Optimal Climate Policy with Incomplete Markets*

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Abstract

How should governments design climate policies in the presence of inequality, uninsurable risk, and fiscal constraints? To address this question, we develop a climate–economy model with incomplete markets and idiosyncratic labor-income risk, where Ricardian equivalence fails and optimal long-run capital taxes are positive. We analytically show that the optimal carbon tax equals the social cost of carbon (SCC) adjusted for fiscal distortions. Calibrating the model to the U.S., we show that these deviations are quantitatively negligible: high levels of household inequality, income risk, and fiscal distortions do not, in themselves, justify lowering climate ambitions. Welfare gains under the optimal policy come almost entirely from efficiency and environmental amenities, with almost no effect on redistribution and insurance, and are fairly evenly distributed across households.

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1 Introduction

Climate change results from what might be "the biggest market failure the world has seen" (Stern, 2008). In an idealized economy with no other distortions, the optimal policy response is straightforward: a Pigouvian carbon tax equal to the social cost of carbon (SCC) would internalize the externality and restore efficiency. In such a first-best world, these efficiency considerations are orthogonal to distributional concerns.

The real world, however, is far from this abstraction. Inequality and uninsurable risk matter for aggregates through labor supply and saving decisions. These decisions, in turn, are influenced by fiscal policy that provides redistribution and insurance. Because some households are borrowing constrained and markets are incomplete, capital is taxed and Ricardian equivalence fails, so the timing of debt and transfers affects the intertemporal allocation of resources—an issue of central importance for climate policy. In such an environment, carbon taxation does not operate in isolation—it coexists with existing fiscal and distributional wedges. How should governments tax carbon in such a world? Should they temper their climate ambitions in response to inequality, risk, and fiscal constraints, or can fiscal policy reconcile climate action with distributional objectives?

To answer these questions, we develop a fiscal climate–economy model with incomplete markets. Our framework builds on the long-standing tradition of neoclassical growth models that integrate the climate and the economy (Nordhaus, 1992; Golosov, Hassler, Krusell, and Tsyvinski, 2014) and on the standard incomplete-markets (SIM) framework featuring uninsurable idiosyncratic risk, as developed by Bewley (1986), Imrohoruglu (1989), Huggett (1993), and Aiyagari (1994). Within this framework, we study a Ramsey problem and characterize—theoretically and quantitatively—the optimal fiscal policy *along the transition path*. We then explore the implications of climate policy for redistribution, insurance, and overall welfare.

The key insight of this paper is that high levels of household inequality, income risk, and fiscal distortions do not, in themselves, justify lowering climate ambitions. Across a wide range of scenarios, we find that optimal carbon taxes hardly deviate from their externality-correcting levels: the welfare gain from moving from the Pigouvian tax to the optimal tax is below 0.01% even in our most constrained economy. This contrasts with influential results from the representative-agent literature, where fiscal distortions typically lead to pollution taxes below the Pigouvian benchmark (Bovenberg and Goulder, 1996; Barrage, 2020). In our framework, where distortionary taxes emerge endogenously as a response to inequality and risk, we find that departures from the Pigouvian rule remain quantitatively minimal. While the government is concerned with inequality and risk, it never chooses to address these concerns through climate policy. The reason, which we formalize in Section 5.2, is that such deviations are justified only insofar as other market failures remain uncorrected—given the constraints on alternative instruments—*and* can be effectively targeted by the carbon tax. Examining the welfare gains from the optimal carbon policy, we find that they stem almost entirely from improvements in efficiency and environmental amenities, and are fairly evenly distributed across households. As a result, using climate policy to pursue redistribution or insurance objectives would entail substantial welfare costs even when redistributive instruments are far from optimized. Our results therefore demonstrate that the Pigouvian principle is remarkably robust: even in the presence of significant fiscal and distributional frictions, very little equity or insurance gains can be obtained by deviating from its path.

Our infinite-horizon economy features heterogeneous households with preferences over consumption, leisure, and environmental amenities. Households face uninsurable idiosyncratic labor productivity risk and can only save in a risk-free asset subject to a borrowing constraint. A final consumption-investment good is produced using capital, labor, and energy. Energy itself is generated in a separate sector that employs capital and labor. Its production results in CO_2 emissions, which firms can mitigate at a cost. Unabated emissions accumulate, driving climate change and affecting households through both production and direct damages to environmental amenities. The government can curb emissions via a carbon tax and finances public expenditures and lump-sum transfers using proportional taxes on capital, labor, energy production, and debt. Within this framework, we solve a Ramsey problem: choosing policy instruments to maximize welfare over the transition.

We first examine this problem theoretically by letting the government jointly optimize over the time paths of linear taxes on carbon, total energy production, capital and labor income, uniform lump-sum transfers, and government debt, taking into account how households and firms respond to changes in fiscal policy. Without further restrictions on the government's instrument set, we find that the optimal carbon tax follows a *modified* Pigouvian rule. The modification occurs because carbon taxes, through their effect on individual consumption streams, interact with incentives to save and supply labor. These interactions, which could call for both higher or lower carbon taxes relative to Pigouvian levels, are driven by income effects on labor supply and, perhaps surprisingly, by the saving decisions of households who are *not* borrowing constrained. We then turn to a quantitative analysis to determine the sign and magnitude of these deviations.

We calibrate our model to the U.S. economy. To align with global scales, we adjust the U.S. economy's emissions and GDP to match those of the world, modifying the population to maintain the U.S. GDP per capita. By doing this, we assume that the rest of the world's emissions adjust in proportion to U.S. emissions, while at the same time global damages are taken into account.[1] In calibrating the climate component, we draw on recent advancements in integrated assessment modeling meant to better capture the impulse response of temperature to emissions. Specifically, we use the climate model of Dietz and Venmans (2019) and calibrate it based on IPCC (2021) and Friedlingstein et al. (2022), with additional parameters taken from Barrage and Nordhaus (2023). The calibration of the macroeconomic aspects of our model is guided by the goal of replicating three sets of statistics: macroeconomic variables, inequality metrics, and measures of idiosyncratic labor-income risk. This quantitative approach ensures that our model accurately reflects the macroeconomic and environmental realities captured in the data.

To quantitatively tackle the optimal policy problem within our framework, we employ numerical techniques developed in Dyrda and Pedroni (2023), approximating the trajectories of fiscal instruments over time using orthogonal polynomials. We examine optimal carbon taxation under different fiscal constraints, considering scenarios with varying degrees of flexibility in policy instruments. We begin with a setting where the government sets the path of the carbon tax but must rebate revenues through lump-sum transfers while keeping income taxes and the debt-to-GDP ratio fixed. We then extend the analysis by allowing the government to choose the path of debt (hence, to choose the timing of lump-sum transfers), as well as the level of labor and capital income taxes.

Across all scenarios, we find that the optimal carbon tax remains extremely close to the Pigouvian benchmark, i.e., virtually equal to the SCC. Thus, while our theoretical results suggest that second-best mechanisms could, in principle, justify significant deviations from the SCC, these effects turn out to be quantitatively negligible: in our most constrained scenario, moving from the simple Pigouvian tax to the optimal one yields welfare gains of only 0.008%, and as the government gains access to additional fiscal instruments, these gains fall further to 0.001%. Moreover, the optimal carbon tax path is remarkably stable across scenarios, even when the government can choose the path of debt, transfers, and the level of income taxes. This invariance is surprising given that, at the optimum, the government's concern for redistribution and insurance results in very high levels of debt and income taxation, with dramatic effects on the macroeconomy. Using our analytical results, we decompose

[1] This approach was the one officially intended by the previous U.S. administration, which claimed that "It is essential that agencies capture the full costs of greenhouse gas emissions as accurately as possible, including by taking global damages into account." (Executive Order 13990 of Jan 20, 2021).

the tax into distinct components and show that while each is influenced by debt and redistributive policies, these effects tend to cancel each other: while higher taxes and debt reduce output and future damages, they also increase the valuation of these damages by increasing the future marginal utility of consumption.

To understand the determinants of the optimal policy, we compute the welfare gains from the optimal carbon tax relative to a carbon tax consistent with the SSP5 scenario (Shared Socioeconomic Pathways, a scenario with limited abatement, see IPCC, 2021). We find average inter-temporal welfare gains of 4.7%. We then decompose these welfare gains into four components: level effects, redistribution, insurance, and environmental amenities. We find that welfare gains stem almost entirely from improvements in aggregate efficiency and environmental amenities, while redistribution and insurance effects play only a negligible role. Hence, using the carbon tax for redistribution or insurance would be highly inefficient: even when fiscal instruments are constrained, the efficiency costs needed to achieve meaningful equity or risk mitigation gains are too high to justify a significant departure from its externality-correction objective. Our results also reveal an important asymmetry in the welfare costs of deviating from the SCC: while taxing carbon above the optimal level has limited welfare effects, under-taxation leads to significant welfare losses. This implies that, even when the government is highly concerned with distributional issues, erring on the side of more ambitious climate policy is preferable to delaying action.

This result does not imply that carbon taxes are necessarily distributionally neutral. In the next part of the paper, we take a positive perspective and examine how the welfare gains from applying the optimal policy in our baseline experiment compare with those under less ambitious climate policies. While the gains are spread fairly evenly across income and wealth groups, with virtually no difference between the first and fourth labor-income and asset quintiles, differences emerge at the top of the distribution: richer households benefit less from higher transfers financed by carbon taxation but more from improved climate amenities. Thus, when we compare the optimal policy to SSP5, a scenario with low carbon taxes, richer households gain more on average: +5.8% for the top labor-income/asset quintile versus +4.7% for the average household. Looking at the distribution of gains over time, most benefits from the climate transition materialize after the 21st century; throughout the 21st century, gains are small though almost always positive.

We show that these results remain robust in a key extension of our model. An important literature examines the *use side* effects of carbon taxes, i.e., their differential impact on household budgets through changes in energy prices (see Pizer and Sexton, 2019, for a review). To capture that lower-income households devote a larger share of their spending to carbon-intensive goods, we extend the model with non-homothetic preferences over a final good and energy and calibrate it to match the cross-sectional distribution of energy expenditure shares. While this specification makes the interaction between carbon taxation and inequality even more explicit, we find that the optimal carbon tax remains virtually equal to the SCC. As for the distribution of welfare gains, they follow the same trend as in the baseline model, i.e., they are homogeneously spread, except for the richest households who benefit more from large environmental improvements.

Related literature. Building on Nordhaus' pioneering work (e.g., Nordhaus, 1992, 2008), Golosov et al. (2014) developed a DSGE formulation of the DICE model to examine optimal policy within a decentralized equilibrium framework. Barrage (2020) further enriched this approach by introducing a more detailed fiscal environment to analyze second-best policies. We contribute to this literature by integrating a fiscal climate–economy model with a heterogeneous-agent framework à la Aiyagari (1994), allowing us to study climate policy in a setting with household inequality and risk. In doing so, our paper advances the understanding of both the normative and positive dimensions of fiscal policy in the presence of environmental externalities and, in particular, climate change.

On the normative side, this paper contributes to an influential public finance literature on optimal taxation of pollution in second-best environments (e.g., Sandmo, 1975; Bovenberg and de Mooij, 1994; Barrage, 2020). Earlier work (Douenne, Hummel, and Pedroni, 2023) considered a setting where household labor productivity is fixed over time, Ricardian equivalence holds, and capital is not taxed in the long run. In that case, we can show analytically that the optimal carbon tax is, on average, Pigouvian.[2] By contrast, here households face risk and occasionally binding borrowing constraints; they are therefore not indifferent to the timing of debt and transfers, and it is optimal to tax capital in the long run (Aiyagari, 1995). Still, we show quantitatively that the optimal carbon tax closely tracks the SCC. This remains true under suboptimal tax systems with constraints on the level and path of other fiscal instruments, including the timing of debt and transfers.

Other studies have since made important progress in advancing our understanding of optimal carbon taxation in incomplete-market economies. However, given the analytical and computational difficulties posed by the introduction of incomplete markets, existing papers have so far considered simplified versions of the optimal taxation problem. For example, Fried, Novan, and Peterman (2024)

[2] As shown in Barrage (2020), the absence of long-run capital taxation means there are no intertemporal distortions that would call for adjusting the rate at which production damages are discounted.

consider a government introducing an exogenous carbon tax within a framework à la Conesa, Kitao, and Krueger (2009a), and search for the optimal mechanism to redistribute the tax revenue in the steady state, combining transfers and reductions in existing income taxes. Several authors (e.g., Belfiori, Carroll, and Hur, 2024; Belfiori and Macera, 2024; Kubler, 2024) study a constrained efficiency problem à la Dávila, Hong, Krusell, and Ríos-Rull (2012), where optimal policy is analyzed under the constraint that no resource transfers occur between agents. In this setting, Belfiori et al. (2024) show that the constrained-efficient carbon tax follows a Pigouvian rule, where aggregate damages are valued at a weighted average of households' marginal utilities of consumption.[3] While the focus on efficiency enhances tractability, this approach does not directly address how a government should design policies to mitigate climate change while accounting for their distributional effects. Another related contribution is Malafry and Brinca (2022) who study optimal carbon taxation in a stylized two-period economy with both idiosyncratic and aggregate risk, where the carbon tax and its lump-sum redistribution are the only available instruments. Closer to our work, Wöhrmüller (2024) introduces an incomplete-markets model à la Aiyagari (1994) where households have Stone-Geary preferences, and computes the value of a constant carbon tax that maximizes welfare in the steady state. In this framework, the optimal carbon tax is sensitive to the level of idiosyncratic risk and to the planner's ability to also tax labor. By contrast, we analyze a general Ramsey problem theoretically and quantitatively, and we study dynamic taxes to maximize welfare over the full transition path. Our approach also allows us to consider a government that can simultaneously choose multiple instruments, reducing the role of arbitrary constraints over the tax system.

Also connected to our work is the literature that studies optimal carbon taxation with inequality across countries (e.g., Lang, 2024; Bourany, 2024). Although their approach bears similarities with ours, these papers do not consider incomplete markets and, in the international context, they study fiscal environments where the only available instruments are carbon taxes and cross-country transfers. Thus, while these papers focus on how global inequality influences the SCC, our interest lies in how a government should implement a carbon tax, in a second-best environment in which the government addresses inequality and risk with income taxes and transfers to its own citizens.

Beyond the environmental economics literature, our paper contributes to a broader literature in macro public finance, which focuses on taxation and optimal policy design in economies with incomplete markets. Early seminal works in this area, exploring fiscal policy under incomplete markets,

[3] Note that despite the differences between the two frameworks, this result coincides with the inequality-adjusted SCC in the complete-market economy of Douenne et al. (2023) (see their Proposition 3).

include studies by Conesa and Krueger (1999), Domeij and Heathcote (2004), Heathcote (2005), and Conesa et al. (2009b). Research on optimal policies in this field has typically focused on analyzing once-and-for-all policy changes, with Boar and Midrigan (2022) and Kina, Slavík, and Yazici (2023) being among the most recent examples. Only recently have studies started to examine dynamic, optimal policies, as seen in Krueger and Ludwig (2016), Itskhoki and Moll (2019), and Dyrda and Pedroni (2023). We contribute to this strand of research by studying optimal, time-varying carbon taxation in an economy blending together state-of-the-art macroeconomic and climate frameworks. From a theoretical perspective, our approach is similar to that of Acikgoz, Hagedorn, Holter, and Wang (2018) and Le Grand and Ragot (2023), who also characterize the solution to a Ramsey problem in a setting with incomplete markets. However, rather than attempting to derive policy rules for taxes on labor and capital income and government debt, our main goal is to obtain an expression for the optimal carbon tax and analyze how it differs from the Pigouvian benchmark.

On the positive side, our paper contributes to the literature on the distributional impacts of climate policy (for a recent example, see Känzig, 2023, and references therein). A growing number of studies have examined this question through the lens of macroeconomic models. Fried, Novan, and Peterman (2018) introduce an OLG model in which households face idiosyncratic productivity shocks and analyze the aggregate and distributional effects of an exogenous carbon tax under various revenue recycling schemes. They find that, compared to lump-sum transfers, reductions in labor and capital taxes improve welfare in the steady state but not during the transition, underscoring the importance of accounting for the full transition path. Benmir and Roman (2022) study the distributional effects of a net-zero emissions policy in the U.S., highlighting the role of credit constraints and the impact of climate policy on the wealth distribution. A few other studies have developed incomplete-market models in which households consume both clean and dirty goods and have non-homothetic preferences to match observed Engel curves (e.g., Le Grand, Oswald, Ragot, and Saussay, 2022; Labrousse and Perdereau, 2024; Kuhn and Schlattmann, 2024). Compared to these papers, we examine the effects of optimal policies throughout the transition and compare them to alternative policy scenarios within a rich quantitative macroeconomic model calibrated to closely reflect key features of the U.S. economy. Our welfare decomposition further allows us to disentangle the respective contributions of aggregate efficiency, redistribution, insurance, and environmental amenities to the overall welfare gains from climate policy.

The remainder of the paper is structured as follows. Section 2 lays out the infinite-horizon model and the planning problem. Section 3 characterizes the optimal carbon tax formula analytically. Section 4 details our calibration. Section 5 presents our numerical solution method and our quantitative results. Finally, Section 6 concludes.

2 Heterogeneous-agent climate–economy model

In this section, we present the dynamic climate–economy model used in our analysis. Household heterogeneity arises from uninsurable idiosyncratic labor-income risk, which gives rise to an endogenous distribution of savings. Our framework builds on the representative-agent model of Barrage (2020), extended by Douenne et al. (2023) to incorporate household heterogeneity. The key new element we introduce is incomplete markets, à la Aiyagari (1994). In this setting, we study the Ramsey problem of a government that uses taxes on labor and capital income, energy, and carbon emissions, together with transfers, to provide redistribution and insurance, and to address climate change.

2.1 Households

Time is discrete and indexed by t. The population at time t is denoted by N_t. Households are *ex ante* heterogeneous: a household of type i is characterized by their initial productivity $e_{i0} \in E = \{e_1, \ldots, e_L\}$ and their initial asset holdings $a_{i0} \in A_0 = \{a_1, \ldots, a_K\}$. Let α_i denote the fraction of type i households, with $\sum_i \alpha_i = 1$. Productivity evolves stochastically, and households can save only in a risk-free asset, subject to a borrowing constraint. In each period, household i draws a productivity level $e_{it} \in E$, and we denote by $\pi_{it}(e_i^t)$ the probability that household i experiences the sequence of productivity realizations $e_i^t = \{e_{i0}, \ldots, e_{it}\}$. Households derive utility from consumption c, disutility from labor effort h, and disutility from a climate variable Z. The preferences of a household of type i are given by

$$\sum_t \beta^t N_t \sum_{e_i^t} \pi_{it}(e_i^t)(u(c_{it}(e_i^t), h_{it}(e_i^t)) - v(Z_t)), \tag{1}$$

where $\beta \in (0, 1)$ is the discount factor, the per-period utility function $u(\cdot)$ satisfies the usual properties, and utility damages $v(\cdot)$ are increasing and convex in the climate variable.

Household per-capita assets evolve according to

$$(1 + n_{t+1})a_{it+1}(e_i^t) = (1 + (1 - \tau_{K,t})(r_t - \delta))a_{it}(e_i^{t-1}) + T_t + w_t(1 - \tau_{H,t})e_{it}h_{it}(e_i^t) - c_{it}(e_i^t), \tag{2}$$

where $n_{t+1} = N_{t+1}/N_t - 1$ is the population growth rate, w_t and r_t denote the before-tax wage and interest rate at time t, $\tau_{H,t}$ and $\tau_{K,t}$ are proportional tax rates on labor and capital income (net of depreciation), and T_t is a *uniform* lump-sum transfer (or tax, if negative). Each period, households choose labor supply and savings to maximize lifetime utility, subject to the intertemporal budget constraint (2) and a borrowing constraint $a_{it+1}(e_i^t) \geq \underline{a}_{t+1}$.[4] For future reference, aggregate consumption and effective labor supply are defined as

$$C_t = N_t \sum_i \alpha_i \sum_{e_i^t} \pi_{it}(e_i^t)\, c_{it}(e_i^t), \quad \text{and} \quad H_t = N_t \sum_i \alpha_i \sum_{e_i^t} \pi_{it}(e_i^t)\, e_{it} h_{it}(e_i^t).$$

2.2 Firms

The economy features two production sectors: a final-good sector and an energy sector. Each is operated by a representative, competitive firm.

2.2.1 Final good sector

The final-good sector produces the consumption-investment good, Y_t, using a constant-returns-to-scale production function $F(\cdot)$ with capital, labor, and energy inputs denoted by $K_{1,t}$, $H_{1,t}$, and E_t. Output is subject to climate damages, $D(Z_t)$, so that

$$Y_{1,t} = \big(1 - D(Z_t)\big) A_{1,t} F(K_{1,t}, H_{1,t}, E_t),$$

where $A_{1,t}$ denotes total factor productivity in the final-good sector. The firm rents capital and hires labor and energy at factor prices r_t, w_t, and $p_{E,t}$, respectively, to maximize profits.

2.2.2 Energy sector

The energy sector produces E_t using a constant-returns-to-scale technology $G(\cdot)$ with capital and labor inputs $K_{2,t}$ and $H_{2,t}$, both of which are fully mobile across sectors. Output is given by

$$E_t = A_{2,t} G(K_{2,t}, H_{2,t}),$$

where $A_{2,t}$ denotes total factor productivity in the energy sector. Energy production generates industrial CO_2 emissions,

$$E_t^M = (1 - \mu_t) E_t,$$

where μ_t represents the share of energy generated using clean technologies. Abating emissions incurs a cost given by $\Theta(\mu_t, E_t) = \theta_t(\mu_t) E_t$, where $\theta_t'(\mu_t), \theta_t''(\mu_t) > 0$, and $\theta_t(0) = 0$.

[4] To ensure the existence of a balanced-growth path given policy, we assume the borrowing limit grows at the rate of technological progress. See Appendix E.1 for details.

The firm's period-t profit is given by revenue net of taxes, factor payments, and abatement costs:

$$\mathcal{P}_t = (p_{E,t} - \tau_{I,t})E_t - \tau_{E,t}(1-\mu_t)E_t - w_t H_{2,t} - r_t K_{2,t} - \theta_t(\mu_t)E_t,$$

where $\tau_{I,t}$ is an excise tax on total energy and $\tau_{E,t}$ is a tax on carbon emissions. The firm chooses capital, labor, and the abatement rate μ_t to maximize profits, which are zero in equilibrium under the given abatement cost structure.

2.3 Government

The government levies proportional income taxes on capital income, $\tau_{K,t}$, and labor income, $\tau_{H,t}$, as well as excise taxes on total energy production, $\tau_{I,t}$, and on carbon emissions, $\tau_{E,t}$. Each period, it uses these instruments to finance an exogenous stream of public expenditures, G_t, and uniform lump-sum transfers, T_t. The government may also issue debt, B_{t+1}, subject to the condition that the debt sequence remains bounded. The government's intertemporal budget constraint is given by

$$G_t + N_t T_t + R_t B_t = \tau_{H,t} w_t H_t + \tau_{K,t}(r_t - \delta)K_t + \tau_{I,t}E_t + \tau_{E,t}E_t^M + B_{t+1}, \tag{3}$$

where H_t and K_t denote aggregate labor supply and capital, respectively, and $R_t \equiv 1 + (1-\tau_{K,t})(r_t - \delta)$ is the gross after-tax interest rate.

