 - T'(z'))\hat{l}(z')w_{T'}(z') \right) h(z') \\ & + \int_{z'}^{z_1} \left( 1 - g(z) + T'(z)z_T(z) + \pi_T(z) + g(z)(1 - T'(z))\hat{l}(z)w_T(z) \right) h(z) dz. \end{aligned} \quad (23)$$

Here, it is explicitly taken into account which terms vary along the income distribution and  $\hat{l}(z)$  denotes the labor effort associated with earnings  $z$ .

*Proof.* See Appendix D. □

Equation (23) gives an optimal tax formula in terms of sufficient statistics that holds at each point in the income distribution. To understand the link with Figure 1, recall that the reform increases the tax curvature below a particular income level and decreases it right above. This reform leads to a local increase in the marginal tax rate and an increase in the tax burden for all individuals with earnings above this level. The first term of equation (23) captures the difference between the welfare effects

of increasing the tax curvature and subsequently decreasing it. Upon making the intervals arbitrarily small, this essentially boils down to taking a derivative. The second term captures the welfare-relevant effects associated with a local increase in the marginal tax rate. Naturally, these effects are proportional to the density of the income distribution at the point where the marginal tax rate is increased. Lastly, the third term integrates the welfare impact of a higher tax burden over all individuals who see their tax burden increase as a result of the reform.

The result from Proposition 3 can be contrasted with the optimal tax formula that holds if labor markets are competitive. See, for instance, Saez (2001) and Golosov et al. (2014). Under perfect competition, labor market outcomes are not affected by a local increase in the curvature of the tax function:  $z_{T''} = w_{T''} = 0$ . Moreover, the wage of an individual with ability  $n$  is  $w(n) = n$  and firms make zero profits:  $\pi(n) = 0$ . Consequently, wages and profits do not respond to a change in the level or slope of the tax function:  $w_T = w_{T'} = \pi_T = \pi_{T'} = 0$ . Equation (23) then becomes:

$$0 = T'(z')z_{T'}(z')h(z') + \int_{z'}^{z_1} \left(1 - g(z) + T'(z)z_T(z)\right)h(z)dz. \quad (24)$$

Apart from differences in presentation, this optimal tax formula coincides with the one derived in Saez (2001) and Golosov et al. (2014), among others.

Compared to the competitive benchmark, the additional ‘sufficient statistics’ that show up in the optimal tax formula (23) are the effects of tax curvature on earnings and hourly wages and the effects of the tax burden and the marginal tax rate on hourly wages and profits. Intuitively, the additional responses of earnings and profits to tax changes have budgetary effects, as the government taxes both labor income and profits. In equation (23), the budgetary effects are captured by  $T'z_{T''}$ ,  $\pi_{T'}$  and  $\pi_T$ . By contrast, the wage responses  $w_{T''}$ ,  $w_{T'}$  and  $w_T$  have an effect on individuals’ disposable incomes. This effect is proportional to  $l$ , the number of hours worked, and the after-tax rate  $1 - T'$ . Because changes in disposable income affect individual utilities, the welfare effects associated with wage changes in equation (23) are weighted by the individual welfare weights  $g$ .

Unfortunately, implementing the optimal tax formula (23) using estimates of sufficient statistics is a very challenging task. To the best of my knowledge, there are no estimates available of the impact of tax curvature on labor market outcomes. Moreover, to implement the optimal tax formula (23), one also requires knowledge of how these statistics vary across the earnings distribution. Despite this, the result does illustrate clearly what forces shape optimal tax policy when firms have labor market power and labor market outcomes are affected by the curvature of the tax function. Moreover, by indicating precisely what statistics determine the welfare effects of tax reforms, the result can also be seen as a motivation for future empirical work.

## 5 Conclusion

If a monopsonist sets hourly wages and individuals choose how many hours to work, labor market outcomes do not only depend on the level and the slope of the tax function, but also on its curvature. I use that insight to obtain the following three results. First, a local increase in the curvature (i.e., the second derivative) of the tax function reduces the hourly wage, hours worked and labor earnings. Intuitively, a more convex or less concave tax schedule lowers the elasticity of labor supply, which

induces the monopsonist to pay lower wages. Second, the optimal tax schedule is concave at the bottom of the income distribution. Declining marginal tax rates at the bottom make low-skilled workers more responsive to wage changes, which leads the monopsonist to pay higher wages. Third, I derive an optimal tax formula that accounts for the impact of tax curvature on labor market outcomes. Compared to existing results in the literature, the additional ‘sufficient statistics’ that characterize the optimal tax system are the impact of tax curvature on earnings and hourly wages, and the impact of the tax burden and the marginal tax rate on hourly wages and profits.