2.4 Climate

The climate variable Z_t is determined by the history of endogenous emissions, $\{E_t^M\}$, together with initial conditions S_0 and other exogenous drivers, $\{\eta_t\}$:

$$Z_t = J(S_0, E_0^M, \ldots, E_t^M, \eta_0, \ldots, \eta_t). \tag{4}$$

For now, we keep the climate model in a general form. A specific formulation is introduced in Section 4.1.

2.5 Competitive equilibrium

In our environment, a competitive equilibrium and a balanced-growth path are defined as follows.

Definition 1 *Given an initial stock of capital K_0 and government debt B_0, a distribution of types i over initial asset holdings a_{i0} and productivity levels e_{i0}, probabilities $\pi_{it}(e_i^t)$, population sizes*

$\{N_t\}_t$, *and a tax policy* $\mathcal{T} \equiv \{\tau_{H,t}, \tau_{K,t}, \tau_{I,t}, \tau_{E,t}, T_t\}_{t=0}^{\infty}$, *a* ***competitive equilibrium*** *consists of individual- and history-specific variables* $X_i \equiv \{a_{it+1}(e_i^t), c_{it}(e_i^t), h_{it}(e_i^t)\}_{i,t,e_i^t}$, *aggregate variables* $X \equiv \{C_t, H_t, H_{1,t}, H_{2,t}, K_{t+1}, K_{1,t}, K_{2,t}, E_t, \mu_t, Z_t, B_{t+1}\}_t$, *and a price system* $P \equiv \{R_t, w_t, r_t, p_{E,t}\}_t$, *such that:*

1. *households and firms optimize given prices and policies;*
2. *the government satisfies its intertemporal budget constraint* (3), *and debt remains bounded;*
3. *temperature evolves according to* (4); *and*
4. *markets clear, i.e., the following conditions are satisfied:*

$$H_t = H_{1,t} + H_{2,t}, \tag{5}$$

$$K_t = K_{1,t} + K_{2,t}, \tag{6}$$

$$C_t + G_t + K_{t+1} + \Theta_t(\mu_t)E_t = \big(1 - D(Z_t)\big)A_{1,t}F(K_{1,t}, H_{1,t}, E_t) + (1-\delta)K_t, \tag{7}$$

$$K_t + B_t = N_t \sum_i \alpha_i \sum_{e_i^{t-1}} \pi_{it-1}(e_i^{t-1})a_{it}(e_i^{t-1}). \tag{8}$$

The economy is on a ***balanced-growth path*** *if all aggregate variables, except temperature* Z_t *and abatement* μ_t, *grow at a constant rate and the competitive equilibrium conditions are satisfied.*

Equations (5)–(8) are the market-clearing conditions for labor, capital, final goods, and assets. Equation (7) shows that increasing abatement, μ_t, reduces the level of resources available, leading to a trade-off between climate mitigation and current consumption.

2.6 Ramsey problem

We assume the planner has utilitarian preferences given by

$$\mathcal{W} = \sum_i \alpha_i \sum_t \beta^t N_t \sum_{e_i^t} \pi_{it}(e_i^t)\big(u(c_{it}(e_i^t), h_{it}(e_i^t)) - v(Z_t)\big) \tag{9}$$

and that it announces and commits to a sequence of policies at time zero.

Definition 2 *Given* K_0, B_0, *and an initial distribution over asset holdings* a_{i0} *and productivity levels* e_{i0}, *for every policy* $\mathcal{T}$, ***equilibrium allocation rules*** $(X_i(\mathcal{T}), X(\mathcal{T}))$ *and* ***equilibrium price rules*** $P(\mathcal{T})$ *are such that* $\{\mathcal{T}, X_i(\mathcal{T}), X(\mathcal{T}), P(\mathcal{T})\}$ *constitute a competitive equilibrium. Given a welfare function* $\mathcal{W}(\mathcal{T})$, *the* ***Ramsey problem*** *is to* $\max_{\mathcal{T}} \mathcal{W}(\mathcal{T})$ *subject to* $(X_i(\mathcal{T}), X(\mathcal{T}))$ *and* $P(\mathcal{T})$ *being equilibrium allocation and price rules.*

We refer to the solution to this Ramsey problem as the *second-best tax system*, and to the implied allocation as the *second-best allocation*—the best outcome achievable given the specified set of tax instruments. In contrast, the *first-best* allocation is defined independently of the tax system, as the allocation that maximizes welfare subject only to resource constraints. We use the term *third-best* to refer to the solution of the Ramsey problem under additional restrictions on the tax system, such as requiring some instruments to remain fixed over time. This distinction will become relevant in Section 5, where we begin with a third-best environment and progressively relax constraints to approximate the second-best.

3 Characterizing Optimal Carbon Taxes

We now characterize optimal carbon taxes in the incomplete-markets environment described above. Because households face occasionally binding borrowing constraints, we cannot apply the primal approach and optimize over allocations subject to resource and implementability constraints (as in, for instance, Chari and Kehoe, 1999, and Werning, 2007). Instead, our formulation follows Acikgoz et al. (2018) and Le Grand and Ragot (2023), who study optimal taxation with incomplete markets but without a climate module. Appendix A presents the formal setup of the Ramsey problem and provides additional details.

As a benchmark, it is useful to first define the first-best, Pigouvian tax, which also corresponds to the social cost of carbon (SCC); we use these terms interchangeably.

Definition 3 *The* ***Pigouvian tax*** *is defined as*

$$\tau_t^{e,Pigou} = \frac{1}{\mathcal{W}_{c,t}} \sum_{j=0}^{\infty} \beta^j \left(\mathcal{W}_{c,t+j} D'_{t+j} A_{1,t+j} F_{t+j} - N_{t+j} \mathcal{W}_{Z,t+j} \right) J_{E_t^M,t+j}, \tag{10}$$

where $\mathcal{W}_{c,t} = \sum_i \alpha_i \sum_{e_i^t} \pi_{it} u_{c,it}$ is the marginal utility of consumption, averaged across household types i and histories e_i^t, and $\mathcal{W}_{Z,t} = -v'(Z_t)$ is the marginal utility of a higher temperature.

The Pigouvian tax is the first-best way to internalize climate externalities: it is set equal to the present value of the damages caused by one additional unit of emissions. An extra unit of emissions in period t raises temperatures in all future periods $t+j$ by $J_{E_t^M,t+j}$, which in turn generates production damages $\mathcal{W}_{c,t+j} D'_{t+j} A_{1,t+j} F_{t+j}$ and utility damages $-N_{t+j}\mathcal{W}_{Z,t+j}$, both expressed in utils and discounted using the period-t average marginal utility of consumption $\mathcal{W}_{c,t}$.[5] The next Proposition characterizes the second-best optimal carbon tax.

[5]With heterogeneous households, there is no unambiguous definition of the SCC, since marginal utilities differ across

Proposition 1 (Optimal carbon tax) *The second-best carbon tax satisfies a modified Pigouvian rule:*

$$\tau_t^{e,SB} = \frac{1}{\nu_t} \sum_{j=0}^{\infty} \beta^j \left(\nu_{t+j} D'_{t+j} A_{1,t+j} F_{t+j} - N_{t+j} \mathcal{W}_{Z,t+j} \right) J_{E_t^M, t+j}, \tag{11}$$

where ν_t is the multiplier on the resource constraint for the final consumption good, which at the optimum satisfies:

$$\nu_t = \mathcal{W}_{c,t} + \sum_i \alpha_i \sum_{e_i^t} \pi_{it}(SD_{it} + LD_{it}), \tag{12}$$

with

$$SD_{i,t} \equiv u_{cc,it} \left(\kappa_{it}^s - \frac{R_t}{1+n_t} \kappa_{it-1}^s \right), \qquad LD_{i,t} \equiv \kappa_{it}^{\ell} \left(u_{cc,it} w_t (1-\tau_{H,t}) e_{it} + u_{ch,it} \right), \tag{13}$$

and where κ^s and κ^{ℓ} are the multipliers associated with the household's modified Euler equation and first-order condition for labor supply, respectively (see Appendix A for details).

The key difference relative to the SCC lies in how climate damages are discounted. In the second-best, damages are not discounted using average marginal utilities of consumption, but instead using the multiplier on the resource constraint, which reflects the shadow value of aggregate resources. Because abatement entails a resource cost, this multiplier captures the opportunity cost of fighting climate change.

In the first-best, this shadow value equals the marginal utility of consumption: the government provides private consumption up to the point where the two are aligned. In the second-best, however, this equivalence breaks down, as saving and labor supply decisions are distorted, and tax and transfer policies affect household incentives along these two margins. The terms SD_{it} and LD_{it} in Proposition 1 explicitly capture how these distortions enter the resource multiplier, through the household's intertemporal (Euler) and intratemporal (labor supply) conditions, respectively. Changes in taxes or transfers can either mitigate or exacerbate these distortions, creating a wedge between the scarcity value of aggregate resources and the average marginal utility of consumption. This wedge calls for a systematic deviation from Pigouvian pricing.

A complementary way to interpret this wedge is through its connection to the marginal cost of public funds (MCF), a key concept in the literature on optimal carbon taxation (see, e.g., Bovenberg and Goulder, 1996, among many others), which measures the welfare cost of raising additional public funds, expressed in terms of the average marginal utility of consumption.

agents. Our choice to value damages using *average* marginal utilities aligns with the utilitarian planner's objective. In the first-best, full redistribution is achieved, and this ambiguity disappears, as marginal utilities of consumption are equalized across households.

Corollary 1 (Link with the MCF) *The second-best carbon tax can be expressed as a modified Pigouvian rule adjusted for the marginal cost of public funds (MCF), which is defined as the ratio between the multiplier on the resource constraint and the average marginal utility of consumption:*

$$\mathrm{MCF}_t \equiv \frac{\nu_t}{\mathcal{W}_{c,t}} = 1 + \frac{\sum_i \alpha_i \sum_{e_i^t} \pi_{it}(SD_{it} + LD_{it})}{\mathcal{W}_{c,t}}. \tag{14}$$

Deviations from Pigouvian pricing arise whenever the MCF differs from one, as in Barrage (2020) and Douenne et al. (2023). In a complete-markets setting, Douenne et al. (2023) show that the MCF averages to one over time under balanced-growth preferences, rendering the second-best carbon tax Pigouvian on average. In our Aiyagari economy, where Ricardian equivalence breaks down, the MCF can persistently lie above or below one, depending on how distortions in saving and labor supply decisions aggregate across households and histories. From a social welfare perspective, it is ambiguous whether households save or work too much or too little, implying that the contribution of SD_{it} and LD_{it} to the MCF can have either sign.[6]

Consequently, at a given allocation, interactions with saving and labor supply decisions can justify either higher or lower carbon taxes relative to the Pigouvian benchmark. However, in some cases, such interactions are absent.

Corollary 2 (Binding borrowing constraints) *If the borrowing constraint for household i binds following both history e_i^{t-1} and history e_i^t, interactions with saving decisions are absent: $SD_{it} = 0$.*

Corollary 2 implies that when household i faces a binding borrowing constraint in both period $t-1$ and period t, its consumption cannot adjust in response to changes in intertemporal incentives. As a result, interactions with saving distortions vanish ($SD_{it} = 0$). In this case, the household's ability to consume or save is determined entirely by the borrowing constraint, rather than by a desire for consumption smoothing. From the planner's perspective, changes in taxes or transfers, through their effects on consumption, neither mitigate nor exacerbate saving distortions for these households. Likewise, in the absence of income effects, changes in consumption streams do not affect labor supply distortions.

Corollary 3 (GHH preferences) *If households have Greenwood-Hercowitz-Huffman (GHH) preferences, such that their per-period utility function takes the form $U(c - \phi(h))$, then $LD_{it} = 0$ for all i and all e_i^t.*

[6] Technically, this means that the multipliers on the modified Euler equation, κ^s, and on the first-order condition for labor supply, κ^ℓ, can be either positive or negative in the optimal tax problem.

Under GHH preferences, labor supply responds only to the after-tax wage and productivity shocks, eliminating any role for income effects. As a result, changes in consumption streams do not interact with labor supply decisions, so the LD_{it} term is zero.

More generally, the presence and magnitude of deviations from Pigouvian taxation depend on whether changes in aggregate resources interact with distortions in household decisions. In the two cases above, such interactions are absent, and the corresponding wedges disappear. In general, the size of the SD_{it} and LD_{it} terms—and hence the deviation from the Pigouvian rule—is determined by the strength of these interactions.

If households have GHH preferences and all are borrowing constrained, the second-best *formula* for optimal carbon taxes coincides with the first-best rule from Definition 3, but the resulting tax rates differ because the formulas are evaluated at different allocations. In the first-best, the government achieves full insurance and redistribution, eliminating all distortionary taxes except for the carbon tax (which equals the SCC). In the second-best, it levies distortionary taxes on capital and labor income to provide partial insurance and redistribution, resulting in differences in aggregate variables. Moreover, while marginal utilities of consumption are equalized across households in the first-best, the second-best equilibrium features dispersion in marginal utilities, which raises the average marginal utility $\mathcal{W}_{c,t}$ when marginal utility is convex. To abstract from these allocative differences and isolate the effects of the distinct terms in the second-best formula, our quantitative analysis compares the second-best carbon tax to the Pigouvian formula evaluated at the second-best allocation.[7] With this comparison, we are interested in assessing how far off a planner would be if they applied the first-best formula in a second-best world.

4 Mapping the Model to the Data

This section describes the details of our calibration. All parameters are summarized in Table III of Appendix B.

Our calibration is based on the U.S. economy. Following Douenne et al. (2023), we scale up the

[7]Since we calibrate the second-best economy to the current tax system, the first-best allocation is quite distant in both aggregates and distributions, and represents a counterfactual much richer economy without distortionary taxes or inequality. We therefore do not find the direct comparison between first-best and second-best rates to be meaningful. One alternative would be to recalibrate the first-best economy to make it comparable (the approach taken in Barrage, 2020), but this introduces attribution challenges. By evaluating the Pigouvian formula at the second-best allocation, we obtain a clean comparison based solely on the objects characterized above.

U.S. so that its emissions and GDP match those of the world, adjusting the population to preserve U.S. GDP per capita. This allows the optimal policy to reflect how the U.S. should adjust its carbon taxation to address domestic inequality, under the assumption that the rest of the world's emissions are proportional to those of the U.S., and that global production and utility damages are internalized as if they affect others analogously to how they affect the U.S. Our approach is intended to ensure that policies reflect the global SCC, in line with the stated objectives of the previous U.S. administration, which asserted that "It is essential that agencies capture the full costs of greenhouse gas emissions as accurately as possible, including by taking global damages into account." (Executive Order 13990 of Jan 20, 2021).[8]

4.1 Climate model

We use the climate model of Dietz and Venmans (2019) to capture two key features of climate dynamics: the temperature response to emissions is almost immediate and persistent, and temperature increases approximately linearly with cumulative emissions.[9] This model provides a tractable and empirically grounded approximation of the impulse response of temperature to emissions, which is essential for a proper quantitative assessment of the welfare effects of climate policy. Formally, global mean surface temperature change relative to pre-industrial levels, Z_t, evolves according to

$$Z_{t+1} = Z_t + \epsilon\big(\zeta\mathcal{E}_t - Z_t\big), \tag{15}$$

where ζ is the transient climate response to cumulative carbon emissions (TCRE) (see Dietz and Venmans, 2019), ϵ governs the speed of temperature adjustment to an emissions pulse, and $\mathcal{E}_t$ denotes cumulative anthropogenic emissions, which evolve as

$$\mathcal{E}_{t+1} = \mathcal{E}_t + E_t^M + E_t^{\text{ex}},$$

with E_t^{ex} representing exogenous land-use emissions.

[8]An alternative is to model only the U.S. economy, assuming foreign emissions scale with domestic ones, but the U.S. considers only the domestic SCC and ignores external damages. This approach is adopted by Benmir and Roman (2022), who impose an emissions cap but do not study the SCC. A third approach, used by Barrage (2020), calibrates the model to the world economy, requiring a global planner to set both carbon and global income taxes.

[9]For a discussion of how these properties appear in other climate–economy models, see Mattauch et al. (2020) and Dietz et al. (2021). For further references on the temperature response to emissions over time, see Joos et al. (2013), Ricke and Caldeira (2014), and references therein. For further references on the linear relationship between cumulative emissions and temperature, see Matthews et al. (2009), Gillett et al. (2013), or the summaries provided in IPCC (2021).

We calibrate the model following IPCC (2021). Initial cumulative carbon emissions are set to $\mathcal{E}_{2020} = 2390$ $GtCO_2$, and the initial temperature change to $Z_{2020} = 1.07$°C. We adopt the IPCC's best estimate of the TCRE, setting $\zeta = 0.00045$ °C/$GtCO_2$. Following Dietz and Venmans (2019), we set the adjustment speed to $\epsilon = 0.5$. Initial industrial and land-use emissions are taken from the Global Carbon Project (Friedlingstein et al., 2022), with $E^M_{2020} = 38.23$ $GtCO_2$/year and $E^{\text{ex}}_{2020} = 4.17$ $GtCO_2$/year. These represent net emissions, i.e., after abatement. As land-use emissions are exogenous in our framework, we follow DICE 2023 (Barrage and Nordhaus, 2023) and assume that gross land-use emissions decline by 10% every five years.

4.2 Damages

We model production damages following Dietz and Venmans (2019), i.e.,

$$D(Z_t) = 1 - \exp\left(-\frac{\alpha_1}{2} Z_t^2\right).$$

This exponential-quadratic specification produces a damage curve similar to that of DICE 2023, although the magnitude of damages is higher under the Dietz and Venmans (2019) calibration: with a baseline parameter of $\alpha_1 = 0.01$, damages amount to 2% of output at 2°C warming and 7.7% at 4°C (compared to 1.4% and 5.5% in DICE 2023). We adopt the central value of $\alpha_1 = 0.01$ from Dietz and Venmans (2019) as a starting point but adjust this parameter to allocate damages between production and utility. Following Barrage (2020), we assume that 74% of damages at 2.5°C warming are due to output losses, while the remaining 26% reflect direct utility impacts. This leads to a calibrated value of $\alpha_1 = 0.0074$, which we use to compute the parameter governing utility damages (α_z, see below).

4.3 Households

In our model, the primary unit of analysis is the *household*, as opposed to the individual. Consequently, we measure all relevant statistics in the data at the household level, applying the equivalence scales recommended by the U.S. Census (Appendix C.2). In the household problem described in Section 2.1, we interpret consumption, hours, and asset positions on a per-capita basis within each household. We discipline both the preference parameters and the labor productivity process faced by households by targeting three sets of statistics: (i) macroeconomic aggregates, (ii) inequality statistics, and (iii) measures of idiosyncratic labor-income risk. We detail our approach below.

Preferences. Households have preferences over consumption, labor, and climate. We assume the following utility function:

$$u\left(c_t, h_t\right) - v(Z_t) = \frac{\left(c^{\gamma}(1-\varsigma h)^{1-\gamma}\right)^{1-\sigma} + \left(1+\alpha_z Z_t^2\right)^{\sigma-1}}{1-\sigma}. \tag{16}$$

We discipline the preference parameters $\{\beta, \gamma, \sigma, \varsigma, \alpha_z\}$ as follows. First, we target a capital-output ratio of 2.6, computed from National Income and Product Accounts (NIPA) data for the period 2009–2019.[10] Second, we set the intertemporal elasticity of substitution (IES) to $1/1.5$, a value well within the range commonly used in the quantitative macroeconomic literature.

To discipline the labor supply margin, we target average hours worked in the population, 0.25, and impose that the average Frisch elasticity equals 1.0.[11]

Since household-level Frisch elasticities depend on each household's labor supply, we compute the intensive-margin average Frisch elasticity as the unweighted average of household-level Frisch elasticities for employed households:

$$\Psi \equiv \int_{h(a,e)\,\geq\,\underline{h}} \left(\gamma + (1-\gamma)\frac{1}{\sigma}\right) \frac{1-h(a,e)}{h(a,e)}\, d\lambda_0(a,e). \tag{17}$$

Finally, we set the parameter α_z to ensure that 26% of total damages are attributed to utility impacts at 2.5°C warming, following Barrage (2020).

Labor productivity. We model household labor productivity as a combination of two components: a persistent component e_P, governed by a Markov matrix Γ_P, and a transitory component e_T, defined by a probability vector P_T.[12] The process features four persistent and six transitory productivity levels. Normalizing average productivity to one leaves 26 free parameters, which we calibrate to match empirical targets related to population structure, inequality, and income risk.

Population. We align the model's household classification with that in the data, using the Survey of Consumer Finances (SCF). Households are grouped into four mutually exclusive categories: workers, business owners, retirees, and non-working households.

[10] Capital is defined as the sum of nonresidential and residential private fixed assets and purchases of consumer durables. For further details, see Appendix C.1.

[11] To compute average hours worked, we use the Current Population Survey (CPS) and calculate average annual hours worked for the entire working-age population, regardless of employment status, which is 1269 hours. Assuming households can work a maximum of 100 hours per capita per week for 52 weeks per year, this corresponds to $1269/(52 \times 100) = 0.25$.

[12] In the model's notation, $\Gamma = \Gamma_P \otimes \mathrm{diag}(P_T)$, and $e = e_P + e_T e_P^{\eta}$. For example, if $\eta = 0$, the transitory shocks are additive; if $\eta = 1$, they are multiplicative.

A household is classified as a business owner if either the head or spouse is engaged in business ownership and business and capital income both exceed labor income. Retirees are households in which both the head and spouse have reported retirement prior to the survey year and are not classified as business owners. Non-working households report no labor income and are neither business owners nor retirees. Remaining households are classified as workers. To simplify the model, we consolidate retirees and non-working households into a single category, *Inactive Households*.

In the model, one persistent productivity state is reserved for business owners, capturing entrepreneurial income (see also Dyrda and Pedroni, 2023). Households with hours worked below the threshold $\underline{h}$ are classified as inactive; all others are classified as workers. We calibrate the model to approximate the observed population shares and the distributions of earnings, income, and wealth across these groups (Table I).

Table I: Population Partitions: Model vs. Data

	Shares			
	Population	**Earnings**	**Income**	**Wealth**
		Workers		
Data	67.2	82.7	69.1	44.9
Model	70.9	86.3	78.7	47.0
		Business Owners		
Data	5.8	13.7	16.1	33.0
Model	6.6	13.7	14.8	31.2
		Inactive Households		
Data	27.0	3.6	14.8	22.2
Model	22.5	0.0	6.5	21.8

Notes: Data comes from 2019 wave of the SCF. Details about the definitions of subgroups of the population can be found in Appendix C.3.

Inequality and Income Risk. We target several key measures of inequality: the shares of wealth, earnings, and hours by quintile; the Gini coefficient; and the shares held by the bottom and top 5% of the distribution. Wealth and earnings data are drawn from the SCF, and hours data from the Current Population Survey (CPS). The model's performance on these dimensions is reported in Table II.