How important the effects of tax curvature are on labor market outcomes is an open question, but one that can be investigated empirically. One approach would be to construct a measure of tax curvature  $T''(z)$  throughout the income distribution, which can then be used as an explanatory variable in a regression framework with wages, hours worked or labor earnings as the dependent variable. Another approach would be to regress any of these outcomes on (instrumented) measures of an individual’s own tax burden, marginal tax rate *and* the marginal tax rate an individual would face if the individual’s earnings increase or decrease. The model from this paper predicts that a higher (lower) marginal tax rate of one’s ‘neighbor’ in the income distribution with slightly higher (lower) earnings negatively affects wages, hours worked and labor earnings. I leave an empirical investigation of this issue as a topic for future research.

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## A Proof Proposition 1

Use the first-order condition (2) to write  $l(n) = l(w(n))$ , or simply  $l = l(w)$ . Substituting this in the firm's objective  $\pi(n) = (n - w)l(w)$  gives an unconstrained optimization problem in the wage  $w$ . The first-order condition is

$$0 = -l(w) + (n - w)l'(w) = -l(w) + (n - w) \times \frac{u''(R(wl(w)))wl(w)R'(wl(w))^2 + u'(R(wl(w)))R'(wl(w)) + u'(R(wl(w)))wl(w)R''(wl(w))}{-u''(R(wl(w)))w^2R'(wl(w))^2 + \phi''(l(w)) - u'(R(wl(w)))w^2R''(wl(w))}, \quad (25)$$

where the term  $l'(w)$  is obtained from implicitly differentiating the first-order condition (2) and  $R(x) = x - T(x)$  denotes the retention function, with  $R'(x) = 1 - T'(x)$  and  $R''(x) = -T''(x)$ . Equation (25) implicitly pins down the equilibrium wage  $w(n)$  and can be used to study how the latter is affected by a local increase in the curvature of the tax function. To that end, introduce a reform parameter  $\gamma$  and define the function

$$\Delta(w, \gamma) = -l(w) + (n - w) \times \frac{u''(R(wl(w)))wl(w)R'(wl(w))^2 + u'(R(wl(w)))R'(wl(w)) + u'(R(wl(w)))wl(w)(R''(wl(w)) - \gamma)}{-u''(R(wl(w)))w^2R'(wl(w))^2 + \phi''(l(w)) - u'(R(wl(w)))w^2(R''(wl(w)) - \gamma)}. \quad (26)$$

Equation (25) implies  $\Delta(w, 0) = 0$ . Because  $R''(x) = -T''(x)$ , an increase in  $\gamma$  can be interpreted as a local increase in the curvature of the tax function. The impact on the equilibrium wage follows from the implicit function theorem:

$$\left. \frac{dw}{d\gamma} \right|_{\gamma=0} = -\frac{\Delta_\gamma(w, 0)}{\Delta_w(w, 0)}. \quad (27)$$

The second-order condition of the profit maximization problem implies  $\Delta_w(w, 0) < 0$ . Consequently, the sign of  $dw/d\gamma$  is the same as the sign of  $\Delta_\gamma(w, 0)$ . Ignoring function arguments to save on notation, the latter is

$$\Delta_\gamma(w, 0) = \frac{(w - n)u'wl}{-u''w^2R'^2 + \phi'' - u'w^2R''} + \frac{(w - n)(u''wR'^2 + u'R' + u'wR'')u'w^2}{(-u''w^2R'^2 + \phi'' - u'w^2R'')^2}. \quad (28)$$

The second-order condition of the utility maximization problem (1) implies that the denominator of the first term is positive. Moreover, at an interior solution with positive profits, the labor supply curve is upward sloping and the wage is below productivity: see equation (4). As a result, the first term of equation (28) is negative. The second term is negative as well. To see why, note that equation (25) implies  $u''wR'^2 + u'R' + u'wR'' > 0$ . It follows that  $\Delta_\gamma(w, 0) < 0$  and hence,  $dw/d\gamma < 0$ . Because the labor supply curve is upward sloping at an interior solution, a local increase in the curvature of the tax function also leads to a reduction in hours worked  $l$  and labor earnings  $z = w \times l$ .

## B Proof Lemma 1

As explained in the proof of Proposition 1, the equilibrium wage  $w(n)$  is pinned down by equation (25). Define the right-hand side of this equation as  $\Omega(w, n)$ . By the implicit function theorem,

$$w'(n) = -\frac{\Omega_n(w, n)}{\Omega_w(w, n)} = -\frac{1}{\Omega_w(w, n)} \frac{l(w)}{n-w}. \quad (29)$$

The second-order condition implies  $\Omega_w(w, n) < 0$ . Consequently, at an interior solution with positive profits,  $w'(n) > 0$ . Because the labor supply curve  $l(w(n))$  is upward sloping at an interior solution (i.e.,  $l'(w) > 0$ ), it follows immediately that labor effort  $l(n)$  and labor earnings  $z(n) = w(n) \times l(n)$  are increasing in ability as well:  $l'(n), z'(n) > 0$ .