To capture the joint distribution of earnings and wealth, we also target their cross-sectional correlation. Our treatment of income risk is guided by the empirical properties of the household labor income process documented in Pruitt and Turner (2020a). We compute and target the variance, Kelly skewness, and Moors kurtosis of labor income growth rates. In calculating these moments in the model, we exclude entrepreneurial households and focus on active households, conditioning on employment status.

4.4 Production

We assume that both sectors feature Cobb-Douglas production functions,

$$F(K_{1,t}, H_{1,t}, E_t) = K_{1,t}^{\alpha} H_{1,t}^{1-\alpha-\nu} E_t^{\nu},$$
$$G(K_{2,t}, H_{2,t}) = K_{2,t}^{\alpha_E} H_{2,t}^{1-\alpha_E},$$

with $\alpha = 0.3$, $\nu = 0.04$ (from Golosov et al., 2014), and $\alpha_E = 0.597$ (from Barrage, 2020). We calibrate initial total factor productivities to match global GDP (World Bank) and aggregate industrial emissions (Friedlingstein et al., 2022), with growth rates taken from DICE 2023.

The abatement cost function and its parametrization are also taken from DICE 2023:

$$\theta_t(\mu_t)E_t = P_t^{\text{back}} \frac{\mu_t^{c_2}}{c_2} E_t,$$

where $c_2 = 2.6$, and P_t^{back} is the backstop price of carbon abatement. The backstop price reflects the marginal cost of fully eliminating emissions through technological substitution or carbon removal; if carbon taxes are set at this level, emissions fall to zero. It starts at 696.2$/tCO$_2$ in 2020, declines by 1% per year until 2050, and by 0.1% per year thereafter.

4.5 Fiscal Policy

For calibration purposes, we introduce a time-invariant consumption tax, τ_C, as a model parameter. We discipline the labor, capital, and consumption tax rates by extending the analysis of Trabandt and Uhlig (2011) through 2019 (Appendix C.4), setting each rate to its average over the 2015–2019 period. This yields an initial capital income tax of $\tau_{K,0} = 33.6\%$, a labor income tax of $\tau_{H,0} = 27.7\%$, and a consumption tax of $\tau_C = 4.2\%$. The initial tax on total energy production, $\tau_{I,0}$, is set to zero, while the initial carbon emissions tax, $\tau_{E,0}$, is set to 6$/tCO$_2$ (Barrage and Nordhaus, 2023).

Table II: Benchmark Model Economy: Target Statistics and Model Counterparts

(1) Macroeconomic aggregates

	Target	Model
Intertemporal elasticity of substitution	0.66	0.66
Capital to output	2.57	2.54
Average Frisch elasticity (Ψ)	1.0	1.0
Average hours worked	0.24	0.25
Transfer to output (%)	14.7	14.7
Debt to output (%)	104.5	104.5
Fraction of hhs with negative net worth (%)	10.8	11.5
Correlation between earnings and wealth	0.51	0.43

(2) Cross-sectional distributions

	Bottom (%)	Quintiles					Top (%)	Gini
	0–5	1st	2nd	3rd	4th	5th	95–100	
Wealth								
Data	−0.5	−0.5	0.8	3.4	8.9	87.4	65.0	0.85
Model	−0.2	0.1	1.7	3.6	6.7	88.1	70.0	0.85
Earnings								
Data	−0.1	−0.1	3.5	10.8	20.6	65.2	35.3	0.65
Model	0.0	0.1	3.6	12.0	17.7	66.6	37.5	0.65
Hours								
Data	0.0	2.7	13.8	19.2	27.9	36.4	11.1	0.34
Model	0.0	0.4	11.4	26.1	28.3	33.9	8.9	0.35

(3) Statistical properties of labor income

	Target	Model
Variance of 1-year growth rate	2.33	2.32
Kelly skewness of 1-year growth rate	−0.12	−0.13
Moors kurtosis of 1-year growth rate	2.65	2.65

We target a government debt-to-GDP ratio of 104.5 percent, matching its 2019 level. Lump-sum transfers are mapped to personal transfer receipts in the NIPA, including Social Security, Medicare, Medicaid, and unemployment insurance. This is consistent with our treatment of retired and unemployed households as unproductive, with transfers providing baseline income. Accordingly, we set the ratio of lump-sum transfers to GDP at 14.7 percent (Appendix C.1).

5 Quantitative results

In this section, we first describe our numerical method for solving dynamic optimal tax problems (Section 5.1). We then present our quantitative results. We begin by showing that the optimal carbon tax closely follows the Pigouvian benchmark and that this trajectory remains remarkably stable across a wide range of fiscal environments (Section 5.2). We then analyze the intra- and intertemporal distribution of the welfare gains generated by the carbon tax (Section 5.3). Finally, we assess the robustness of these findings in an extended model featuring non-homothetic household preferences over final-good and energy consumption (Section 5.4).

5.1 Solution method

Our solution method builds on Dyrda and Pedroni (2023). To convert the infinite-dimensional Ramsey problem defined above into a finite-dimensional one, we assume the existence of a Ramsey balanced growth path—in the long run, all optimal fiscal instruments, including government debt, grow at a constant rate, and the economy converges to a new balanced growth path. To reduce the dimensionality of the problem, we approximate the time paths of fiscal instruments using a combination of orthogonal polynomials:

$$x_t = \left(\sum_{i=0}^{m_{x0}} \alpha_i^x P_i(t) \right) \exp\left(-\lambda^x t\right) + \big(1 - \exp\left(-\lambda^x t\right)\big) \left(\sum_{i=0}^{m_{xF}} \beta_i^x P_i(t) \right), \quad t \in \{0, \dots, t_F\}, \tag{18}$$

where x_t denotes any of the fiscal instruments $\{\tau_{H,t}, \tau_{K,t}, \tau_{E,t}, T_t\}$; $\{P_i(t)\}_{i=0}^{m_{x0}}$ and $\{P_i(t)\}_{i=0}^{m_{xF}}$ are families of Chebyshev polynomials; $\{\alpha_i^x\}_{i=0}^{m_{x0}}$ and $\{\beta_i^x\}_{i=0}^{m_{xF}}$ are the corresponding weights; λ^x controls the convergence rate of the fiscal instrument; and t_F is the period after which the instrument remains constant. The orders of the polynomial approximations are given by m_{x0} and m_{xF} for the short-run and long-run dynamics, respectively. Given this approximation, we optimize the parameters to maximize

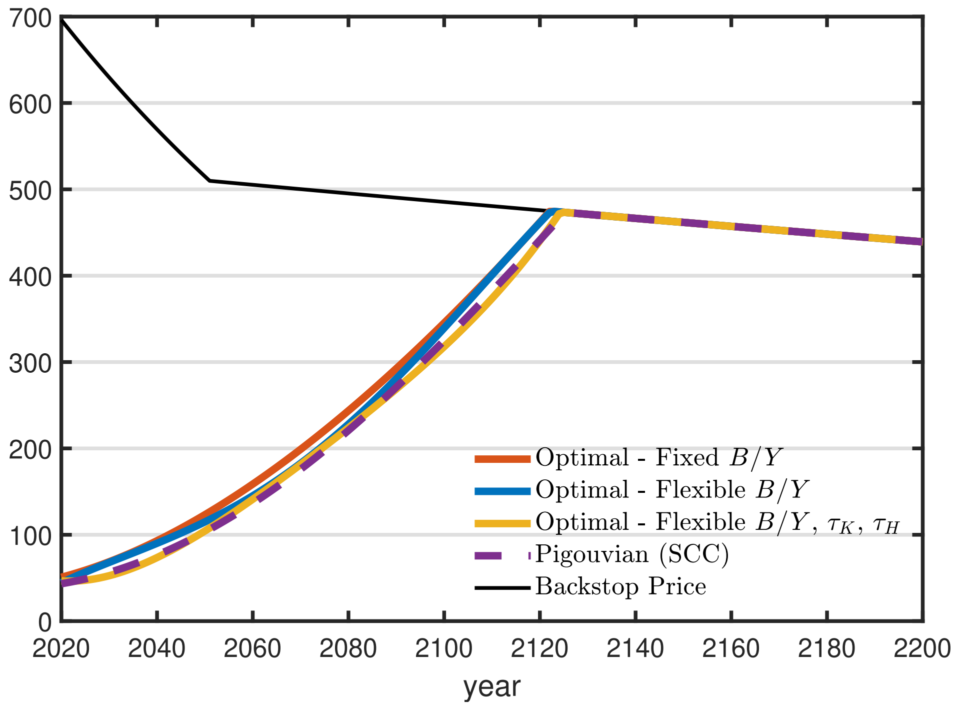


Figure 1: Optimal Carbon Taxes and Backstop Price (in \$/tCO_2).

Note: This figure plots the paths of optimal carbon taxes in the baseline scenario where the debt-to-GDP ratio and income taxes are set at their current level (red), in the second scenario where the government can choose the path of debt (blue), and in the third scenario where it can also choose income taxes (yellow). The purple dashed line represents the SCC evaluated at the allocation of the third scenario, and the black line plots the backstop price. All values are expressed in \$/tCO_2.

welfare over the transition between balanced growth paths.[13] See Appendix E for more details.

5.2 Optimal carbon taxes

Figure 1 plots the optimal carbon tax paths across three scenarios. In the first scenario (red line, our baseline), the government treats income taxes and the debt-to-GDP ratio as fixed over time, choosing only the carbon tax path and rebating its revenue lump sum. In the second scenario (blue line), income taxes remain fixed, but the government controls the path of debt, determining the timing of lump-sum transfers. In the third scenario (yellow line), the government additionally sets labor and capital income

[13]We assume that every policy instrument becomes constant after period $t_{F,1}$, and that the detrended economy has converged to a steady state by period $t_{F,2} > t_{F,1}$. Given this assumption, we compute the optimal policy along the transition. If instruments were allowed to move indefinitely, the transition could continue for some time before reaching a different steady state—or possibly none at all (see Auclert et al., 2024). This would not have material welfare consequences as of period zero, however, since the horizons $t_{F,1}$ and $t_{F,2}$ are chosen such that extending them further has a negligible effect on welfare along the transition.

taxes. The figure also displays the Pigouvian tax path (dashed purple line)—the social cost of carbon (SCC)—along with the backstop price (black line).

There are two main takeaways from this figure. First, across all scenarios, the optimal carbon tax is very close to the SCC. This result holds even in settings where the government faces strong constraints on its ability to smooth consumption and provide redistribution and insurance (see also Figure G.1 in the appendix). Second, although the different scenarios lead to stark differences in policies for debt, transfers, and income taxes, the paths of carbon taxes remain strikingly similar. Sections 5.2.1 and 5.2.2 examine, in turn, the mechanisms behind these results.

5.2.1 Deviations from Pigou

Optimal vs. SCC The results in Figure 1 show that in an incomplete-markets economy with significant inequality, risk, borrowing constraints, and fiscal distortions—conditions under which Ricardian equivalence fails—the optimal carbon tax remains nearly identical to the SCC.[14] Another way to see that the distance between the optimal carbon tax and the SCC is small, and decreases even further as constraints are removed, is to look at the welfare gains from shifting from the SCC to each scenario's optimal carbon tax path: the gain is 0.008% in the baseline scenario and falls to 0.001% in the other two scenarios. While we established in Section 3 that deviations from Pigou were theoretically possible, they turn out to be quantitatively negligible.

A question of targeting Why are deviations from the Pigouvian benchmark so small? Intuitively, deviations from the Pigouvian rule are warranted only if the carbon tax can *effectively* address other market failures. To formalize this idea, consider a planner maximizing welfare $f(x, y)$ by choosing two instruments: a tax on carbon emissions, x, and a lump-sum transfer financed with distortionary taxes on labor and capital income, y. Suppose the planner faces a constraint on y. A first-order Taylor expansion around the unconstrained optimum (x^*, y^*) yields

$$x^{**} - x^* \;\approx\; \frac{f_{xy}(x^*, y^*)}{f_{xx}(x^*, y^*)}\,(y^* - \bar{y}),$$

[14]The Pigouvian taxes in Figure 1 are computed by evaluating equation (10) at the allocation from the third scenario, which allows the most flexibility in policy instruments. Figure G.1 in the appendix shows the Pigouvian tax paths for the allocations in the other two scenarios. Figure G.2 in the appendix also reports the Pigouvian tax evaluated at the marginal utility of the (risk-free) representative agent. Ignoring the dispersion in marginal utilities yields a slightly higher SCC, though the difference is small: about 7–10% in the first scenario, where the government cannot redistribute, and 2–4% in the other two scenarios.

with x^{**} the constrained optimal value of x. When y can be freely optimized, the envelope theorem implies that the optimal carbon tax is unaffected by concerns related to y. When y is constrained, however, the deviation in x depends on both the distance from the unconstrained optimum, $(y^* - \bar{y})$, and the ratio of the cross- to own-second derivatives, f_{xy}/f_{xx}. In economic terms, this means that departures from the Pigouvian benchmark arise only when fiscal constraints are quantitatively important and the carbon tax and the lump-sum transfer are sufficiently complementary (i.e., f_{xy} is large relative to f_{xx}). In what follows, we use this heuristic to interpret the magnitude of deviations from the Pigouvian benchmark in our results and to compare them with findings in the literature.

Fiscal constraint tightness Our finding that deviations of the optimal carbon tax from Pigou are negligible contrasts sharply with results from the representative-agent literature, where distortionary taxation typically implies taxing below the SCC (e.g., Bovenberg and Goulder, 1996; Barrage, 2020). In these economies, the planner would ideally set large lump-sum taxes to finance all government spending, which is forbidden by assumption, so $(y^* - \bar{y})$ is high by construction. The government must then rely on distortionary taxes that reduce efficiency without providing redistribution or insurance benefits. When carbon taxes further reduce incentives to work and invest, they erode the tax base and strongly interact with these wedges, so f_{xy}/f_{xx} is also high. This rationalizes the relatively large effect of the MCF on optimal carbon taxes in these economies.

In heterogeneous-agent economies, by contrast, lump-sum *taxes* are not ruled out: the planner endogenously chooses to finance *transfers* to provide redistribution and insurance. As discussed in Douenne et al. (2023), in a setting with heterogeneity but complete markets, the optimal policy equates the marginal cost of distortions with the marginal benefit of redistribution. Hence, the envelope argument described above applies: when the tax system is optimized, even under the constraint of linear taxes, the shadow costs of fiscal distortions—captured in the current setting by the terms SD and LD in our Proposition 1—are in fact negligible.[15] Consequently, the distance between the second-best and first-best tax-transfer systems, $(y^* - \bar{y})$, is small, leading to only minor deviations from the Pigouvian benchmark. Hence, under the flexibility of second-best fiscal instruments, Pigouvian results hold almost exactly. The striking feature is that this remains true even when fiscal constraints are severe, which can only be the case if f_{xy}/f_{xx} is also small.

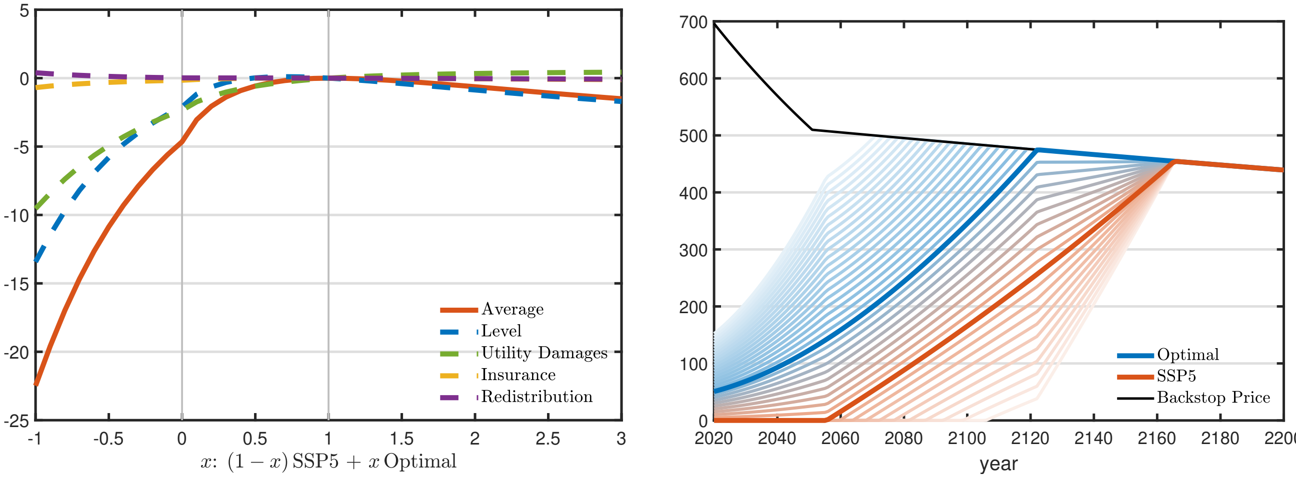
[15] From Corollary 1, these deviations can equivalently be summarized by the MCF. In a complete-market setting, Douenne et al. (2023) shows that the MCF averages to 1 over time. With incomplete markets, we cannot establish this result analytically, but we confirm that the correction term induced by the MCF remains quantitatively negligible.

In our baseline third-best scenario, income taxes are fixed and lump-sum transfers are not allowed to vary over time. Since the planner would like to substantially increase redistribution and front-load transfers (see Section 5.2.2), these are highly binding constraints, making $(y^* - \bar{y})$ relatively large. Yet the deviations from Pigou remain small, implying that f_{xy}/f_{xx} must be correspondingly low: the carbon tax offers little complementarity to transfers in providing redistribution and insurance. We provide direct evidence of this in the welfare decomposition below.

Welfare decomposition To explore this complementarity in our quantitative model, we decompose the welfare gains from the carbon tax into four components: level effects, redistribution, insurance, and environmental amenities (from the utility term directly dependent on temperature). Our approach extends the method of Dyrda and Pedroni (2023) by incorporating the fourth component, as detailed in Appendix D. Figure 2a compares the optimal policy in our baseline experiment to a scenario where all other instruments are held fixed, but the carbon tax is adjusted to replicate the temperature trajectory of the SSP5 scenario (IPCC, 2021), with differences in tax revenue offset by adjustments in lump-sum transfers. The SSP5 scenario represents a future with weak climate policies; Figure 2b shows its corresponding tax path (see Figure G.3a in appendix for the associated temperature trajectory). To assess welfare changes across a broad set of policy scenarios, the figure plots welfare gains for convex combinations of the optimal policy and SSP5.

Figure 2a shows that virtually all welfare gains from higher carbon taxes arise from aggregate efficiency improvements (blue line) and reduced amenity-related damages (green line); the effects on redistribution and insurance (purple and yellow lines) are negligible. Looking at the slopes near the optimum, we also see that f_{xy} is much smaller than f_{xx}. This implies that even under stringent fiscal constraints—i.e., in our third-best scenarios where $(y^* - \bar{y})$ is large—carbon taxes are ineffective tools for addressing inequality and risk. Deviating from the externality-correction objective of the tax to achieve redistribution or insurance would therefore be extremely costly.

An extreme scenario Figure G.4 in the appendix considers an extreme case where the level of the carbon tax is closely tied to how efficiently public spending is financed. In this scenario, carbon tax revenues are directed towards wasteful spending: the optimal policy then delays climate action by two decades, before rapidly accelerating the transition. Although the carbon tax represents a small share of total government revenue (see Figure G.5 in the appendix), the inefficiency it introduces at



(a) Welfare Gains and Decomposition

(b) Carbon Tax (in \$/tCO_2).

Figure 2: Welfare Gains and Carbon Tax Path: Optimal Policy vs. SSP5.

Note: The figure on the left plots the welfare losses in percentage of consumption required to make the government indifferent between the optimal policy and policy x (red line). The dashed lines represent the different components of our welfare decomposition, illustrating how transitioning from policy x to the optimal policy improves welfare through each channel: level effects (blue), environmental amenities (green), insurance (yellow), and redistribution (purple). The figure on the right represents the corresponding carbon tax path for policies x computed as convex combinations (shaded lines) of the optimal policy in our baseline (blue) and the SSP5 scenario (red), where all other instruments remain at their baseline levels except for the carbon tax and lump-sum transfers.

the margin is sufficient to significantly alter the optimal climate policy path.[16]

Asymmetric welfare losses There is another striking finding that emerges from Figure 2a: welfare losses from deviations are highly asymmetric. Exceeding the optimal tax level results in minor welfare losses, whereas undershooting it leads to sharp welfare declines. Thus, given uncertainties around the SCC (see e.g., Cai and Lontzek, 2019), and despite substantial distributional concerns, it is preferable for policymakers to err on the side of overly ambitious climate policies.

[16] Intuitively, the revenue generated by the carbon tax follows what we could call a "carbon Laffer curve". It grows as the carbon price increases, but starts to decline as emissions converge towards zero. When the carbon tax revenue is used for wasteful spending, the opportunity cost of foregone revenue is lower, hence the government adopts a more radical strategy, not taxing carbon for a while and setting very high rates later on. It is interesting to note that even then, long-term climate objectives remain largely unchanged: while cumulative climate damages increase, the later policy acceleration results in similar—even slightly lower—long-run temperature outcomes.

5.2.2 Invariance across scenarios

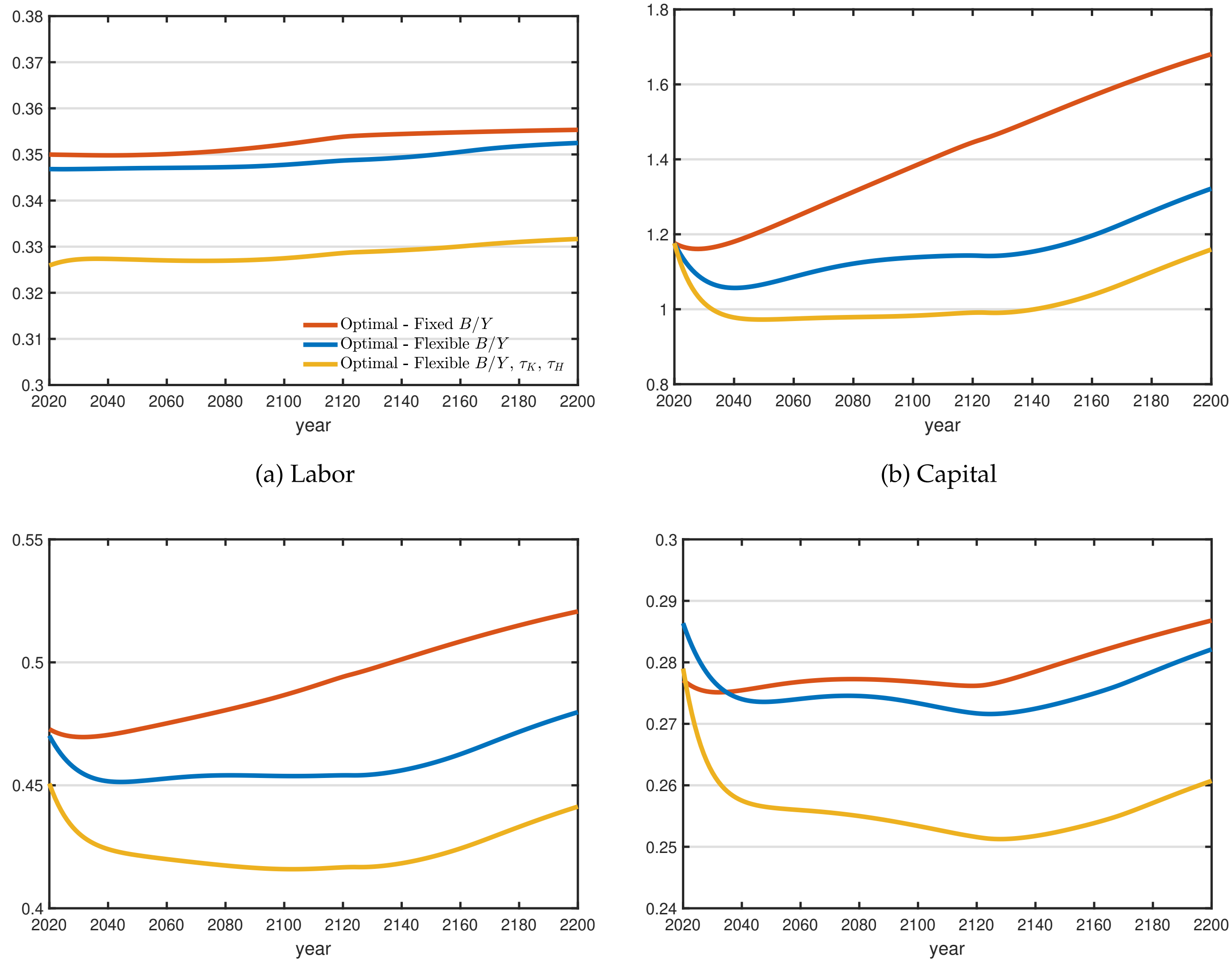
The three scenarios depicted in Figure 1 differ in the extent to which the government can adjust fiscal instruments. In the second scenario, where the government can adjust the debt trajectory, it borrows heavily to front-load transfers to households, as in Dyrda and Pedroni (2023). As illustrated in Figure G.6 in the appendix, the ratio of transfers to GDP (T/Y) initially rises from 14.6% to 21.9%, accompanied by a substantial increase in the debt-to-GDP ratio. These transfers reduce inequality and mitigate household risk by facilitating consumption smoothing. Further, by relaxing credit constraints, they move the economy closer to Ricardian equivalence. In the third scenario, where the government can additionally adjust income tax rates, it increases taxes on labor (from 27.7% to 41.5%) and capital (from 33.6% to 44.7%), enabling even larger transfers, which exceed 25% of GDP throughout the 21st century before gradually declining.

These policy changes have substantial consequences for the economy. Figure 3 shows that higher taxes and increased debt significantly reduce labor supply due to substitution effects toward leisure and income effects from increased transfers. Capital accumulation is adversely affected through multiple channels: reduced labor supply decreases output, higher public debt crowds out private capital, and higher transfers reduce precautionary savings. Consequently, both production and consumption decline. The contraction in labor supply triggers an immediate 4.6% drop in GDP, with the gap widening over time as capital adjusts. After a century, GDP in the third scenario, in which the government sets both the debt trajectory and income tax levels, is 15.7% lower than in the baseline case. However, inequality is substantially reduced: for instance, the Gini coefficient of consumption in the initial period after the reform declines from 0.287 in the baseline scenario to 0.241 in the second scenario and further to 0.237 in the third scenario.

It is therefore striking that, despite these differences, we obtain very similar paths for the optimal carbon tax. To explore the mechanisms behind this robustness, we leverage the fact that the optimal carbon tax closely tracks the SCC and use our analytical expression for the SCC derived in Section 3.[17] From equation (10) in Definition 3, we can decompose the Pigouvian tax (i.e., the SCC) as follows:

$$\tau_t^{e,Pigou} = \frac{1}{\lambda_t^A} \sum_{j=0}^{\infty} \beta^j \left(\lambda_{t+j}^A \lambda_{t+j}^B - \lambda_{t+j}^C \right) J_{E_t^M, t+j}, \tag{19}$$

[17] The advantage of using the SCC over our second-best tax formula is that it depends only on objects that can be directly observed in equilibrium, i.e., it does not require computing the entire path of the multiplier ν_t. It also enables us to study the impact of other fiscal instruments on climate policy beyond the effect of second- and third-best mechanisms.



(a) Labor

(b) Capital

(c) Production

(d) Consumption

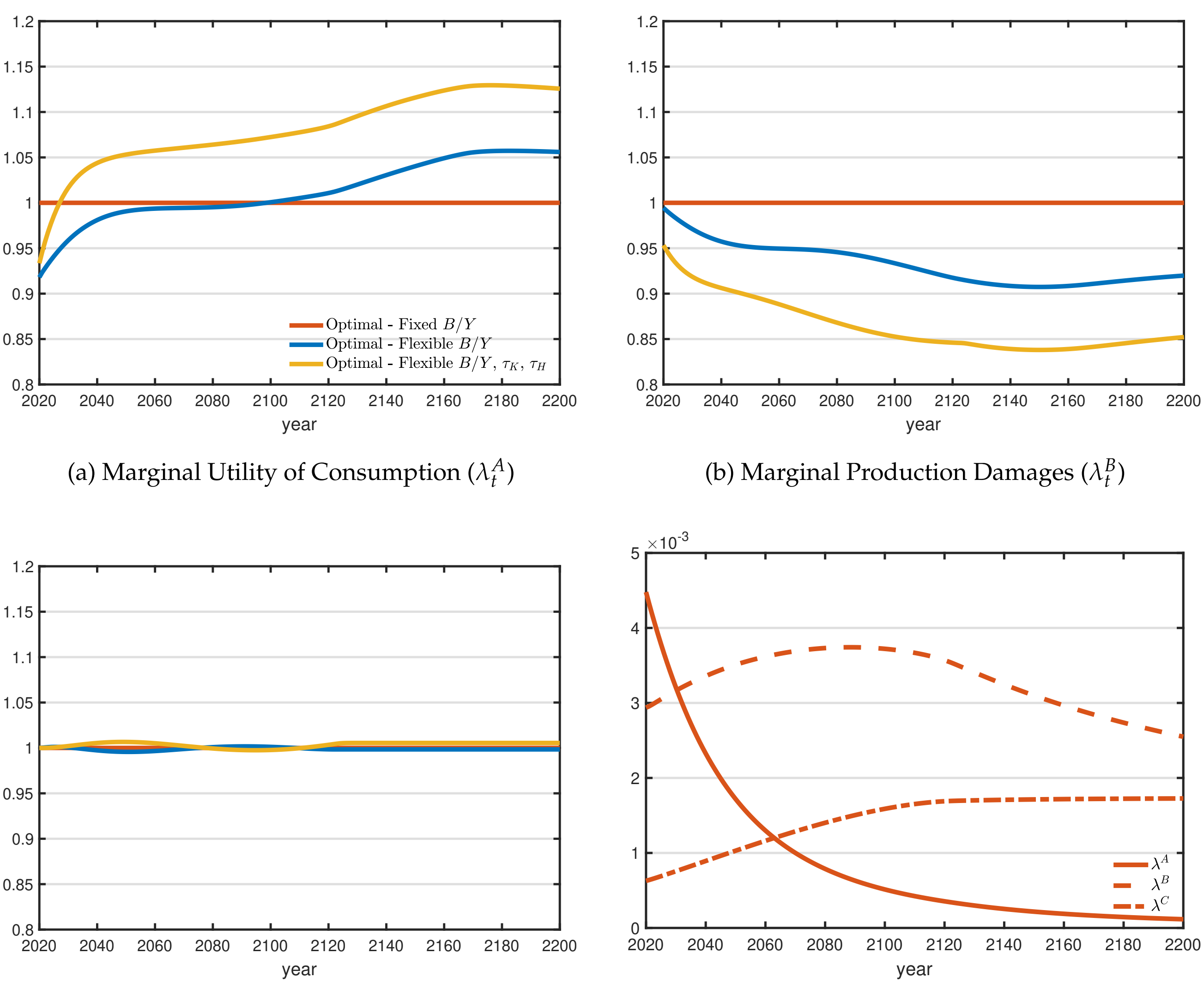
Figure 3: Path of Aggregate Economic Variables

Note: The figure plots the paths of aggregate economic variables in the baseline scenario where the debt-to-GDP ratio and income taxes are set at their current level (red), in the second scenario where the government can choose the path of debt (blue), and in the third scenario where it can also choose income taxes (yellow). The top-left figure plots the path of labor supply, the top-right figure plots the path of capital, the bottom-left figure plots the path of production, and the bottom-right figure plots the path of consumption.

with

$$\lambda_t^A = \mathcal{W}_{c,t}, \qquad \lambda_t^B = D_t' A_{1,t} F_t, \qquad \lambda_t^C = \mathcal{W}_{Z,t}. \tag{20}$$

Figure 4 plots the evolution of each component over time for the three scenarios. The third component, λ_t^C, remains largely unchanged across scenarios due to the similar temperature trajectories (shown in Figure G.7 in the appendix). The first two components, however, are more sensitive to



(a) Marginal Utility of Consumption (λ_t^A)

(b) Marginal Production Damages (λ_t^B)

(c) Marginal Utility from Climate Amenities (λ_t^C)

(d) Scale of Components

Figure 4: Path of Components of the SCC

Note: Panels (a), (b), and (c) plot the time path of each of the components of the optimal carbon tax specified in equation (19) for three scenarios, normalized based on the value of the baseline scenario. The scenarios correspond to the baseline where the debt-to-GDP ratio and income taxes are set at their current level (red), the second scenario where the government can choose the path of debt (blue), and the third scenario where it can also choose income taxes (yellow). Panel (d) plots the time path of each of these components (in absolute values) for the baseline scenario.

policy changes, as they depend on the paths of consumption (λ_t^A) and production (λ_t^B). When the government increases debt to finance transfers, the marginal utility of consumption initially declines. Over time, as the economy becomes poorer due to debt-induced crowding out of capital and wealth effects from transfers, the marginal utility of consumption rises relative to the baseline scenario. This intertemporal shift reduces the discounting of future damages, implying a higher carbon tax. On the other hand, the contraction of the economy lowers the absolute level of climate damages, which tends to reduce the carbon tax. Although these two effects do not fully offset each other, the negative co-movement of λ_t^A and λ_t^B keeps the damages path relatively stable across scenarios. After adjusting by the denominator λ_t^A, higher debt and income taxes result in a slightly higher SCC in early periods and a lower one later (see Figure G.1 in the appendix), but the overall quantitative differences remain small.

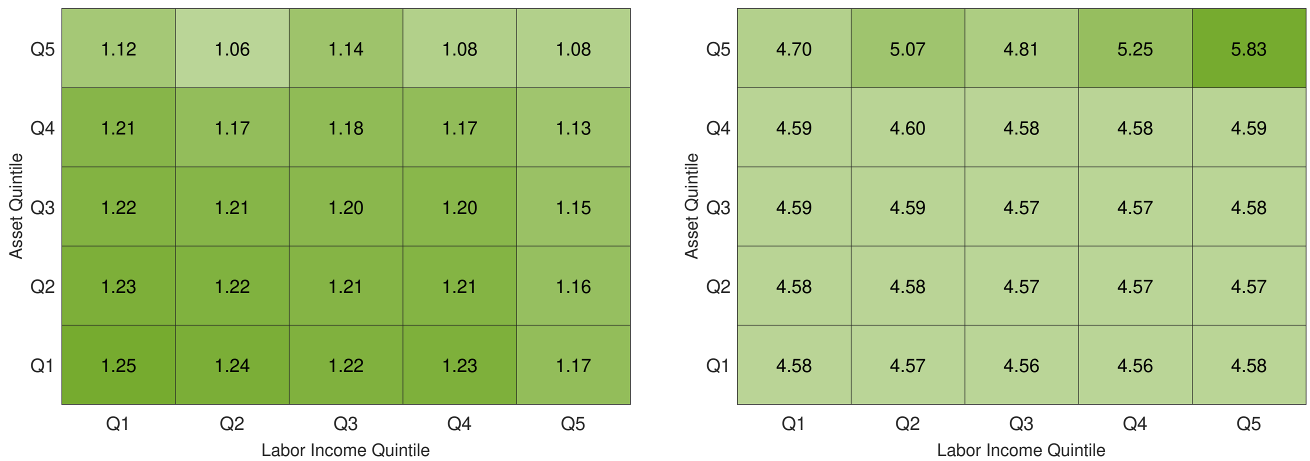
5.3 Distributional consequences

The results presented above indicate that redistribution and insurance considerations play only a minor role in shaping optimal climate policy. However, this does not necessarily imply that climate policy is distributionally neutral. To illustrate this point, we calculate the distribution of welfare gains associated with changes in the carbon tax.

Figure 5 shows how these gains are distributed between households according to their position in the distribution of labor income and assets. Specifically, Figure 5b compares the gains from the optimal policy in our baseline scenario to those under a counterfactual policy where all taxes are set at the same level, except for the carbon tax, whose path is chosen to replicate the temperature trajectory in the SSP5 scenario. Figure 5a conducts a similar exercise with the SSP2 scenario as its counterfactual, i.e., a climate trajectory more ambitious than SSP5 that can be described as "Middle of the Road"; see IPCC (2021) and Figure G.3a in the appendix for the corresponding temperature path.

The average welfare gains from adopting the optimal policy, relative to SSP2 and SSP5, are 1.2% and 4.7%, respectively. Figure 5a shows that with a moderate increase in carbon taxes, welfare gains are distributed relatively evenly across households. In this case, lump-sum redistribution of carbon tax revenues makes the fiscal system slightly more progressive. Because emissions are proportional to consumption in our benchmark model, poorer households receive transfers that exceed their carbon tax payments. Still, the effect is limited: households in the bottom income and wealth quintiles gain 1.3%, only 15.7% more than those in the top quintiles.

In contrast, when the optimal policy is compared to SSP5, the distributional pattern reverses. Richer households benefit more from higher carbon taxes, with welfare gains of 5.8%, which are 27.3% higher than those of the poorest households. As shown in Douenne et al. (2023), this reflects their higher willingness to pay for climate amenities. Because climate damages are convex, this effect becomes increasingly important at higher temperatures and eventually outweighs the modest redis-



(a) Optimal vs. SSP2

(b) Optimal vs. SSP5

Figure 5: Distribution of Welfare Gains by Assets and Labor Income

Note: The figure plots the welfare gains expressed as the percentage increase in consumption required under policy SSP2 (left) and SSP5 (right) to make different groups of households indifferent between this policy and the optimal one. The groups correspond to quintiles of labor income further divided into assets quintiles. The optimal policy is the one of the baseline with fixed debt-to-GDP and income taxes. In the SSP scenarios, all instruments remain at their baseline levels except for the carbon tax and lump-sum transfers.

tributive impact of tax changes.

Figure G.3 in the appendix illustrates the evolution of welfare gains over time. Although most gains from the optimal policy materialize in the next century, the optimal policy generates immediate benefits compared to both SSP2 and SSP5 scenarios. In each period, these gains are relatively evenly distributed across households. To the extent that differences across households exist, they arise primarily because higher carbon taxes result in lower temperatures, which benefit relatively more the rich, and increased transfers, which benefit poorer households more.

5.4 Extension: non-homothetic preferences

A well-documented empirical regularity in the literature using micro-data is the relationship between income and carbon emissions: although poorer households emit less carbon overall, their consumption is typically more carbon-intensive (see e.g., Sager, 2019). This pattern arises because certain carbon-intensive goods, such as energy, exhibit characteristics of necessity goods, resulting in budget shares that decline as income increases. To capture this feature, we extend our model so that agents consume not only the final good but also a second, dirtier consumption good produced using only energy. To replicate the observed distribution of energy consumption, we introduce non-homothetic preferences using a Stone-Geary utility function (as in Fried et al., 2018; Douenne et al., 2023; Fried et al., 2024; Wöhrmüller, 2024; Belfiori et al., 2024). We then recalibrate the model, additionally targeting the distribution of energy budget shares (see Appendix F.2 for details).

Figure 6 plots the optimal carbon tax path and the Pigouvian tax under two scenarios. The first corresponds to our baseline scenario, in which both debt and income taxes are fixed. The second maintains fixed debt but allows the government to set the level of income taxes, as well as taxes on energy production, τ_I, and taxes directly on household energy-good consumption, τ_D, two policy instruments introduced to capture potential sectoral wedges (see Douenne et al., 2023).

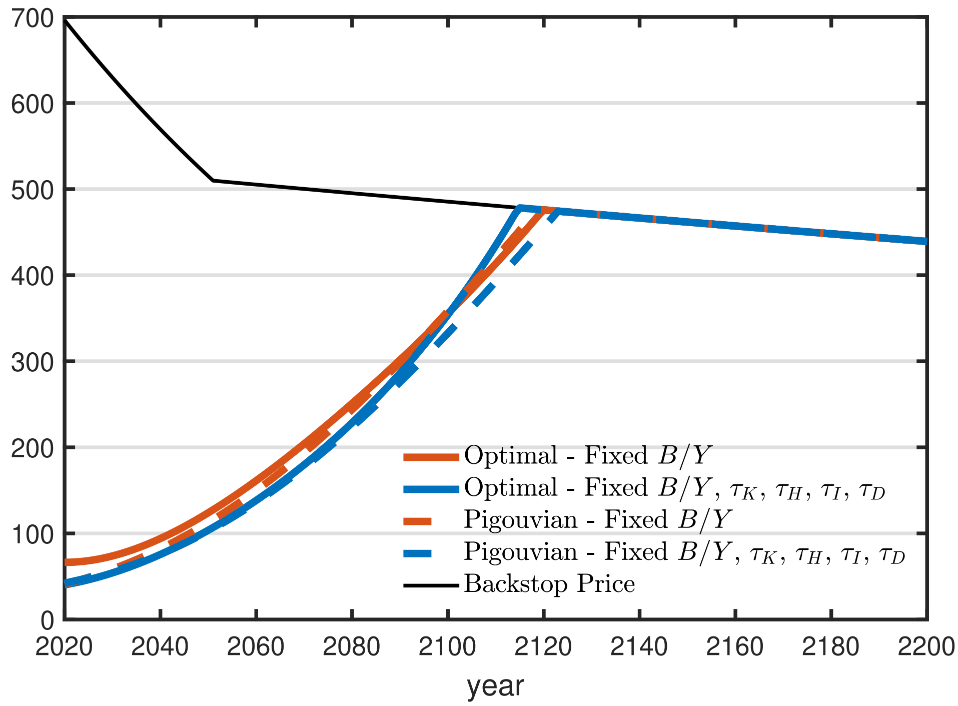


Figure 6: Optimal Carbon Taxes and Backstop Price with Stone-Geary utility (in $/t$CO_2$).

Note: This figure plots the paths of optimal carbon taxes in the extended economy with Stone-Geary preferences, in a scenario where the debt-to-GDP ratio and income and energy taxes are set at their current level (red), and in a scenario where debt remains fixed but the government can choose the level of income and energy taxes (blue). The dashed lines represent the corresponding SCC, and the black line plots the backstop price. All values are expressed in $/t$CO_2$.

Despite the explicit link that this model introduces between household carbon emissions and their economic resources, our main result remains unchanged: in both scenarios, the optimal carbon tax remains very close to the SCC. As in our benchmark, the welfare cost of doing *simply Pigou* instead of implementing the optimal tax is extremely low, at 0.008% in the first experiment, and 0.001% in the second one, exactly analogous to our baseline findings.

Figure G.8 in the appendix reproduces the welfare results from Figure 5b using our extended model with non-homothetic preferences. The overall patterns remain similar. For moderate increases in the carbon tax (relative to SSP2), welfare gains are slightly progressive, whereas for larger increases (relative to SSP5) they become more regressive, with particularly high gains for the richest households. Overall, however, welfare improvements remain fairly evenly distributed across households. The main difference with the baseline experiment is that the magnitude of the gains is smaller under non-homothetic preferences. Because energy is more of a necessity in this setting, the transition is costlier, and welfare gains are correspondingly lower.

6 Conclusion

In this paper, we study optimal climate policy in the presence of inequality and uninsurable labor-income risk. We extend a standard integrated assessment model to an Aiyagari (1994) economy and solve the associated Ramsey problem, i.e., we characterize the path of fiscal policies that maximize social welfare over the transition.

We begin with a theoretical analysis. We derive an analytical expression for the optimal carbon tax in an incomplete-markets setting where the government lacks access to state-contingent assets and individualized taxes and transfers. We show that the optimal carbon tax follows a modified Pigouvian rule, adjusted to capture the interaction between carbon pricing and pre-existing distortions in labor supply and saving decisions. We show that these terms arise due to income effects on labor supply and the savings behavior of unconstrained households. To explore the sign and magnitude of these deviations, we then turn to a quantitative analysis.

We calibrate the model to the U.S. economy, targeting key macroeconomic aggregates, inequality statistics, and measures of idiosyncratic income risk. Using numerical techniques developed by Dyrda and Pedroni (2023), we solve for the optimal policy path. We find that, even in the presence of significant inequality, risk, and fiscal constraints, the optimal carbon tax remains very close to the SCC: optimal deviations from the Pigouvian rule are very small and result in negligible welfare gains. To better understand this result, we decompose the welfare gains associated with higher carbon taxation. Our decomposition reveals that deviations from the SCC entail large aggregate welfare losses, but yield negligible gains in terms of redistribution and insurance. Thus, even if the government is highly concerned by inequality and risk, it never finds it optimal to use the carbon tax to deal with these issues.

Overall, our findings suggest that concerns about inequality, risk, and fiscal constraints should not be used as arguments against ambitious carbon pricing. Future work could extend this framework to incorporate heterogeneous exposure to climate damages, differences in ability to adapt, or interactions with aggregate climate risk.

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Appendices

A Ramsey Problem

This Appendix provides additional details regarding our main theoretical results. We start by characterizing the solution to the household problem. The household maximizes lifetime utility (1) subject to the sequence of budget constraints (2) and the borrowing constraint. The first-order necessary conditions are:

$$-u_c(c_{it}(e_i^t), h_{it}(e_i^t)) + \xi_{it}(e_i^t) + \beta R_{t+1} \sum_{e_i^{t+1}} \pi_{it+1}(e_i^{t+1}|e_i^t) u_c(c_{it+1}(e_i^{t+1}), h_{it+1}(e_i^{t+1})) = 0, \tag{21}$$

$$\xi_{it}(e_i^t)(a_{it+1}(e_i^t) - \underline{a}_{t+1}) = 0, \quad \xi_{it}(e_i^t) \geq 0, \quad a_{it+1}(e_i^t) \geq \underline{a}_{t+1}, \tag{22}$$

$$u_c(c_{it}(e_i^t), h_{it}(e_i^t)) w_t (1 - \tau_{H,t}) e_{it} + u_h(c_{it}(e_i^t), h_{it}(e_i^t)) = 0, \tag{23}$$

$$(1 + n_{t+1}) a_{it+1}(e_i^t) = R_t a_{it}(e_i^{t-1}) + T_t + w_t(1 - \tau_{H,t}) e_{it} h_{it}(e_i^t) - c_{it}(e_i^t), \tag{24}$$

where $\pi_{it+1}(e_i^{t+1}|e_i^t)$ denotes the probability that a households draws the sequence e_i^{t+1} conditional on having drawn e_i^t, and $\xi_{it}(e_i^t) \geq 0$ is the multiplier on the borrowing constraint $a_{it+1}(e_i^t) \geq \underline{a}_{t+1}$. These are the (modified) Euler equation, the borrowing constraint, the labor supply equation, and the household's per-period budget constraint, written in terms of per-capita variables. Assuming the second-order conditions are satisfied, for given prices and given policy, these equations pin down the paths for household assets, consumption, labor supply, and the multiplier ξ for every household type i and every possible realization of shocks e_i^t.