## C Proof Proposition 2

The key behind the result from Proposition 2 lies in demonstrating that the constraint  $\pi(n_0) \geq 0$  is binding. To do so, write the optimal tax problem in terms of allocation variables:

$$\begin{aligned} & \max_{[v(n), \pi(n), l(n), b(n)]_{n_0}^{n_1}} \mathcal{W} = \int_{n_0}^{n_1} \Psi(v(n)) f(n) dn, & (30) \\ \text{s.t. } & \int_{n_0}^{n_1} \left[ nl(n) - u^{-1}(v(n) + \phi(l(n))) \right] f(n) dn = G, \\ & \forall n : v'(n) = \phi'(l(n)) \left( \frac{\pi(n)}{nl(n) - \pi(n)} \right) b(n), \\ & \forall n : \pi'(n) = l(n), \\ & \forall n : l'(n) = b(n), \\ & \forall n : b(n) \geq 0, \\ & \pi(n_0) \geq 0. \end{aligned}$$

Using integration by parts on the incentive constraints, the Lagrangian is given by

$$\begin{aligned} \mathcal{L} = & \int_{n_0}^{n_1} \left[ \left( \Psi(v(n)) + \eta \left( nl(n) - u^{-1}(v(n) + \phi(l(n))) - G \right) \right) f(n) \right. \\ & + \lambda(n) \phi'(l(n)) \left( \frac{\pi(n)}{nl(n) - \pi(n)} \right) b(n) + \lambda'(n) v(n) + \mu(n) l(n) + \mu'(n) \pi(n) + \chi(n) b(n) \\ & + \chi'(n) l(n) + \psi(n) b(n) \left. \right] dn + \lambda(n_0) v(n_0) - \lambda(n_1) v(n_1) + \mu(n_0) \pi(n_0) - \mu(n_1) \pi(n_1) \\ & + \chi(n_0) l(n_0) - \chi(n_1) l(n_1) + \xi \pi(n_0). \end{aligned} \quad (31)$$

The first-order necessary conditions with respect to the states  $v(n)$ ,  $\pi(n)$ ,  $l(n)$  and the control  $b(n)$  are:

$$v(n) : \left( \Psi'(v(n)) - \frac{\eta}{u'(c(n))} \right) f(n) + \lambda'(n) = 0, \quad (32)$$

$$\pi(n) : \lambda(n) \phi'(l(n)) \frac{nl(n)}{(nl(n) - \pi(n))^2} b(n) + \mu'(n) = 0, \quad (33)$$

$$\begin{aligned}
l(n) : \quad & \eta \left( n - \frac{\phi'(l(n))}{u'(c(n))} \right) f(n) + \mu(n) + \chi'(n) \\
& + \lambda(n) \left[ \phi''(l(n)) \frac{\pi(n)}{nl(n) - \pi(n)} - \phi'(l(n)) n \frac{\pi(n)}{(nl(n) - \pi(n))^2} \right] b(n) = 0, \quad (34)
\end{aligned}$$

$$b(n) : \quad \lambda(n) \phi'(l(n)) \frac{\pi(n)}{nl(n) - \pi(n)} + \chi(n) + \psi(n) = 0. \quad (35)$$

Here,  $c(n) = u^{-1}(v(n) + \phi(l(n)))$  denotes the consumption of an individual with ability  $n$ . The latter is increasing in ability, because monotonicity of labor earnings, i.e.,  $b(n) \geq 0$ , implies labor effort  $l(n)$  and utility  $v(n)$  are increasing in ability as well. As with the utility and profit maximization problems (1) and (3), I assume the second-order conditions are satisfied.<sup>10</sup>

The transversality conditions imply  $\lambda(n_0) = \lambda(n_1) = \mu(n_1) = 0$  and  $\mu(n_0) + \xi = 0$ . Because  $\Psi(\cdot)$  and  $u(\cdot)$  are concave and because  $v(n)$  and  $c(n)$  are increasing in ability, equation (32) implies that  $\lambda(n) \leq 0$  for all  $n$  with a strict equality only at the end-points.<sup>11</sup> Equation (33) then implies  $\mu(n)$  is increasing in ability. From  $\mu(n_1) = 0$ , it follows that  $\mu(n_0) < 0$  and hence,  $\xi > 0$ . The constraint  $\pi(n_0) \geq 0$  thus holds with equality.