Turning to the profit maximization problem, the first-order conditions in the final goods sector are:

$$r_t = \big(1 - D(Z_t)\big) A_{1,t} F_K(K_{1,t}, H_{1,t}, E_t), \tag{25}$$

$$w_t = \big(1 - D(Z_t)\big) A_{1,t} F_H(K_{1,t}, H_{1,t}, E_t), \tag{26}$$

$$p_{E,t} = \big(1 - D(Z_t)\big) A_{1,t} F_E(K_{1,t}, H_{1,t}, E_t). \tag{27}$$

The first-order conditions in the energy sector are:

$$r_t = \big(p_{E,t} - \tau_{I,t} - \tau_{E,t}(1 - \mu_t) - \Theta_{E,t}\big) A_{2,t} G_K(K_{2,t}, H_{2,t}), \tag{28}$$

$$w_t = \big(p_{E,t} - \tau_{I,t} - \tau_{E,t}(1 - \mu_t) - \Theta_{E,t}\big) A_{2,t} G_H(K_{2,t}, H_{2,t}), \tag{29}$$

$$\tau_{E,t} = \theta_t'(\mu_t). \tag{30}$$

Combined with the market clearing conditions, the first-order conditions of households and firms characterize a competitive equilibrium for a given policy.

The government's problem is to choose the policy that decentralizes as a competitive equilibrium the one that delivers the highest level of social welfare

$$\mathcal{W} = \sum_i \alpha_i \sum_t \beta^t N_t \sum_{e_i^t} \pi_{it}(e_i^t)\big(u(c_{it}(e_i^t), h_{it}(e_i^t)) - v(Z_t)\big). \tag{31}$$

As stated, in addition to the households' first-order conditions (21)–(24), the planner faces as constraints the firms' first-order conditions and market clearing conditions.

To reduce the dimensionality of the optimal tax problem, we first use the first-order conditions of firms in the final good sector and the relationship $R_t = 1 + (1-\tau_{K,t})(r_t - \delta)$ to substitute out for prices in the household's first-order conditions (21)–(24). Next, we use the modified Euler equation (21) to substitute out for the multiplier $\xi_{it}(e_i^t)$ in equation (22). Then, to reduce the number of instances where the tax rates show up, it is convenient to employ Walras' law and replace the government's budget constraint by the following aggregate resource constraint (cf. Barrage, 2020):

$$\big(1 - D(Z_t)\big)A_{1,t}F(K_{1,t}, H_{1,t}, E_t) - \theta_t(\mu_t)E_t = C_t + G_t + K_{t+1} - (1-\delta)K_t. \tag{32}$$

In words, output net of abatement costs is used either for consumption by households, the government, or by firms (in the form of investment). Lastly, to further reduce instances where the tax rates show up, we employ the property that, since tax rates cannot be conditioned on sector, marginal rates of technical substitution between capital and labor are equalized across sectors:

$$F_{K,t}G_{H,t} = G_{K,t}F_{H,t}. \tag{33}$$

Combining the above, we can formulate the optimal tax problem as follows:

$$\max_{\substack{\{C_t, H_t, H_{1,t}, H_{2,t}, K_t, K_{1,t}, K_{2,t},\\ E_t, \mu_t, Z_t, \tau_{H,t}, \tau_{K,t}, \tau_{I,t}, \tau_{E,t}, T_t, B_t\}_t,\\ \{c_{it}(e_i^t), h_{it}(e_i^t), a_{it+1}(e_i^t)\}_{i,t,e_i^t}}} \mathcal{W} = \sum_i \alpha_i \sum_t \beta^t N_t \sum_{e_i^t} \pi_{it}(e_i^t)\big(u(c_{it}(e_i^t), h_{it}(e_i^t)) - v(Z_t)\big),$$

$$\begin{aligned}
\text{s.t. } \forall i, t, e_i^t:\ & \Big[u_c(c_{it}(e_i^t), h_{it}(e_i^t)) - \beta\big(1 + (1-\tau_{K,t+1})\big(\big(1 - D(Z_{t+1})\big)A_{1,t+1}F_{K,t+1} - \delta\big)\big) \\
& \times \sum_{e_i^{t+1}} \pi_{it+1}(e_i^{t+1}|e_i^t) u_c(c_{it+1}(e_i^{t+1}), h_{it+1}(e_i^{t+1}))\Big](a_{it+1}(e_i^t) - \underline{a}_{t+1}) = 0, \\
\forall i, t, e_i^t:\ & u_c(c_{it}(e_i^t), h_{it}(e_i^t)) \geq \beta\big(1 + (1-\tau_{K,t+1})\big(\big(1 - D(Z_{t+1})\big)A_{1,t+1}F_{K,t+1} - \delta\big)\big) \\
& \times \sum_{e_i^{t+1}} \pi_{it+1}(e_i^{t+1}|e_i^t) u_c(c_{it+1}(e_i^{t+1}), h_{it+1}(e_i^{t+1})),
\end{aligned}$$

$$
\begin{aligned}
&\forall i,t,e_i^t:\ a_{it+1}(e_i^t) \geq \underline{a}_{t+1},\\
&\forall i,t,e_i^t:\ u_c(c_{it}(e_i^t),h_{it}(e_i^t))\big(1-D(Z_t)\big)A_{1,t}F_{H,t}(1-\tau_{H,t})e_{it} + u_h(c_{it}(e_i^t),h_{it}(e_i^t)) = 0,\\
&\forall i,t,e_i^t:\ (1+n_{t+1})a_{it+1}(e_i^t) = (1+(1-\tau_{K,t})(\big(1-D(Z_t)\big)A_{1,t}F_{K,t}-\delta))a_{it}(e_i^{t-1}) + T_t\\
&\qquad\qquad + \big(1-D(Z_t)\big)A_{1,t}F_{H,t}(1-\tau_{H,t})e_{it}h_{it}(e_i^t) - c_{it}(e_i^t),\\
&\qquad \forall t:\ C_t = N_t\sum_i \alpha_i \sum_{e_i^t}\pi_{it}(e_i^t)c_{it}(e_i^t),\\
&\qquad \forall t:\ H_t = N_t\sum_i \alpha_i \sum_{e_i^t}\pi_{it}(e_i^t)e_{it}h_{it}(e_i^t),\\
&\qquad \forall t:\ H_t = H_{1,t} + H_{2,t},\\
&\qquad \forall t:\ K_t + B_t = N_t\sum_i \alpha_i \sum_{e_i^{t-1}}\pi_{it-1}(e_i^{t-1})a_{it}(e_i^{t-1}),\\
&\qquad \forall t:\ K_t = K_{1,t} + K_{2,t},\\
&\qquad \forall t:\ E_t = A_{2,t}G\big(K_{2,t},H_{2,t}\big),\\
&\qquad \forall t:\ \big(1-D(Z_t)\big)A_{1,t}F(K_{1,t},H_{1,t},E_t) - \theta_t(\mu_t)E_t = C_t + G_t + K_{t+1} - (1-\delta)K_t,\\
&\qquad \forall t:\ Z_t = J\left(S_0,(1-\mu_0)E_0,...,(1-\mu_t)E_t,\eta_0,...,\eta_t\right),\\
&\qquad \forall t:\ F_{K,t}G_{H,t} = G_{K,t}F_{H,t},\\
&\qquad \forall t:\ \tau_{E,t} = \theta_t'(\mu_t),\\
&\qquad \forall t:\ (1-D(Z_t))A_{1,t}F_{H,t} = ((1-D(Z_t))A_{1,t}F_{E,t} - \tau_{I,t} - (1-\mu_t)\tau_{E,t} - \theta_t(\mu_t))A_{2,t}G_{H,t}.
\end{aligned}
$$

Here, we suppress function arguments of the derivatives of the production function in both sectors to save on notation. The choice variables are the policy instruments, i.e., taxes, transfers and government debt, and the (aggregate and individual- and history-specific) allocation variables (note that the prices w_t, r_t, R_t, and $p_{E,t}$ have been substituted out for). Turning to the constraints, the first five restate the households' first-order conditions, the next eight are market clearing and resource constraints, and the final three follow from the first-order conditions of firms in both sectors. An allocation that satisfies the above constraints is part of a competitive equilibrium given policy (where prices follow from the first-order conditions of firms). Conversely, if an allocation satisfies the above constraints, it can be decentralized as part of a competitive equilibrium given the instrument set under consideration. In what follows, we denote by $\alpha_i\beta^t N_t\pi_{it}(e_i^t)\kappa_{it}^1(e_i^t),..,\alpha_i\beta^t N_t\pi_{it}(e_i^t)\kappa_{it}^5(e_i^t)$ the multipliers on the constraints that vary with household types i and the history of shocks e_i^t and by $\beta^t\lambda_{1t},...,\beta^t\lambda_{11t}$ the multipliers on the constraints that vary over time but not across types or histories.

To derive a formula for the optimal carbon tax, as in Barrage (2020) and Douenne et al. (2023), the first step is to combine the first-order conditions for temperature Z_t and abatement μ_t. The one for abatement specifies a relationship between the multipliers λ_7, λ_8 (for different periods), λ_{10}, and λ_{11}. This relationship simplifies considerably using the first-order condition for the carbon tax $\tau_{E,t}$, which implies $\lambda_{10t} = 0$, and the intermediate-goods tax $\tau_{I,t}$, which, given that $\lambda_{10t} = 0$, implies $\lambda_{11t} = 0$. After canceling terms, the remaining multipliers are linked through

$$-\lambda_{7t}\theta_t' + \sum_{j=0}^{\infty} \beta^j \lambda_{8t+j} J_{E_t^M,t+j} = 0. \tag{34}$$

Using the property $\theta_t'(\mu_t) = \tau_{E,t}$, we obtain the following expression for the carbon tax:

$$\tau_{E,t} = \frac{1}{\lambda_{7t}} \sum_{j=0}^{\infty} \beta^j \lambda_{8t+j} J_{E_t^M,t+j}. \tag{35}$$

To derive an expression for both λ_{7t} and λ_{8t} (in different periods), consider the first-order condition for temperature Z_t. In the optimal tax problem, temperature shows up in the objective, the households' first-order conditions, the aggregate resource constraint, the climate module, and the final first-order condition of firms. To simplify, we use the property that, if $\tau_{I,t}$ and $\tau_{E,t}$ are optimized, the multiplier $\lambda_{11t} = 0$. Under the assumption that utility damages from climate change enter the preference specification separately, temperature Z_t affects the households' first-order conditions *only* through its effect on wages w_t and interest rates R_t (since these depend on production damages $D(Z_t)$). In the first-order condition with respect to temperature, the effects of Z_t on wages w_t and interest rates R_t cancel out by virtue of the first-order conditions for taxes on labor and capital income, $\tau_{H,t}$ and $\tau_{K,t}$ (this is an application of the Envelope theorem).[18] Combining the above, the first-order condition for temperature Z_t is,

$$N_t \mathcal{W}_{Z,t} - \lambda_{7t} D_t' A_{1,t} F_t + \lambda_{8t} = 0, \tag{36}$$

where $F_t = F(K_{1,t}, H_{1,t}, E_t)$ and $\mathcal{W}_{Z,t} = -v'(Z_t)$ denotes the marginal utility of a higher temperature. Substituting out for λ_{8t+j} in equation (35) gives

$$\tau_{E,t} = \frac{1}{\lambda_{7t}} \sum_{j=0}^{\infty} \beta^j (\lambda_{7t+j} D_{t+j}' A_{1,t+j} F_{t+j} - N_{t+j} \mathcal{W}_{Z,t+j}) J_{E_t^M,t+j}, \tag{37}$$

which coincides with equation (11) from Proposition 1, where we use the notation ν_t instead of λ_{7t} to refer to the multiplier on the aggregate resource constraint (32). At the optimum, the carbon tax equals

[18]Note that, in the current problem formulation, $\tau_{H,t}$ and $\tau_{K,t}$ show up *only* in the household's first-order conditions and only by affecting after-tax wages and interest rates. Hence, by choosing $\tau_{H,t}$ and $\tau_{K,t}$, it is *as if* the planner can directly optimize over after-tax wages and interest rates. Consequently, the effect of temperature on after-tax wages and interest rates (through production damages) is not welfare-relevant if capital and labor taxes are optimized.

the present value of production damages (first term) and utility damages (second term). Importantly, to compute the present value, the planner uses the multiplier λ_{7t} on the market clearing condition for final goods, which captures the scarcity of aggregate resources.

To obtain an expression for λ_{7t}, note that the first-order condition with respect to aggregate consumption C_t implies $\lambda_{7t} = \lambda_{1t}$. An expression for λ_{1t}, in turn, can be obtained using the first-order condition with respect to consumption of household i after history of shocks e_i^t:

$$
\begin{aligned}
&\alpha_i \beta^t N_t \pi_{it}(e_i^t)(u_c(c_{it}(e_i^t), h_{it}(e_i^t)) - \lambda_{1t}) + \alpha_i \beta^t N_t \pi_{it}(e_i^t) \kappa_{it}^1(e_i^t) u_{cc}(c_{it}(e_i^t), h_{it}(e_i^t))(a_{it+1}(e_i^t) - \underline{a}_{t+1}) \\
&+ \alpha_i \beta^t N_t \pi_{it}(e_i^t) \kappa_{it}^2(e_i^t) u_{cc}(c_{it}(e_i^t), h_{it}(e_i^t)) + \alpha_i \beta^t N_t \pi_{it}(e_i^t) \kappa_{it}^4(e_i^t)(u_{cc}(c_{it}(e_i^t), h_{it}(e_i^t)) \\
&\times (1 - D(Z_t)) A_{1,t} F_{H,t}(1 - \tau_{H,t}) e_{it} + u_{ch}(c_{it}(e_i^t), h_{it}(e_i^t))) - \alpha_i \beta^t N_t \pi_{it}(e_i^t) \kappa_{it}^5(e_i^t) \\
&- \alpha_i \beta^{t-1} N_{t-1} \pi_{it-1}(e_i^{t-1}) \kappa_{it-1}^1(e_i^{t-1}) \beta R_t \pi_{it}(e_i^t | e_i^{t-1}) u_{cc}(c_{it}(e_i^t), h_{it}(e_i^t))(a_{it}(e_i^{t-1}) - \underline{a}_t) \\
&- \alpha_i \beta^{t-1} N_{t-1} \pi_{it-1}(e_i^{t-1}) \kappa_{it-1}^2(e_i^{t-1}) \beta R_t \pi_{it}(e_i^t | e_i^{t-1}) u_{cc}(c_{it}(e_i^t), h_{it}(e_i^t)) = 0, \qquad (38)
\end{aligned}
$$

where we used the expression for the gross, after-tax interest rate R_t. This equation can be simplified in a number of steps. First, divide by $\beta^t N_t$, use the property $\pi_{it}(e_i^t) = \pi_{it-1}(e_i^{t-1}) \pi_{it}(e_i^t | e_i^{t-1})$ and the first-order condition (26) to substitute out for the wage w_t. Next, define the composite multiplier

$$
\kappa_{it}^s(e_i^t) = \kappa_{it}^1(e_i^t)(a_{it+1}(e_i^t) - \underline{a}_{t+1}) + \kappa_{it}^2(e_i^t), \qquad (39)
$$

which captures the change in welfare due to a tightening or relaxation of the household's Euler equation $u_{c,it} \geq \beta R_{t+1} \mathbb{E}_t u_{c,it+1}$. This composite multiplier captures the welfare effect that operates through changes in household's savings behavior (hence, the superscript s). Similarly, denote by $\kappa_{it}^\ell(e_i^t) = \kappa_{it}^4(e_i^t)$ the multiplier on the household's first-order condition for labor supply (hence, the superscript ℓ). Combining the above, equation (38) can be written as

$$
\begin{aligned}
&\alpha_i \pi_{it}(e_i^t)(u_c(c_{it}(e_i^t), h_{it}(e_i^t)) - \lambda_{1t}) + \alpha_i \pi_{it}(e_i^t) u_{cc}(c_{it}(e_i^t), h_{it}(e_i^t)) \left(\kappa_{it}^s(e_i^t) - \frac{R_t}{1 + n_t} \kappa_{it-1}^s(e_i^{t-1}) \right) \\
&+ \alpha_i \pi_{it}(e_i^t) \kappa_{it}^\ell(e_i^t)(u_{cc}(c_{it}(e_i^t), h_{it}(e_i^t)) w_t (1 - \tau_{H,t}) e_{it} + u_{ch}(c_{it}(e_i^t), h_{it}(e_i^t))) - \alpha_i \pi_{it}(e_i^t) \kappa_{it}^5(e_i^t) = 0. \qquad (40)
\end{aligned}
$$

The above must hold for each household type i and each history e_i^t. Summing across types and histories of shocks gives, after rearranging:

$$
\begin{aligned}
&\sum_i \alpha_i \sum_{e_i^t} \pi_{it}(e_i^t) \Big[u_c(c_{it}(e_i^t), h_{it}(e_i^t)) + u_{cc}(c_{it}(e_i^t), h_{it}(e_i^t))(\kappa_{it}^s(e_i^t) - R_t \kappa_{it-1}^s(e_i^{t-1})/(1 + n_t)) \qquad (41) \\
&+ \kappa_{it}^\ell(e_i^t)(u_{cc}(c_{it}(e_i^t), h_{it}(e_i^t)) w_t (1 - \tau_{H,t}) e_{it} + u_{ch}(c_{it}(e_i^t), h_{it}(e_i^t))) \Big] = \lambda_{1t} + \sum_i \alpha_i \sum_{e_i^t} \pi_{it}(e_i^t) \kappa_{it}^5(e_i^t).
\end{aligned}
$$

The second term on the right-hand side cancels by virtue of the first-order condition for the lump-sum transfer T_t:

$$\sum_i \alpha_i \beta^t N_t \sum_{e_i^t} \pi_{it}(e_i^t) \kappa_{it}^5(e_i^t) = 0. \tag{42}$$

Combining the above, we obtain the following expression for λ_{1t}:

$$\lambda_{1t} = \sum_i \alpha_i \sum_{e_i^t} \pi_{it}(e_i^t) \psi_{it}(e_i^t), \tag{43}$$

where the term

$$\begin{aligned} \psi_{it}(e_i^t) &= u_c(c_{it}(e_i^t), h_{it}(e_i^t)) + u_{cc}(c_{it}(e_i^t), h_{it}(e_i^t))(\kappa_{it}^s(e_i^t) - R_t \kappa_{it-1}^s(e_i^{t-1})/(1+n_t)) \\ &\quad + \kappa_{it}^\ell(e_i^t)(u_{cc}(c_{it}(e_i^t), h_{it}(e_i^t)) w_t (1-\tau_{H,t}) e_{it} + u_{ch}(c_{it}(e_i^t), h_{it}(e_i^t))) \end{aligned} \tag{44}$$

is closely related to what Le Grand and Ragot (2023) refer to as the marginal social valuation of liquidity for an agent i after history of shocks e_i^t. It measures the change in social welfare associated with increasing the consumption of household i after history of shocks e_i^t. It consists of the marginal utility of consumption (first term) and the welfare effect associated with a tightening or relaxation of the first-order condition for savings (second term) and labor supply (third term). Note that, because $c_{it}(e_i^t)$ shows up in the Euler equation for two periods, the multiplier κ^s shows up twice.

Using equation (37), the property that $\lambda_{7t} = \lambda_{1t}$ and the expression for λ_{1t}, we arrive at the following expression for the optimal carbon tax:

$$\tau_{E,t} = \sum_{j=0}^{\infty} \beta^j \left[\frac{\lambda_{1t+j}}{\lambda_{1t}} D'_{t+j} A_{1,t+j} F_{t+j} - \frac{N_{t+j} \mathcal{W}_{Z,t+j}}{\lambda_{1t}} \right] J_{E_t^M, t+j}, \tag{45}$$

where the multiplier λ_{1t} satisfies

$$\lambda_{1t} = \mathcal{W}_{c,t} + \sum_i \alpha_i \sum_{e_i^t} \pi_{it}(SD_{it} + LD_{it}) \tag{46}$$

with $\mathcal{W}_{c,t} = \sum_i \alpha_i \sum_{e_i^t} \pi_{it} u_{c,it}$ denoting the marginal utility of consumption averaged across household types and histories, and, ignoring function arguments to save on notation,

$$SD_{it} \equiv u_{cc,it}(\kappa_{it}^s - R_t \kappa_{it-1}^s/(1+n_t)), \tag{47}$$

$$LD_{it} \equiv \kappa_{it}^\ell (u_{cc,it} w_t (1-\tau_{H,t}) e_{it} + u_{ch,it}), \tag{48}$$

are terms related to savings and labor supply distortions, which capture the welfare effect associated with a tightening or relaxation of the households' first-order condition for savings and labor supply due to an increase in $c_{it}(e_i^t)$.

It is worth highlighting that (cf. Corollary 1), if a household of type i is borrowing constrained both after history e_i^{t-1} and after history e_i^t (i.e., for two consecutive periods), changes in $c_{it}(e_i^t)$ do not interact with savings decisions: $SD_{it} = 0$. To see this, consider equation (39) for the composite multiplier κ^s. If the borrowing constraint binds after a particular history (so that assets equal their lower bound), the first term is equal to zero. The second term is zero as well because, with a binding borrowing constraint, the Euler equation holds with a strict inequality, i.e., $u_{c,it} > \beta R_{t+1} E_t u_{c,it+1}$. Consequently, the composite multiplier equals zero. If that is true after both history e_i^{t-1} and history e_i^t, it follows that $SD_{it} = 0$. Intuitively, if households are hand-to-mouth for two consecutive periods, changes in consumption do not affect households' savings decisions in either that period or the one before.

Turning to the labor distortion, the latter is equal to zero if there are no income effects on labor supply, that is, if households have GHH preferences (cf. Corollary 2). To see this, suppose the per-period utility function over consumption and labor supply (recall: climate damages enter the preference specification separately) is given by

$$U(c - \phi(h)). \tag{49}$$

Substituting in equation (48) gives

$$\begin{aligned} LD_{it} &= \kappa_{it}^{\ell} \left[U''(c_{it} - \phi(h_{it})) w_t (1 - \tau_{H,t}) e_{it} - U''(c_{it} - \phi(h_{it})) \phi'(h_{it}) \right] \\ &= \kappa_{it}^{\ell} U''(c_{it} - \phi(h_{it})) (w_t (1 - \tau_{H,t}) e_{it} - \phi'(h_{it})). \end{aligned} \tag{50}$$

The first-order condition for labor supply, in turn, is $w_t(1 - \tau_{H,t}) e_{it} = \phi'(h_{it})$. Hence, if households have GHH preferences, it follows that $LD_{it} = 0$. Intuitively, without income effects on labor supply, changes in consumption $c_{it}(e_i^t)$ do not affect incentives to supply labor and hence, do not tighten or relax the households' first-order condition for labor supply.

B Calibration Details

Table III presents the model parameters calibrated to match the moments described in Section 4 (summarized in Tables I and II). Table IV reports the parameters set exogenously. More precisely, the initial levels of output Y_{2020}, energy E_{2020}, and abatement μ_{2020} are not directly set exogenously, but are obtained by appropriately adjusting the productivities $A_{1,2020}$ and $A_{2,2020}$ and the initial carbon tax $\tau_{E,2020}$, respectively.

Table III: Calibrated Model Parameters

Description	Parameter	Value
Preferences and technology		
Consumption share	γ	0.74
Preference curvature	σ	1.69
Discount factor	β	0.995
Weight on leisure	ς	1.979
Weight on damages in utility	α_Z	2.16×10^{-4}
Borrowing constraint	$\underline{a}$	-0.080
Ratio of TFPs	A_2/A_1	6.831
Fiscal policy		
Government expenditure	G	0.069
Transfers	T	0.088
Labor productivity process		
Productivity process curvature	η	1.12

Persistent shock

$$\Gamma_P = \begin{bmatrix} 0.994 & 0.002 & 0.004 & 3\text{E}{-5} \\ 0.019 & 0.979 & 0.001 & 9\text{E}{-5} \\ 0.023 & 0.000 & 0.977 & 5\text{E}{-5} \\ 0.000 & 0.000 & 0.012 & 0.987 \end{bmatrix} \quad e_P = \begin{bmatrix} 0.185 \\ 0.305 \\ 0.537 \\ 27.223 \end{bmatrix}$$

Transitory shock

$$P_T = \begin{bmatrix} 0.357 \\ 0.002 \\ 0.467 \\ 0.004 \\ 0.025 \\ 0.176 \end{bmatrix} \quad e_T = \begin{bmatrix} 0.07 \\ 0.09 \\ 3.12 \\ 3.16 \\ 7.80 \\ 9.51 \end{bmatrix}$$

Table IV: Exogenously Imposed Parameters

Parameter	Description	Value	Source
Production first sector $[Y_{1,t} = (1 - D(Z_t))A_{1,t}K_{1,t}^{\alpha}H_{1,t}^{1-\alpha-\nu}E_t^{\nu}$, and $D(Z_t) = 1 - e^{-\frac{a_1}{2}Z_t^2}]$			
α	Return to scale on labor sector 1	0.3	DICE 2023
ν	Return to scale on energy sector 1	0.04	Golosov et al. (2014)
δ	Depreciation rate on capital (per year)	0.1	DICE 2023
a_1	Damage coefficient	0.0074	Dietz et al. (2019) and Barrage (2020)
Y_{2020}	Initial output (in trillions 2023 USD)	83.48	World Bank (2016-2020)
Production second sector $[E_t = A_{2,t}K_{2,t}^{\alpha_E}H_{2,t}^{1-\alpha_E}]$			
α_E	Return to scale on capital sector 2	0.597	Barrage (2020)
E_{2020}	Init. gross indus. emissions ($GtCO_2$ per year)	38.23	Friedlingstein et al. (2022)
Climate $[Z_{t+1} = Z_t + \epsilon(\zeta\mathcal{E}_t - Z_t)$, $\mathcal{E}_{t+1} = \mathcal{E}_t + E_t^M + E_t^{ex}$, and $E_{t+1}^{ex} = (1 + g_{E^{ex}})\, E_t^{ex}]$			
$\mathcal{E}_{2020}$	Initial cumulative carbon emissions (in GtCO2)	2390	IPCC (2021)
Z_{2020}	Initial atmos. temp. change ($^\circ C$ since 1900)	1.07	IPCC (2021)
ϵ	Initial pulse-adjustment timescale	0.5	Dietz et al. (2019)
ζ	Trans. clim. resp. to cum. emissions (TCRE)	0.00045	IPCC (2021)
E_{2020}^{ex}	Init. gross CO_2 emis. land ($GtCO_2$ per year)	4.17	Friedlingstein et al. (2022)
$g_{E^{ex}}$	Ex. growth rate of gross land emissions (per year)	−0.021	DICE 2023
Abatement costs $[P_t^{back} = P_{t-1}^{back}(1 - g^{P^{back}})$, and $\theta_t(\mu_t) = P_t^{back}\frac{\mu_t^{c_2}}{c_2}]$			
P_{2020}^{back}	Backstop price in 2020 (in $\$/tCO_2$)	696.2	DICE 2023
$g^{P^{back}}$	Decline rate backstop price (per year)	0.01 [2020–2050], 0.001 [2051–]	DICE 2023
c_2	Exponent abatement cost function	2.6	DICE 2023
μ_{2020}	Initial abatement share	0.051	DICE 2023
Exogenous growth parameters $[g_{A_i,t} = g_{A_i,2020}\, e^{-gg_{A_i}(t-2020)}$, $A_{i,t} = (1 + g_{A_i,t})\, A_{i,t-1}$, and $N_t = N_{t-1}(N_{\max}/N_{t-1})^{g_N}]$			
$g_{A_{1,2020}}, g_{A_{2,2020}}$	Initial TFP growth rate sectors 1 and 2 (per year)	0.0159	DICE 2023
$gg_{A_{1,t}}, gg_{A_{2,t}}$	Decline rate TFP growth sectors 1 and 2 (per year)	0.0072	DICE 2023
N_{2020}	Initial population (in billions)	1.368	World Bank US-adjusted
$N_{\max}$	Asymptotic population (in billions)	1.910	DICE 2023 US-adjusted
g_N	Rate of convergence of population	0.029	DICE 2023
Fiscal Policy			
τ_K	Capital income tax	0.336	Appendix C.4
τ_H	Labor income tax	0.277	Appendix C.4
τ_C	Consumption tax	0.042	Appendix C.4
$\tau_{I,2020}$	Energy tax	0.000	Appendix C.4
$\tau_{E,2020}$	Initial carbon tax	0.006	Appendix C.4

C Data

In what follows, we describe our procedure to obtain macroeconomic data and cross-sectional moments at the household level. We use the Current Population Survey (CPS) to construct the cross-sectional moments for hours, the Survey of Consumer Finances (SCF) to construct the cross-sectional moments for wealth, earnings, and income, and the Consumption Expenditure Survey (CEX) to construct the cross-sectional moments for consumption. Finally, we discuss the computation of our targets for the statistical properties of the labor income process based on the data provided by Pruitt and Turner (2020b) from the Internal Revenue Service.

C.1 National Income and Product Accounts (NIPA)

Following Aiyagari and McGrattan (1998), we define physical capital as the sum of nonresidential and residential private fixed assets and purchases of consumer durables. Therefore, our definition excludes government fixed assets. We compute the average capital-output ratio, following the outlined definition, for the period 2009-2019 using two tables provided by the U.S. Bureau of Economic Analysis: Table 1.1. Current-Cost Net Stock of Fixed Assets and Consumer Durable Goods for the capital series and Table 1.1.5. Gross Domestic Product for the GDP series. We obtain the ratio of 2.57.

We define the investment-output ratio in a way consistent with the capital-output ratio, that is, we compute investment as the sum of nonresidential and residential private fixed assets and purchases of consumer durables, and exclude the government's investment. We compute the average investment-output ratio, following the outlined definition, for the 2009-2019 period using two tables provided by the U.S. Bureau of Economic Analysis: Table 1.5. Investment in Fixed Assets and Consumer Durable Goods for the capital series and Table 1.1.5. Gross Domestic Product for the GDP series.

The third statistic we discipline using data from the NIPA tables is the transfers-to-output ratio. We define transfers in the data as personal current transfer receipts, which include social security transfers, Medicare, Medicaid, unemployment benefits, and veteran benefits. We choose this for two reasons: First, we include retired and unemployed households in our inequality moments. Second, lump-sum transfers in the model can be interpreted as a basic income in the case of not working. We compute the transfers to output ratio, following the outlined definition, for the 2009-2019 period using two tables provided by the U.S. Bureau of Economic Analysis: Table 2.1. Personal Income and Its Disposition for the construction of the transfers and Table 1.1.5. Gross Domestic Product for the

GDP series. We obtain the average ratio of 0.147.

C.2 Equivalence Scale

We construct cross-sectional statistics at the household level. To account for the distribution of household sizes, we use an equivalence scale. This way, we take into consideration the number of people living in the household and how these people share resources and take advantage of economies of scale. We use an equivalence adjustment based on a three-parameter scale that reflects and follows the procedure of the U.S. Census Bureau:[19]

1. On average, children consume less than adults.
2. As family size increases, expenses do not increase at the same rate.
3. The increase in expenses is larger for a first child of a single-parent family than for the first child of a two-adult family.

The three-parameter scale is calculated in the following way:

- One and two adults: scale = $(\text{number of adults})^{0.5}$
- Single parents: scale = $(\text{number of adults} + (0.8 \times \text{first child}) + 0.5 \times \text{other children})^{0.7}$
- All other families: scale = $(\text{number of adults} + 0.5 \times \text{number of children})^{0.7}$

We apply the same equivalence measures to the SCF and the CEX.

C.3 The Survey of Consumer Finances (SCF).

C.3.1 Partition of the Population

We partition the groups of households in the SCF into four categories: workers, business owners, retirees, and non-working households. The partition is mutually exclusive and exhaustive. The following table summarizes the shares for each of the household types in the 2019 SCF sample.

	Workers	Business Owners	Retirees	Non-working
2019 Share (%)	67.19	5.84	9.02	17.95

[19] See the link here: https://www.census.gov/topics/income-poverty/income-inequality/about/metrics/equivalence.html

Business Owners. Business owner households are defined as (1) one of the heads or the spouse of the household is an active business owner, and (2) total household labor income is less than both the total household business income and the total household capital income.

Retirees. A household is defined as a retiree household if (1) both the head and the spouse of the household declared a retirement year prior to the survey year, and (2) the household is not a business owner household.

Non-working. A household is non-working if (1) the household is not a business owner household, (2) the household is not a retiree household, and (3) the household earns no labor income.

Workers. All households that do not fall into the above three categories are classified as workers.

C.4 Time Series for Tax Rates

In this section, we provide a description of the procedure we use to obtain average, effective tax rates for the United States by updating and extending the approach by Trabandt and Uhlig (2011). There are four rates computed: the average effective personal income tax rate, the average effective consumption tax rate, the average effective capital tax rate, and the average effective labor income tax rate. There are three main sources of data: the OECD database, the AMECO database, and the BEA statistics.

Variable Names and Associated Dataset. There are a total of two tables (T11000 from section 1 and T60200 from section 6) used from BEA, two tables (*simplified non-financial accounts table* and *revenue statistics for tax revenue table*) from OECD, and two variables (*private final consumption expenditure* and *total final consumption expenditure of general government*) from AMECO. In particular, the T11000 table is downloaded from Section 1 and T60200 from Section 6. We extract *"Gross wages and salaries"* and *"Net Operating Surplus"*, corresponding to line 3 and line 9 from table T11000. We extract the variable for compensation of employees from *"Government"* that includes the federal and state amounts from table T60200. As a result of a modification in industry classification, the table layout changes over time, resulting in the existence of four Excel sheets, with no fixed line number assigned to this variable. For reference purposes, we will utilize line 76 for this variable, as it corresponds to the line number in the statistics for the period from 1948 to 1987.

For the OECD data, the catalogue webpage provides a search function, which allows us to locate the tables of interest. The simplified non-financial accounts table is downloaded for the USA, transaction sector Households and non-profit institutions serving households (*SS14_S15*), in the national currency unit. The variables (with the associated variable code) used are: Consumption of fixed capital (SK1R), Received property income (SD4R), Paid property income (SD4P), and Gross operating surplus and mixed income (SB2G_B3G).

Similarly, the revenue statistics for the tax revenue table are downloaded for the USA, sector Total, in the national currency unit. The variables (along with their associated variable codes) used are: Taxes on financial and capital transaction (4400), General taxes (5110), Excises (5121), Taxes on individual income, profits and capital gain (1100), Taxes on corporate income (1200), Social security contributions (2000), Taxes from Employers (2200), Taxes on payroll and workforce (3000), and Recurrent taxes on immovable property (4100).

The annual macro-economic database of the European Commission's Directorate General for Economic and Financial Affairs (AMECO) is accessed from Ameco Online. The variables are acquired via the search function in the Ameco Online platform by choosing the USA as the country and the national currency as the unit. We will refer to the variable for private final consumption expenditure as *PFCE* and total final consumption expenditure of general government as *GFCE*.

Every variable is encoded in national current currencies in millions of dollars. The following equations are used to calculate the effective tax rates, utilizing variable codes for ease of reference. *Lines* correspond to tables from BEA, *codes* refer to tables from OECD, and *variable names* pertain to variables from AMECO or those further calculated in the text. For each year, the effective tax rates are determined using the following equations. After obtaining the tax rates for each year, we calculate the average of the tax rates from 1995 to 2019.

Personal Income Tax Rate (PITR) The effective personal income tax rate is calculated by

$$\frac{1100}{\text{line 4} + (\text{OSPUE + PEI})},$$

with OSPUE + PEI calculated as

$$\text{OSPUE} + \text{PEI} = \text{SB2G\_B3G} + \text{SD4R} - \text{SD4P} - \mathbf{1} \times \text{SK1R}.$$

We follow the practice by Trabandt and Uhlig (2011) to set the indicator to 1, i.e., we subtract the consumption of fixed capital from the operating surplus and mixed income.

Consumption Tax Rate The effective consumption tax rate is calculated by

$$\frac{5110 + 5121}{\text{PFCE} + \text{GFCE} - \text{line } 76 - 5110 - 5121}.$$

Labor Income Tax Rate The effective labor income tax rate is calculated by

$$\frac{\text{PITR} + \text{line } 4 + 2000 + 3000}{\text{line } 4 + 2200}.$$

Capital Tax Rate The effective capital tax rate is calculated by

$$\frac{\text{PITR} \times (\text{OSPUE} + \text{PEI}) + 4400 + 4100 + 1200}{\text{line } 11}.$$

D Welfare Decomposition

This appendix provides details related to the welfare decomposition. Most of it closely follows the corresponding appendix in Dyrda and Pedroni (2023), with adjustments to account for the inclusion of climate. We include this material for completeness and the reader's convenience.

D.1 Definitions

Average welfare gain Consider a policy reform and denote by $\{c_t^j, h_t^j, Z_t^j\}$ the equilibrium consumption, labor, and temperature paths of a household with and without the reform, with $j = R$ or $j = NR$ respectively. The average welfare gain, Δ, from implementing the reform is defined as the constant (over time and across agents) percentage increase to c_t^{NR} that equalizes utilitarian welfare to the value under the reform:

$$\int \mathbb{E}_0 \left[U\left(\left\{ (1+\Delta)\, c_t^{NR}, n_t^{NR} \right\} \right) \right] d\lambda_0 - V\left(\left\{ Z^{NR} \right\} \right) = \int \mathbb{E}_0 \left[U\left(\left\{ c_t^{R}, n_t^{R} \right\} \right) \right] d\lambda_0 - V\left(\left\{ Z^{R} \right\} \right), \quad (51)$$

where λ_0 is the initial distribution over states (a_0, e_0),

$$U\left(\left\{ c_t^j, n_t^j \right\} \right) \equiv \sum_{t=0}^{\infty} \beta^t N_t u\left(c_t^j, n_t^j \right), \quad \text{and} \quad V\left(\left\{ Z^j \right\} \right) \equiv \sum_{t=0}^{\infty} \beta^t N_t v\left(Z_t^j \right), \quad \text{for } j = R, NR.$$

These welfare gains can be decomposed into four components:

1. Climate-utility effect Let $\widetilde{\Delta}$ be the welfare gain ignoring climate utility damages:

$$\int E_0 \left[U\left(\left\{ \left(1+\widetilde{\Delta}\right) c_t^{NR}, n_t^{NR} \right\} \right) \right] d\lambda_0 = \int E_0 \left[U\left(\left\{ c_t^{R}, n_t^{R} \right\} \right) \right] d\lambda_0, \quad (52)$$

and define the climate-utility component, Δ_C, such that:

$$1 + \Delta_C \equiv \frac{1+\Delta}{1+\widetilde{\Delta}}.$$

2. Level effect Let the aggregate consumption and labor at each t be:

$$C_t^j \equiv \int c_t^j d\lambda_t^j, \quad \text{and} \quad H_t^j \equiv \int h_t^j d\lambda_t^j,$$

where λ_t^j is the distribution over (a_0, e^t) conditional on whether or not the reform is implemented. Then, the level effect Δ_L satisfies:

$$\sum_{t=0}^{\infty} \beta^t u\left((1+\Delta_L) C_t^{NR}, H_t^{NR} \right) = \sum_{t=0}^{\infty} \beta^t u\left(C_t^{R}, H_t^{R} \right). \quad (53)$$

3. Insurance effect Let $\{\bar{c}_t^j(a_0, e_0), \bar{h}_t^j(a_0, e_0)\}$ denote a certainty-equivalent sequence of consumption and labor conditional on a household's initial state satisfying:

$$\sum_{t=0}^{\infty} \beta^t u\left(\bar{c}_t^j(a_0, e_0), \bar{h}_t^j(a_0, e_0)\right) = \mathbb{E}_0\left[\sum_{t=0}^{\infty} \beta^t u\left(c_t^j, h_t^j\right)\right]. \tag{54}$$

Next, let $\bar{C}_t^j$ and $\bar{H}_t^j$ denote aggregate certainty equivalents, that is

$$\bar{C}_t^j = \int \bar{c}_t^j(a_0, e_0) d\lambda_0, \quad \text{and} \quad \bar{H}_t^j = \int \bar{h}_t^j(a_0, e_0) d\lambda_0, \quad \text{for } j = R, NR. \tag{55}$$

The insurance effect, Δ_I, is defined as:

$$1 + \Delta_I \equiv \frac{1 - p_{risk}^R}{1 - p_{risk}^{NR}}, \quad \text{where} \quad \sum_{t=0}^{\infty} \beta^t u\left((1 - p_{risk}^j) C_t^j, H_t^j\right) = \sum_{t=0}^{\infty} \beta^t u\left(\bar{C}_t^j, \bar{H}_t^j\right). \tag{56}$$

Here, p_{risk}^j is the welfare cost of risk in the economies with and without reform.

4. Redistribution effect Define the redistribution effect, Δ_R, by:

$$1 + \Delta_R \equiv \frac{1 - p_{ineq}^R}{1 - p_{ineq}^{NR}}, \quad \text{where} \quad \sum_{t=0}^{\infty} \beta^t u\left((1 - p_{ineq}^j) \bar{C}_t^j, \bar{H}_t^j\right) = \int \sum_{t=0}^{\infty} \beta^t u\left(\bar{c}_t^j(a_0, e_0), \bar{h}_t^j(a_0, e_0)\right) d\lambda_0. \tag{57}$$

Analogously to p_{risk}^j, p_{ineq}^j denotes the cost of inequality.

Choice of certainty equivalents. There can be many certainty-equivalent paths that satisfy equation (54), potentially differing over time and across consumption-labor pairs. In general, these choices can affect the decomposition components. However, if the certainty equivalents follow parallel paths, these choices are inconsequential: The certainty equivalents display *parallel patterns* if $\bar{c}_t^j(a_0, e_0) = \eta^j(a_0, e_0)\tilde{C}_t^j$, and $1 - \bar{h}_t^j(a_0, e_0) = \eta^j(a_0, e_0)(1 - \tilde{H}_t^j)$, for some function $\eta^j(a_0, e_0)$ and paths $\{\tilde{C}_t^j\}$, and $\{\tilde{H}_t^j\}$. This assumption imposes two restrictions: (i) certainty equivalents of households with different initial conditions differ only by a proportional factor $\eta^j(a_0, e_0)$ (parallel patterns); and (ii) the same proportionality factor applies to both consumption and leisure. Deviations from (i) generally lead to small changes in the decomposition. However, deviations from (ii) are more consequential, as they affect the decomposition of curvature between consumption and leisure, which in turn influences the insurance and redistribution effects.[20] Importantly, the level effect remains invariant regardless of this choice. In particular, one may assume that the certainty-equivalent paths follow their respective aggregates, i.e., $\tilde{C}_t^j = C_t^j$ and $\tilde{H}_t^j = H_t^j$. Under Assumption D.1, this choice is immaterial (see Dyrda and Pedroni (2023) for a proof).

[20] For example, applying the proportionality only to consumption gives $(\eta c)^\gamma (1-h)^{1-\gamma} = \eta^\gamma c^\gamma (1-h)^{1-\gamma}$, whereas applying it to both consumption and leisure as in Assumption D.1 yields $\eta c^\gamma (1-h)^{1-\gamma}$.

D.2 Proofs

Proposition 2 *If preferences satisfy the property that for any scalar x, $u(xc, h) = g(x)u(c, h)$ for some totally multiplicative function $g(\cdot)$, then:*

$$1 + \widetilde{\Delta} = (1 + \Delta_L)(1 + \Delta_I)(1 + \Delta_R).$$

Notice that U, defined above, inherits the property from u that, for any scalar x,

$$U\left(\{xc_t, h_t\}\right) = g(x)U\left(\{c_t, h_t\}\right). \tag{58}$$

The result follows by applying this identity iteratively across the expressions defined in equations (53) through (57):

$$\begin{aligned}
\int \mathbb{E}_0\left[U\left(\left\{c_t^R, h_t^R\right\}\right)\right] d\lambda_0 &\overset{(54)}{=} \int U\left(\left\{\bar{c}_t^R, h_t^R\right\}\right) d\lambda_0 \overset{(57)}{=} U\left(\left\{\left(1 - p_{ineq}^R\right)\bar{C}_t^R, \bar{H}_t^R\right\}\right) \\
&\overset{(58)}{=} g\left(1 - p_{ineq}^R\right) U\left(\left\{\bar{C}_t^R, \bar{H}_t^R\right\}\right) \\
&\overset{(56)}{=} g\left(1 - p_{ineq}^R\right) U\left(\left\{\left(1 - p_{risk}^R\right) C_t^R, H_t^R\right\}\right) \\
&\overset{(58)}{=} g\left(\left(1 - p_{ineq}^R\right)\left(1 - p_{risk}^R\right)\right) U\left(\left\{C_t^R, H_t^R\right\}\right) \\
&\overset{(53)}{=} g\left(\left(1 - p_{ineq}^R\right)\left(1 - p_{risk}^R\right)\right) U\left(\left\{\left(1 + \Delta_L\right) C_t^{NR}, H_t^{NR}\right\}\right) \\
&\overset{(58)}{=} g\left(\left(1 + \Delta_L\right)\left(1 - p_{ineq}^R\right)\left(1 - p_{risk}^R\right)\right) U\left(\left\{C_t^{NR}, H_t^{NR}\right\}\right) \\
&\overset{(58)}{=} g\left(\left(1 + \Delta_L\right)\left(1 - p_{ineq}^R\right)\frac{\left(1 - p_{risk}^R\right)}{\left(1 - p_{risk}^{NR}\right)}\right) U\left(\left\{\left(1 - p_{risk}^{NR}\right) C_t^{NR}, H_t^{NR}\right\}\right) \\
&\overset{(56)}{=} g\left(\left(1 + \Delta_L\right)\left(1 + \Delta_I\right)\left(1 - p_{ineq}^R\right)\right) U\left(\left\{\bar{C}_t^{NR}, \bar{H}_t^{NR}\right\}\right) \\
&\overset{(58)}{=} g\left(\left(1 + \Delta_L\right)\left(1 + \Delta_I\right)\frac{\left(1 - p_{ineq}^R\right)}{\left(1 - p_{ineq}^{NR}\right)}\right) U\left(\left\{\left(1 - p_{ineq}^{NR}\right)\bar{C}_t^{NR}, \bar{H}_t^{NR}\right\}\right) \\
&\overset{(57)}{=} g\left(\left(1 + \Delta_L\right)\left(1 + \Delta_I\right)\left(1 + \Delta_R\right)\right) \int U\left(\left\{\bar{c}_t^{NR}, \bar{h}_t^{NR}\right\}\right) d\lambda_0 \\
&\overset{(56)}{=} g\left(\left(1 + \Delta_L\right)\left(1 + \Delta_I\right)\left(1 + \Delta_R\right)\right) \int \mathbb{E}_0\left[U\left(\left\{c_t^{NR}, h_t^{NR}\right\}\right)\right] d\lambda_0 \\
&\overset{(58)}{=} \int \mathbb{E}_0\left[U\left(\left\{\left(1 + \Delta_L\right)\left(1 + \Delta_I\right)\left(1 + \Delta_R\right) c_t^{NR}, h_t^{NR}\right\}\right)\right] d\lambda_0.
\end{aligned}$$

The definition of $\widetilde{\Delta}$ in equation (52), then, implies the result.