If  $\pi(n_0) = 0$ , it follows that  $w(n_0) = n_0$  at the optimal tax system if  $l(n_0) > 0$ . Equation (4) then implies  $e_{lw}(n_0) \rightarrow \infty$ . This requires that the denominator in equation (5) approaches zero. Therefore, at the lowest skill level  $-u''w^2(1 - T')^2 + \phi'' + u'w^2T'' \rightarrow 0$ . Because  $u'' \leq 0$  and  $\phi'' > 0$ , it follows that the tax function is concave at the bottom of the income distribution:  $T'' < 0$ .

## D Proof Proposition 3

Add equations (17), (18), (20) and (22) and set the resulting expression equal to zero. Dividing by  $\eta \times \zeta \times dT''$  gives

$$\begin{aligned}
& \frac{1}{\zeta} \int_{z'}^{z'+\zeta} \left[ (T' z_{T''} + g(1 - T') l w_{T''}) h \right] (z) dz - \frac{1}{\zeta} \int_{z'+\delta}^{z'+\delta+\zeta} \left[ (T' z_{T''} + g(1 - T') l w_{T''}) h \right] (z) dz \\
& + \int_{z'+\zeta}^{z'+\delta} \left[ (T' z_{T'} + \pi_{T'} + g(1 - T') l w_{T'}) h \right] (z) dz \\
& + \int_{z'+\delta+\zeta}^{z_1} \left[ (1 - g + T' z_T + \pi_T + g(1 - T') l w_T) h \right] (z) dz \times \delta = 0. \quad (36)
\end{aligned}$$

Next, take the limit as  $\zeta \rightarrow 0$ :

$$\begin{aligned}
& \left[ (T' z_{T''} + g(1 - T') l w_{T''}) h \right] (z') - \left[ (T' z_{T''} + g(1 - T') l w_{T''}) h \right] (z' + \delta) \\
& + \int_{z'}^{z'+\delta} \left[ (T' z_{T'} + \pi_{T'} + g(1 - T') l w_{T'}) h \right] (z) dz
\end{aligned}$$

<sup>10</sup>As explained before, these conditions can be checked *ex post* after having solved the optimal tax problem. Numerical examples for which they hold are available upon request.

<sup>11</sup>It can be verified that no solution exists where  $v(n)$  and  $c(n)$  do not vary with ability and hence,  $\lambda(n) = 0$  for all  $n$ . This would require  $b(n) = 0$  for all  $n$ , in which case  $l(n)$  is constant as well, cf. equation (11). Equation (33) and the condition  $\mu(n_1) = 0$  then imply  $\mu(n) = 0$  for all  $n$ . From equation (34) and the transversality conditions  $\chi(n_0) = \chi(n_1) = 0$  it follows that  $\chi(n) \geq 0$  with a strict equality only at the end-points. Equation (35) then contradicts the requirement that the multiplier  $\psi(n) \geq 0$ .

$$+ \int_{z'+\delta}^{z_1} \left[ (1 - g + T'z_T + \pi_T + g(1 - T')lw_T)h \right] (z) dz \times \delta = 0. \quad (37)$$

The first term in square brackets is evaluated at earnings  $z'$  and the second term is evaluated at earnings  $z' + \delta$ . To proceed, divide equation (37) by  $\delta$  and rearrange

$$\begin{aligned} & - \frac{[(T'z_{T''} + g(1 - T')lw_{T''})h](z' + \delta) - [(T'z_{T''} + g(1 - T')lw_{T''})h](z')}{\delta} \\ & + \frac{1}{\delta} \int_{z'}^{z'+\delta} \left[ (T'z_{T'} + \pi_{T'} + g(1 - T')lw_{T'})h \right] (z) dz \\ & + \int_{z'+\delta}^{z_1} \left[ (1 - g + T'z_T + \pi_T + g(1 - T')lw_T)h \right] (z) dz = 0. \end{aligned} \quad (38)$$

Taking the limit as  $\delta \rightarrow 0$ :

$$\begin{aligned} & - \frac{d}{dz} \left[ (T'z_{T''} + g(1 - T')lw_{T''})h \right] (z') + \left[ (T'z_{T'} + \pi_{T'} + g(1 - T')lw_{T'})h \right] (z') \\ & + \int_{z'}^{z_1} \left[ (1 - g + T'z_T + \pi_T + g(1 - T')lw_T)h \right] (z) dz = 0. \end{aligned} \quad (39)$$

The first term is the derivative of the expression in square brackets with respect to earnings, evaluated at  $z'$ . To arrive at the result from Proposition 3, note that equation (39) must hold for a reform considered at each income level  $z' \in [z_0, z_1]$ . As a final step, equation (23) makes explicit which terms vary across the earnings distribution.