E Numerical Methods

This appendix describes the numerical procedures used to solve the model and compute the solution to the optimal fiscal policy problems. We begin by showing how to transform the original balanced-growth economy into a stationary representation that can be addressed using time-invariant numerical methods. We then detail the numerical algorithm used to solve for equilibrium in the model. Finally, we discuss the global optimization procedure employed to determine the optimal fiscal policies.

E.1 Detrending the Model

Population evolves according to $N_{t+1} = (1 + n_{t+1})N_t$, while labor-augmenting productivity grows according to $\Psi_{t+1} = (1 + g_{t+1})\Psi_t$, with $\Psi_0 = 1$. A tilde denotes the stationary version of a non-stationary variable.

E.1.1 Households

Optimization problem. Given initial assets a_0 and idiosyncratic productivity e_0, a household of size N_t maximizes

$$\max_{\{c_t, h_t, a_{t+1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t N_t \left[u(c_t, h_t) - v(Z_t)\right]$$

subject to the per-capita budget constraint

$$(1 - \tau_t^c)\, c_t + (1 + n_{t+1})\, a_{t+1} = (1 - \tau_t^h)\, w_t e_t h_t + \left[1 + (1 - \tau_t^k)\, r_t\right] a_t + T_t, \quad \text{and} \quad a_t \geq \bar{a}\, \Psi_t. \tag{59}$$

The term $(1 + n_{t+1})$ converts assets chosen in period t into the larger cohort that begins period $t + 1$.

Stationary formulation. Let

$$\tilde{c}_t \equiv \frac{c_t}{\Psi_t}, \quad \tilde{a}_t \equiv \frac{a_t}{\Psi_t}, \quad \tilde{w}_t \equiv \frac{w_t}{\Psi_t}, \quad \text{and} \quad \tilde{T}_t \equiv \frac{T_t}{\Psi_t}.$$

Dividing (59) by Ψ_t and using $\Psi_{t+1} = (1 + g_{t+1})\Psi_t$ the household problem can be rewritten as that of maximizing

$$\max_{\{\tilde{c}_t, h_t, \tilde{a}_{t+1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t N_t \left[\Psi_t^{\gamma(1-\sigma)} u(\tilde{c}_t, h_t) - v(Z_t)\right],$$

subject to

$$(1 - \tau_t^c)\tilde{c}_t + (1 + n_{t+1})(1 + g_{t+1})\tilde{a}_{t+1} = (1 - \tau_t^h)\tilde{w}_t e_t h_t + \left[1 + (1 - \tau_t^k)\, r_t\right]\tilde{a}_t + \tilde{T}_t, \quad \text{and} \quad \tilde{a}_t \geq \bar{a}.$$

E.1.2 Production and prices

Final-good production. The production function is specified as

$$Y_t = A_{1,t} K_{1,t}^{\alpha} H_{1,t}^{1-\alpha-\nu} E_t^{\nu},$$

so that labor-augmenting technology is given by

$$\Psi_t \equiv A_{1,t}^{1/(1-\alpha-\nu)}.$$

The firm's optimality conditions, then, imply

$$\begin{aligned}
r_t &= (1 - D(Z_t)) A_{1,t}\, \alpha K_{1,t}^{\alpha-1} H_{1,t}^{1-\alpha-\nu} E_t^{\nu},\\
w_t &= (1 - D(Z_t)) A_{1,t} (1-\alpha-\nu) K_{1,t}^{\alpha} H_{1,t}^{-\alpha-\nu} E_t^{\nu},\\
p_{E,t} &= (1 - D(Z_t)) A_{1,t}\, \nu K_{1,t}^{\alpha} H_{1,t}^{1-\alpha-\nu} E_t^{\nu-1}.
\end{aligned}$$

Defining

$$\tilde{Y}_t \equiv \frac{Y_t}{\Psi_t N_t}, \quad \tilde{K}_t \equiv \frac{K_t}{\Psi_t N_t}, \quad \tilde{H}_t \equiv \frac{H_t}{N_t}, \quad \text{and} \quad \tilde{E}_t \equiv \frac{E_t}{\Psi_t N_t},$$

we can rewrite these equations as

$$\tilde{Y}_t = \tilde{K}_{1,t}^{\alpha} \tilde{H}_{1,t}^{1-\alpha-\nu} \tilde{E}_t^{\nu}. \tag{60}$$

$$r_t = (1 - D(Z_t)) \alpha \tilde{K}_{1,t}^{\alpha-1} \tilde{H}_{1,t}^{1-\alpha-\nu} \tilde{E}_t^{\nu}, \tag{61}$$

$$\tilde{w}_t = (1 - D(Z_t))(1-\alpha-\nu) \tilde{K}_{1,t}^{\alpha} \tilde{H}_{1,t}^{-\alpha-\nu} \tilde{E}_t^{\nu}, \tag{62}$$

$$p_{E,t} = (1 - D(Z_t)) \nu \tilde{K}_{1,t}^{\alpha} \tilde{H}_{1,t}^{1-\alpha-\nu} \tilde{E}_t^{\nu-1}. \tag{63}$$

Energy production. Energy is produced using

$$E_t = A_{2,t} K_{2,t}^{1-\alpha_E} H_{2,t}^{\alpha_E},$$

with first order conditions given by

$$\begin{aligned}
r_t &= \left[p_{E,t} - \tau_t^i - \tau_t^e (1-\mu_t) - \Theta_t(\mu_t) \right] \, A_{2,t} \,(1-\alpha_E)\, K_{2,t}^{-\alpha_E} H_{2,t}^{\alpha_E},\\
w_t &= \left[p_{E,t} - \tau_t^i - \tau_t^e (1-\mu_t) - \Theta_t(\mu_t) \right] \, A_{2,t}\, \alpha_E\, K_{2,t}^{1-\alpha_E} H_{2,t}^{\alpha_E-1},\\
\tau_t^e &= \Theta_{\mu,t}(\mu_t).
\end{aligned}$$

Assuming a constant relative productivity

$$\bar{A}_{21} \equiv A_{2,t} A_{1,t}^{-\alpha_E/(1-\alpha-\nu)},$$

we obtain

$$\tilde{E}_t = \bar{A}_{21}\,\tilde{K}_{2,t}^{1-\alpha_E}\tilde{H}_{2,t}^{\alpha_E}, \tag{64}$$

$$r_t = \left[p_{E,t} - \tau_t^i - \tau_t^e(1-\mu_t) - \Theta_t(\mu_t)\right]\;\bar{A}_{21}\,(1-\alpha_E)\,\tilde{K}_{2,t}^{-\alpha_E}\tilde{H}_{2,t}^{\alpha_E}, \tag{65}$$

$$\tilde{w}_t = \left[p_{E,t} - \tau_t^i - \tau_t^e(1-\mu_t) - \Theta_t(\mu_t)\right]\;\bar{A}_{21}\,\alpha_E\,\tilde{K}_{2,t}^{1-\alpha_E}\tilde{H}_{2,t}^{\alpha_E-1}. \tag{66}$$

E.1.3 Static allocation of capital and labor

Define shares

$$\kappa_t \equiv \frac{\tilde{K}_{1,t}}{\tilde{K}_t}, \quad \text{and} \quad \eta_t \equiv \frac{\tilde{H}_{1,t}}{\tilde{H}_t}.$$

Equating the two expressions for the rental rate (equations (61) and (65)), and doing the same for the wage expressions (equations (62) and (66)), we obtain the system

$$\frac{\kappa_t}{1-\kappa_t} = \frac{\alpha}{1-\alpha-\nu}\,\frac{\alpha_E}{1-\alpha_E}\,\frac{\eta_t}{1-\eta_t}, \tag{67}$$

$$\frac{1-\alpha-\nu}{\alpha_E} = \left(\nu - \frac{\tau_t^i + \tau_t^e(1-\mu_t) + \Theta_t(\mu_t)}{(1-D(Z_t))\tilde{K}_{1,t}^{\alpha}\tilde{H}_{1,t}^{1-\alpha-\nu}\tilde{E}_t^{\nu-1}}\right)\frac{\eta_t}{1-\eta_t}, \tag{68}$$

which can be solved, each period, for κ_t and η_t.

E.1.4 Abatement technology and energy tax

Assuming

$$\Theta_t(\mu_t)E_t = \frac{P_{b,t}}{a_2}\mu_t^{a_2}E_t,$$

the marginal abatement cost is given by $\Theta_{\mu,t}(\mu_t) = P_{b,t}\mu_t^{a_2-1}$, and the policy rule $\tau_t^e = \Theta_{\mu,t}(\mu_t)$ implies

$$\mu_t = \left(\tau_t^e / P_{b,t}\right)^{1/(a_2-1)}. \tag{69}$$

E.1.5 Government budget constraint

Scaling the level constraint by $\Psi_t N_t$ yields

$$\tau_t^c\tilde{C}_t + \tau_t^h\tilde{w}_t\tilde{H}_t + \tau_t^k r_t\tilde{A}_t + \tau_t^e(1-\mu_t)\tilde{E}_t + \tau_t^i\tilde{E}_t + (1+g_{t+1})(1+n_{t+1})\tilde{B}_{t+1} - (1+r_t)\tilde{B}_t = \tilde{G}_t + \tilde{T}_t. \tag{70}$$

The scaling factor in front of $\tilde{B}_{t+1}$ converts next-period government debtexpressed in tomorrows efficient unitsinto todays units, ensuring consistency with the detrended resource constraint.

E.2 Computing Equilibrium given Policy

We compute the equilibrium in three steps: first, we obtain the initial stationary equilibrium; second, we obtain the final stationary equilibrium; and third, we solve for the transition path between the two.

Algorithm: Equilibrium Computation

Step 1: Initial Stationary Equilibrium

- **Exogenous inputs:** Fix the tax vector $\left(\tau_0^h, \tau_0^k, \tau_0^e, \tau_0^i, t_0 = \tilde{T}_0/\tilde{Y}_0, b_0 = \tilde{B}_0/\tilde{Y}_0\right)$; the environmental block $\left(\Theta_0(\cdot), Z_0\right)$; productivity and population levels $(A_{1,0}, A_{2,0}, N_0)$; and growth rates (g_0, n_0).
- **Guess:** Initial values $(\tilde{K}_0, \tilde{H}_0)$.
- **Solve for aggregates:** Solve equations (69), (67), (68), (64), (61), (62), (63), and (60) to obtain, in order:[a]
$$(\mu_0, \kappa_0, \eta_0, \tilde{E}_0, r_0, \tilde{w}_0, p_{E,0}, \tilde{Y}_0, \tilde{T}_0 = t_0\tilde{Y}_0,\ \tilde{B}_0 = b_0\tilde{Y}_0).$$
- **Solve household problem:** Given $(r_0, \tilde{w}_0)$, find the decision rules $\tilde{c}_0(\tilde{a}, e)$, $h_0(\tilde{a}, e)$, $\tilde{a}_0'(\tilde{a}, e)$ and the stationary distribution $\lambda_0(\tilde{a}, e)$.
- **Update guess:**
$$\tilde{H}_0 = \int e\, h_0\, d\lambda_0, \qquad \tilde{K}_0 = \int \tilde{a}\, d\lambda_0 - \tilde{B}_0.$$
Repeat the previous three sub-steps until convergence.
- **Government budget:** Compute $\tilde{G}_0$ residually from the government budget constraint (70).

Step 2: Final Stationary Equilibrium

- Fix $b_F = \tilde{B}_F/\tilde{Y}_F$ and Z_F, and assume that after period T_F all other exogenous variables are either constant or grow at their respective long-run rates.
- Repeat **Step 1** with terminal values to obtain
$$(\mu_F, \kappa_F, \eta_F, \tilde{E}_F, r_F, \tilde{w}_F, p_{E,F}, \tilde{Y}_F, \tilde{H}_F, \tilde{K}_F, \tilde{T}_F, \tilde{B}_F).$$

Step 3: Transition Path

- Given the policy path $\{\tau^h_t, \tau^k_t, \tau^e_t, \tau^i_t, t_t, \tilde{G}_t\}_{t=0}^{T_F}$,[b] guess the aggregate paths $\{\tilde{C}_t, \tilde{A}_t, \tilde{K}_t, \tilde{H}_t, Z_t\}_{t=0}^{T_F}$.

- For each $t \in \{0, \ldots, T_F\}$, solve (69), (67), (68), (64), (61), (62), (63), and (60) for

$$(\mu_t, \kappa_t, \eta_t, \tilde{E}_t, r_t, \tilde{w}_t, p_{E,t}, \tilde{Y}_t).$$

- Starting from $\tilde{B}_0$, iterate the government budget constraint (70) forward to obtain $\{\tilde{B}_t\}_{t=0}^{T_F}$ and thus $b_F = \tilde{B}_F/\tilde{Y}_F$.

- Update the climate block: with $\{\tilde{E}_t\}_{t=0}^{T_F}$, compute $\{Z_t\}_{t=0}^{T_F}$ and obtain Z_F.

- With (b_F, Z_F), obtain the final equilibrium via **Step 2**; solve backward for the decision rules $\{\tilde{c}_t, h_t, \tilde{a}'_t\}_{t=0}^{T_F}$ and forward for the distributions $\{\lambda_t\}_{t=0}^{T_F}$.

- Update aggregate paths:

$$\tilde{C}_t = \int \tilde{c}_t \, d\lambda_t, \quad \tilde{A}_t = \int \tilde{a} \, d\lambda_t, \quad \tilde{H}_t = \int e \, h_t \, d\lambda_t, \quad \tilde{K}_t = \tilde{A}_t - \tilde{B}_t.$$

  Repeat until convergence.

[a] Except for the system for κ_0 and η_0, the listed equations admit analytical solutions.

[b] We parametrize time-varying policy paths using equation (18). For lump-sum transfers, we set $m_{t,0} = 1$ and $m_{t,F} = 0$. For carbon taxes, policy after the backstop price is reached is irrelevant, so we focus on the "short-run" policy by setting $\lambda^{\tau_E} = 0$, $m_{\tau_E,0} = 3$, and $m_{\tau_E,F} = 0$.

E.3 Global Optimization

We use a global optimization procedure developed and described in detail in Dyrda and Pedroni (2023) (see Appendix D.3). The algorithm is implemented in modern Fortran language using MPI library and the experiments were conducted at the Niagara cluster (see Ponce et al. (2019) and Loken et al. (2010) for more details on the cluster), part of Digital Research Alliance of Canada and located at the University of Toronto.

F Energy-good Economy

F.1 Environment

This Appendix describes how the baseline model is adjusted to capture that low-income households devote a larger share of their spending to carbon-intensive goods. As in Douenne et al. (2023), we assume households consume energy directly (the 'dirty' good d), which enters the preference specification as a necessity (see Appendix F.2). In addition to the consumer price $p_{D,t}$, households pay a tax $\tau_{D,t}$ per unit of energy consumption. Optimally splitting consumption between the final and the dirty good leads to the following intratemporal optimality condition:

$$\frac{u_d(c_{it}(e_i^t), d_{it}(e_i^t), h_{it}(e_i^t))}{u_c(c_{it}(e_i^t), d_{it}(e_i^t), h_{it}(e_i^t))} = p_{D,t} + \tau_{D,t}. \tag{71}$$

The first-order conditions for consumption of the final good, labor supply, and savings are the same as before (see Appendix A).

Turning to the production side, firms in the energy sector sell part of their total output $E_t = A_{2,t}G(K_{2,t}, H_{2,t})$ to households at a price $p_{D,t}$ and the remainder to firms in the final good sector at a price $p_{E,t}$. Consequently,

$$E_t = E_{1,t} + D_t, \tag{72}$$

where $E_{1,t}$ denotes the energy input used by firms in the final good sector and $D_t = N_t \sum_i \sum_{e_i^t} \pi_{it}(e_i^t) d_{it}(e_i^t)$ denotes total consumption of the dirty good by households.

As will be explained in Section F.2, to jointly match total emissions, the share of emissions by firms, and household spending on energy, we introduce a parameter ι that governs the *emission intensity* of energy consumption by households. Specifically, we assume total emissions are proportional to $E_{1,t} + \iota D_t$. If $\iota = 1$, energy consumption by households and firms is equally polluting and total emissions are proportional to energy production E_t, as in the baseline. By contrast, a value of $\iota < 1$ ($\iota > 1$) implies that every unit of energy consumption by households generates fewer (more) emissions than a unit of energy consumption by firms in the final good sector. Unabated emissions are then given by $E_t^M = (1 - \mu_t)(E_{1,t} + \iota D_t)$ and the total costs of abatement are equal to $\theta_t(\mu_t)(E_{1,t} + \iota D_t)$. Combining the above, profits in the energy sector are

$$\mathcal{P}_t = p_{E,t}E_{1,t} + p_{D,t}D_t - \tau_{I,t}E_t - \tau_{E,t}(1 - \mu_t)(E_{1,t} + \iota D_t) - w_t H_{2,t} - r_t K_{2,t} - \theta_t(\mu_t)(E_{1,t} + \iota D_t). \tag{73}$$

Energy producers choose how much capital and labor to hire, what fraction of emissions to abate and how much of their output to sell to households and firms in the final good sector to maximize profits, subject to the constraints $E_t = A_{2,t}G(K_{2,t}, H_{2,t})$ and $E_t = E_{1,t} + D_t$. The first-order conditions for $H_{2,t}$, $K_{2,t}$, and μ_t are the same as before (see Appendix A). Furthermore, for energy producers to be willing to sell to both households and firms in the final goods sector, the following must hold:

$$p_{D,t} = p_{E,t} + (\iota - 1)(\tau_{E,t}(1-\mu_t) + \theta_t(\mu_t)). \tag{74}$$

As in the baseline, under the given abatement cost structure, profits in the energy sector are zero in equilibrium.

The final modification is in the government's budget constraint, which now includes revenues from taxing household energy consumption:

$$G_t + N_t T_t + R_t B_t = \tau_{H,t} w_t H_t + \tau_{K,t}(r_t - \delta)K_t + \tau_{I,t}E_t + \tau_{E,t}E_t^M + \tau_{D,t}D_t + B_{t+1}. \tag{75}$$

For brevity, we restate the optimal tax problem but do not characterize the optimum (the derivations are similar to those from Appendix A):

$$\max_{\substack{\{C_t, D_t, H_t, H_{1,t}, H_{2,t}, K_t, K_{1,t}, K_{2,t},\\ E_t, \mu_t, Z_t, \tau_{H,t}, \tau_{K,t}, \tau_{I,t}, \tau_{E,t}, \tau_{D,t}, T_t, B_t\}_t,\\ \{c_{it}(e_i^t), d_{it}(e_i^t), h_{it}(e_i^t), a_{it+1}(e_i^t)\}_{i,t,e_i^t}}} \mathcal{W} = \sum_i \alpha_i \sum_t \beta^t N_t \sum_{e_i^t} \pi_{it}(e_i^t)(u(c_{it}(e_i^t), d_{it}(e_i^t), h_{it}(e_i^t)) - v(Z_t)),$$

$$\begin{aligned}
\text{s.t. } \forall i,t,e_i^t:\ & \Big[u_c(c_{it}(e_i^t), d_{it}(e_i^t), h_{it}(e_i^t)) - \beta(1 + (1-\tau_{K,t+1})(\big(1 - D(Z_{t+1})\big)A_{1,t+1}F_{K,t+1} - \delta)) \\
& \times \sum_{e_i^{t+1}} \pi_{it+1}(e_i^{t+1}|e_i^t) u_c(c_{it+1}(e_i^{t+1}), d_{it+1}(e_i^{t+1}), h_{it+1}(e_i^{t+1}))\Big](a_{it+1}(e_i^t) - \underline{a}_{t+1}) = 0, \\
\forall i,t,e_i^t:\ & u_c(c_{it}(e_i^t), d_{it}(e_i^t), h_{it}(e_i^t)) \geq \beta(1 + (1-\tau_{K,t+1})(\big(1 - D(Z_{t+1})\big)A_{1,t+1}F_{K,t+1} - \delta)) \\
& \times \sum_{e_i^{t+1}} \pi_{it+1}(e_i^{t+1}|e_i^t) u_c(c_{it+1}(e_i^{t+1}), d_{it+1}(e_i^{t+1}), h_{it+1}(e_i^{t+1})), \\
\forall i,t,e_i^t:\ & a_{it+1}(e_i^t) \geq \underline{a}_{t+1}, \\
\forall i,t,e_i^t:\ & u_c(c_{it}(e_i^t), d_{it}(e_i^t), h_{it}(e_i^t))\big(1 - D(Z_t)\big)A_{1,t}F_{H,t}(1-\tau_{H,t})e_{it} + u_h(c_{it}(e_i^t), d_{it}(e_i^t), h_{it}(e_i^t)) = 0, \\
\forall i,t,e_i^t:\ & u_d(c_{it}(e_i^t), d_{it}(e_i^t), h_{it}(e_i^t)) = u_c(c_{it}(e_i^t), d_{it}(e_i^t), h_{it}(e_i^t)) \\
& \times ((1 - D(Z_t))A_{1,t}F_{E,t} + (\iota - 1)(\tau_{E,t}(1-\mu_t) + \theta_t(\mu_t)) + \tau_{D,t}), \\
\forall i,t,e_i^t:\ & (1 + n_{t+1})a_{it+1}(e_i^t) = (1 + (1-\tau_{K,t})(\big(1 - D(Z_t)\big)A_{1,t}F_{K,t} - \delta))a_{it}(e_i^{t-1}) + T_t \\
& + \big(1 - D(Z_t)\big)A_{1,t}F_{H,t}(1-\tau_{H,t})e_{it}h_{it}(e_i^t) - c_{it}(e_i^t) \\
& - ((1 - D(Z_t))A_{1,t}F_{E,t} + (\iota - 1)(\tau_{E,t}(1-\mu_t) + \theta_t(\mu_t)) + \tau_{D,t})d_{it}(e_i^t), \\
\forall t:\ & C_t = N_t \sum_i \alpha_i \sum_{e_i^t} \pi_{it}(e_i^t) c_{it}(e_i^t),
\end{aligned}$$

$$\forall t: \; D_t = N_t \sum_i \alpha_i \sum_{e_i^t} \pi_{it}(e_i^t) d_{it}(e_i^t),$$

$$\forall t: \; H_t = N_t \sum_i \alpha_i \sum_{e_i^t} \pi_{it}(e_i^t) e_{it} h_{it}(e_i^t),$$

$$\forall t: \; H_t = H_{1,t} + H_{2,t},$$

$$\forall t: \; K_t + B_t = N_t \sum_i \alpha_i \sum_{e_i^{t-1}} \pi_{it-1}(e_i^{t-1}) a_{it}(e_i^{t-1}),$$

$$\forall t: \; K_t = K_{1,t} + K_{2,t},$$

$$\forall t: \; E_t = A_{2,t} G\big(K_{2,t}, H_{2,t}\big),$$

$$\forall t: \; E_t = E_{1,t} + D_t,$$

$$\forall t: \; \big(1 - D(Z_t)\big) A_{1,t} F(K_{1,t}, H_{1,t}, E_{1,t}) - \theta_t(\mu_t)(E_{1,t} + \iota D_t) = C_t + G_t + K_{t+1} - (1-\delta)K_t,$$

$$\forall t: \; Z_t = J\left(S_0, (1-\mu_0)(E_{1,0} + \iota D_0), ..., (1-\mu_t)(E_{1,t} + \iota D_t), \eta_0, ..., \eta_t\right),$$

$$\forall t: \; F_{K,t} G_{H,t} = G_{K,t} F_{H,t},$$

$$\forall t: \; \tau_{E,t} = \theta_t'(\mu_t),$$

$$\forall t: \; (1 - D(Z_t)) A_{1,t} F_{H,t} = ((1 - D(Z_t)) A_{1,t} F_{E,t} - \tau_{I,t} - (1-\mu_t)\tau_{E,t} - \theta_t(\mu_t)) A_{2,t} G_{H,t}.$$

F.2 Calibration

Preferences In the model, energy consumption enters utility with a Stone-Geary specification, i.e., period sub-utility over consumption and leisure is given by

$$u_t(c_{it}, d_{it}, h_{it}) = \frac{\left(c_{it}(d_{it} - \bar{d}_t)^{\epsilon}(1 - \varsigma h_{it})^{\gamma}\right)^{1-\sigma}}{1-\sigma},$$

with d_{it} the energy consumption of household i at time t and $\bar{d}_t$ denotes the subsistence consumption of energy. To ensure existence of a balanced-growth path, the latter is assumed to grow at the rate of labor-augmenting productivity: $\bar{d}_t = \bar{d}\Psi_t$. The parameter ϵ, in turn, captures the relative preference for energy d vs. the final good c.

From the household's problem, we see that how much energy a given household consumes depends on an intra-temporal condition (cf. equation (71)), where households split their total expenditures between c and d. Thus, $\forall i, j$ and $\forall t$, $c_{i,t} > c_{j,t}$ *iff* $d_{i,t} > d_{j,t}$. This implies that whoever consumes the most final good in the model will also consume the most energy. The goal of the calibration is therefore to reproduce how much energy people consume on average at different levels of total consumption.

Empirical approach We use data from the Consumer Expenditure Surveys (CEX) to compute the distribution of average energy consumption as a function of total consumption.[21] For calibration purposes, this distribution is discretized: in the calibration we choose $\bar{d}$ and ϵ such that we can match the share of aggregate household energy consumption consumed by different quintiles of households (who differ in income and wealth but who can be ranked in terms of their consumption). We additionally try to match the energy consumption share of the bottom and top 5%, as well as the Gini coefficient. Our two last targets are the share of aggregate emissions coming directly from household energy consumption (as opposed to firms producing the final good c), and the share of their total expenditures that households spend on energy.

Practical implementation Our model starts in 2020. Because energy consumption in 2020 displays unusual patterns due to the COVID-2019 pandemic, we instead use the 2019 survey.[22] We first pool the four waves of the survey for 2019, rank households by their total expenditures, and divide them into 5 expenditure quintiles. We then compute the share of aggregate household energy consumption consumed by each of these quintiles.[23] The figures are the following, from the bottom to the top expenditure quintile: 11.3%; 16.6%; 20.0%; 23.3%; 28.9%. These figures mean that the 20% of households who have the lowest total expenditures consume 11.3% of all energy consumed by households, while the 20% who have the highest total expenditures consume 28.9% of it. Thus, energy is clearly a necessity, as it is far less unequally distributed than income, wealth, or total consumption.

Top and bottom 5% shares To compute the share of aggregate household energy expenditure coming from the top and bottom 5%, we apply the same methodology as above. We find that the 5% poorest households (in terms of lowest total expenditures) consume 1.8% of aggregate household energy consumption, while the 5% richest consume 8.5% of it.

[21] Averaging across people with similar levels of total consumption is necessary for two reasons. First, because it is consistent with the model that does not feature preference heterogeneity. If we were to ignore this aspect, we would overestimate the energy share of the richest households by imputing them high levels of energy consumption that are partly driven by preferences. Second, because energy consumption is reported over short periods of time in the survey, hence at the household level we only observe a noisy measure of actual energy consumption levels. With our approach, these measurement errors average out.

[22] If we use previous years, or pool together multiple years, we obtain very similar distributions.

[23] To account for heterogeneous household size, total expenditures and energy expenditures are normalized by the equivalence scale of the household.

Gini of energy consumption by total expenditure To compute the Gini of energy consumption, we split households in 100 percentiles of total expenditures, from the poorest to the richest. For each group we compute their share of aggregate household energy expenditure. We then use Brown's formula to compute the associated Gini coefficient:

$$\text{Gini} = 1 - 0.01 \sum_{k=1}^{100} (Y_k + Y_{k-1}),$$

where Y denotes the cumulative share of energy consumption. The resulting Gini coefficient is 0.178 for the year 2019, showing once again that energy consumption is far less concentrated than other economic variables, and in particular, less concentrated than total consumption.[24]

Other targets There are two other key targets necessary to calibrate the parameters of the Stone-Geary utility function and the emission intensity of energy consumption by households (see Appendix F): the share of their total expenditures that households spend in energy, and the share of total energy (alternatively, emissions) that is directly consumed by households.From the CEX, we find an energy consumption share for households of 7.0% for the year 2019. This number barely moves if we consider the average over the period 2015-2019, but it is significantly lower than in previous periods.[25]

For the share of aggregate emissions coming from households, we follow and update the calibration in Douenne et al. (2023), based on the latest version of the emissions inventory report of the U.S. EPA. From Table 2-12 of the report, we see that 15.3% of U.S. gross emissions came from the residential sector in 2020, and 27.2% from the transportation sector. Out of these 27.2% of transport emissions, not all should be imputed to households' direct energy consumption. Table 2-13 informs us about how emissions from the transportation sector are distributed across types of vehicles. 21.0% and 37.8% of all emissions from the transport sector come from passenger cars and light-duty trucks (such as pickups, minivans, and SUVs): together, they represent 15.9% of U.S. gross emissions. Assuming households are responsible for the most part of these emissions, we can target a share of 30% of all emissions coming from households' direct energy consumption.[26]

[24] Note that because this number is computed from the distribution of average energy spending at different levels of total consumption, it excludes the undesirable variability introduced by the measurement error in the CEX, and it ignores preference heterogeneity within consumption groups. It is therefore significantly lower than the Gini obtained from the raw data, but it is consistent with our model that assumes preference homogeneity.

[25] In our previous paper, for the period 2011-2015, and focusing on the sample of working households, it was 10.8% (and 9.1% over the full sample).

[26] In this case, the implicit assumption is that 92% of emissions from passenger cars and light-duty trucks come from households, i.e., they emit only 14.7%=30%-15.3% of total gross U.S. emissions with their vehicles instead of 15.9%.

Summary To calibrate the extended model, we follow the same approach as in the benchmark (see Section 4 and Appendix B). The exogenously imposed parameters are the same as before, cf. Table IV. Table V, in turn, shows the internally calibrated parameters. The target statistics and model counterparts are summarized in Tables VI and VII.

Table V: Calibrated Model Parameters: Energy-good Economy

Description	Parameter	Value
Preferences and technology		
Consumption share	γ	1.739
Preference curvature	σ	0.677
Discount factor	β	0.990
Energy subsistence consumption	$\bar{d}$	0.033
Weight on energy	ϵ	0.027
Weight on leisure	ς	1.833
Weight on damages in utility	α_Z	1.64×10^{-4}
Borrowing constraint	$\underline{a}$	-0.160
Ratio of TFPs	A_2/A_1	3.539
Household emission intensity	ι	0.477
Fiscal policy		
Government expenditure	G	0.053
Transfers	T	0.067
Labor productivity process		
Productivity process curvature	η	0.610

Persistent shock

$$\Gamma_P = \begin{bmatrix} 0.999 & 9\text{E}{-4} & 4\text{E}{-4} & 2\text{E}{-5} \\ 0.016 & 0.946 & 0.037 & 0.002 \\ 0.017 & 0.009 & 0.969 & 0.005 \\ 0.008 & 0.003 & 0.026 & 0.963 \end{bmatrix} \quad e_P = \begin{bmatrix} 0.440 \\ 0.687 \\ 2.459 \\ 10.864 \end{bmatrix}$$

Transitory shock

$$P_T = \begin{bmatrix} 0.323 \\ 0.505 \\ 0.165 \\ 0.001 \end{bmatrix} \quad e_T = \begin{bmatrix} -0.34 \\ 0.30 \\ 2.36 \\ 15.71 \end{bmatrix}$$

Table VI: Population Partitions: Energy-good Economy Model vs. Data

	Shares			
	Population	**Earnings**	**Income**	**Wealth**
	Workers			
Data	67.2	44.9	69.1	82.7
Model	74.0	47.7	76.4	83.2
	Business Owners			
Data	5.8	33.0	16.1	13.7
Model	4.6	27.5	16.6	16.8
	Inactive Households			
Data	27.0	22.2	14.8	3.6
Model	21.4	24.8	7.0	0.1

Notes: Data comes from 2019 wave of the SCF. Details about the definitions of subgroups of the population can be found in Appendix C.3.

Table VII: Energy-good Economy: Target Statistics and Model Counterparts

(1) Macroeconomic aggregates

	Target	**Model**
Intertemporal elasticity of substitution	0.66	0.66
Capital to output	2.57	2.57
Average Frisch elasticity (Ψ)	1.0	1.0
Average hours worked	0.24	0.25
Transfer to output (%)	14.7	14.6
Debt to output (%)	104.5	103.0
Fraction of hhs with negative net worth (%)	10.8	21.6
Correlation between earnings and wealth	0.51	0.40
Energy exp. share	0.07	0.06
HH. share in emissions	0.30	0.29

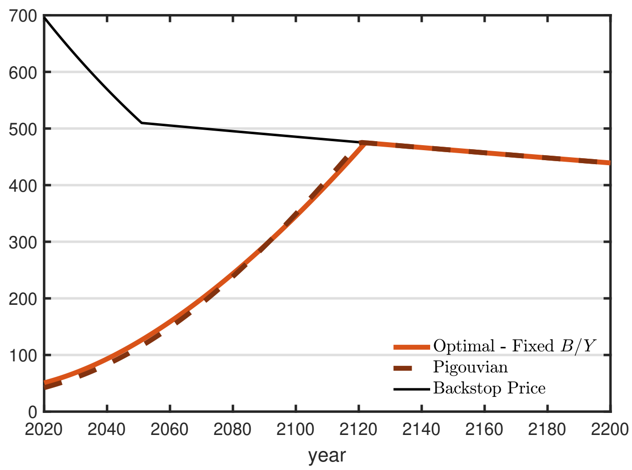
(2) Cross-sectional distributions

	Bottom (%)	**Quintiles**					**Top (%)**	**Gini**
	0–5	**1st**	**2nd**	**3rd**	**4th**	**5th**	**95–100**	
				Wealth				
Data	−0.5	− 0.5	0.8	3.4	9.0	87.4	64.9	0.85
Model	−0.5	− 1.2	1.2	3.8	8.4	87.8	66.0	0.86
				Earnings				
Data	− 0.1	− 0.1	3.5	10.8	20.6	65.2	35.2	0.65
Model	0.0	0.0	4.4	10.1	13.3	72.2	38.2	0.67
				Hours				
Data	0.0	2.7	13.8	19.2	27.9	36.4	11.1	0.34
Model	0.0	0.2	13.6	23.7	27.1	35.4	9.4	0.35
				Energy Consumption by Total Expenditure				
Data	1.8	11.3	16.6	20.0	23.3	28.9	8.5	0.18
Model	3.2	14.5	16.4	17.7	19.5	32.0	14.0	0.17

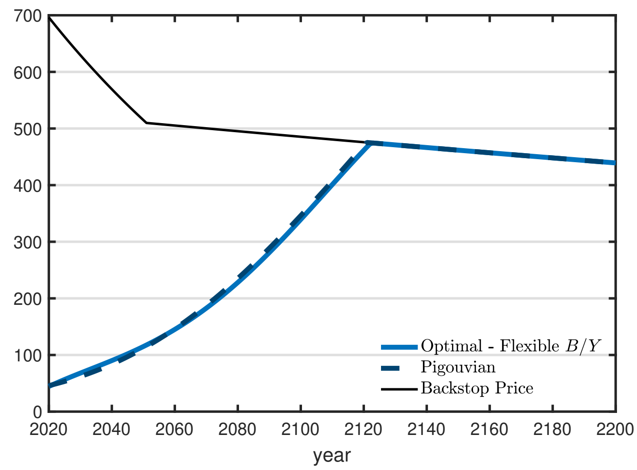
(3) Statistical properties of labor income

	Target	**Model**
Variance of 1-year growth rate	2.33	2.31
Kelly skewness of 1-year growth rate	−0.12	−0.06
Moors kurtosis of 1-year growth rate	2.65	2.14

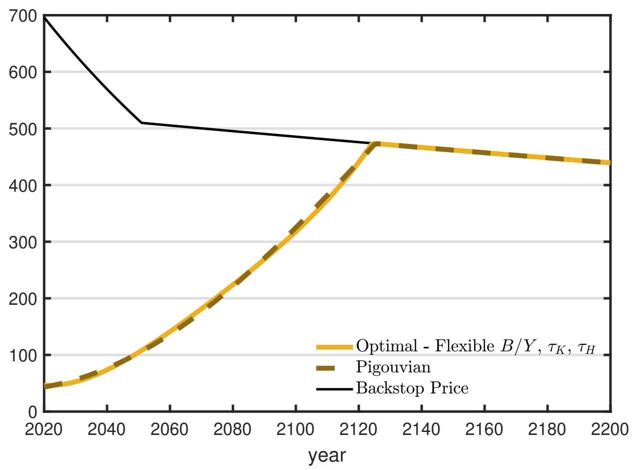
G Additional Figures



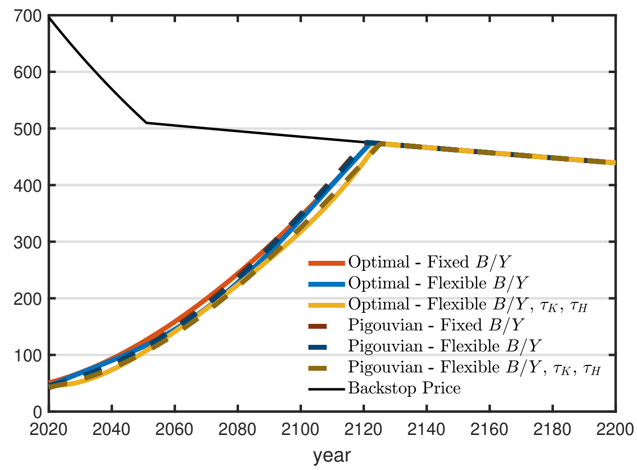
(a) Fixed B/Y



(b) Flexible B/Y



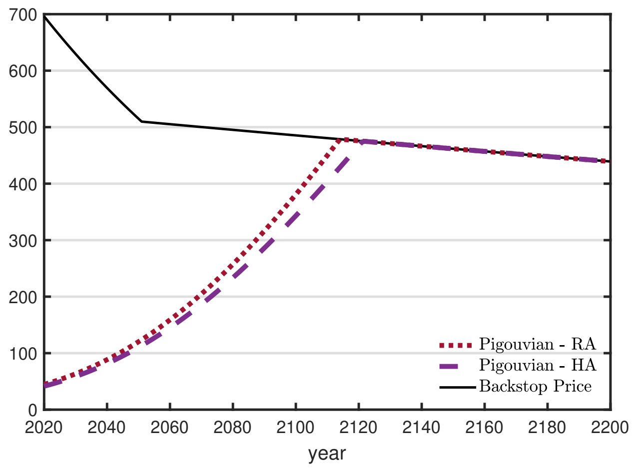
(c) Flexible B/Y, τ_K, and τ_H



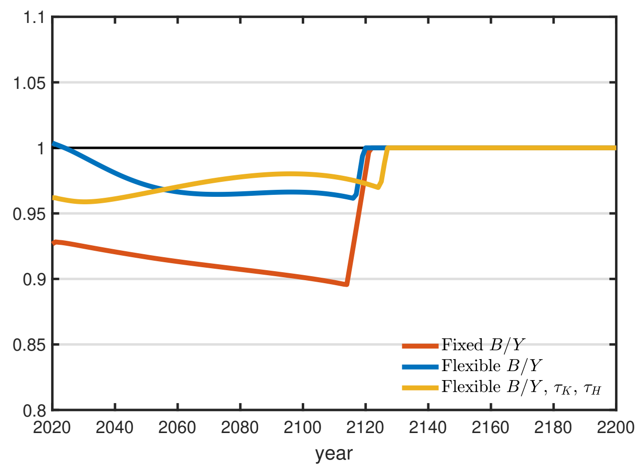
(d) All experiments together

Figure G.1: Optimal Carbon Taxes versus SCC (in \$/tCO_2)

Note: This figure plots the paths of optimal carbon taxes in the baseline scenario where the debt-to-GDP ratio and income taxes are set at their current level (Panels (a) and (d) in red), in the second scenario where the government can choose the path of debt (Panels (b) and (d) in blue), and in the third scenario where it can also choose income taxes (Panels (c) and (d) in yellow). The dashed lines represent the corresponding SCC, and the black line plots the backstop price. All values are expressed in \$/tCO_2.



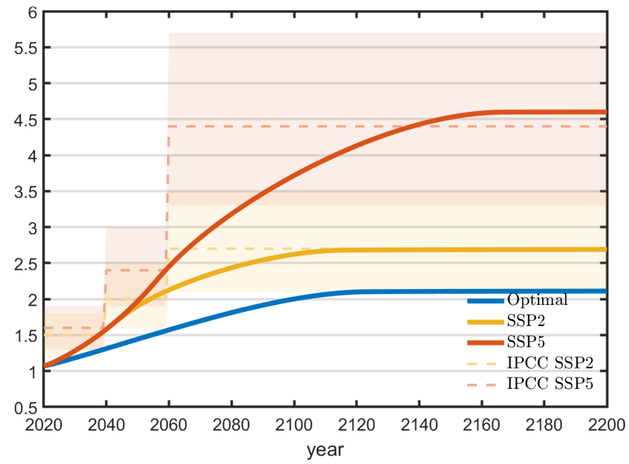
(a) Levels in the baseline (in \$/tCO_2)



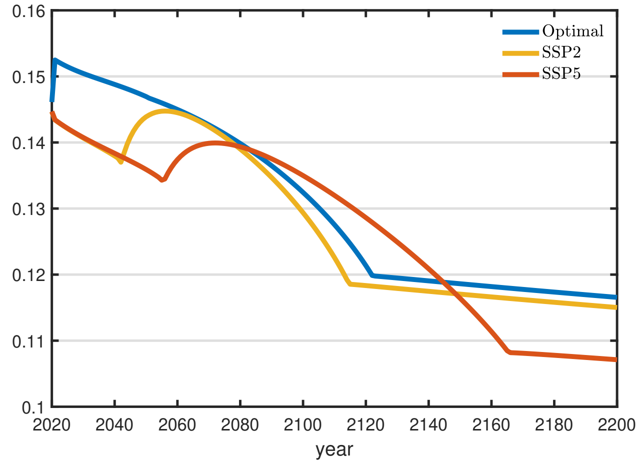
(b) Ratios across scenarios

Figure G.2: Social Cost of Carbon for Representative vs. Heterogeneous Agents.

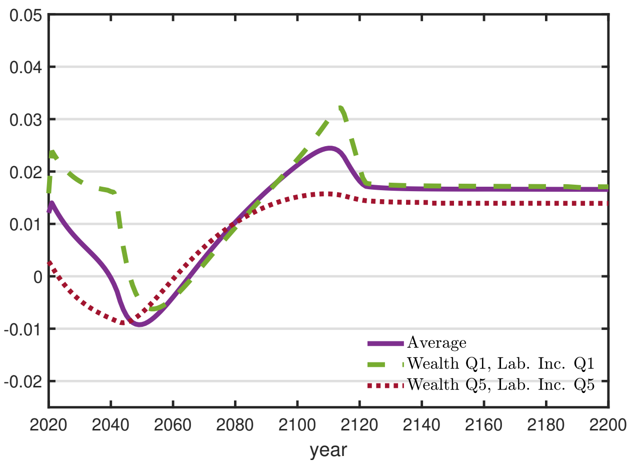
Note: The left figure plots the SCC (i.e., the Pigouvian tax) in the baseline scenario where the debt-to-GDP ratio and income taxes are set at their current level. The purple dashed line corresponds to the SCC evaluated at the average of marginal utilities of consumption; the purple dotted line corresponds to the SCC evaluated at the marginal utility of the representative agent. The right figure plots the ratio of these two versions of the SCC in the baseline scenario (red), in the second scenario where the government can choose the path of debt (blue), and in the third scenario where it can also choose income taxes (yellow).



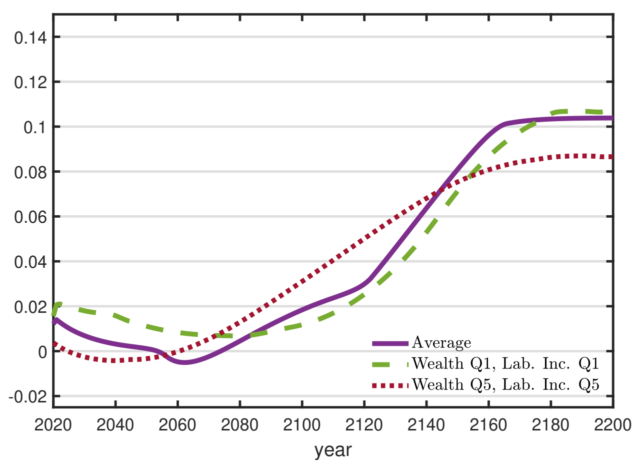
(a) Temperature Path (in °C)



(b) Transfers over GDP (T/Y)



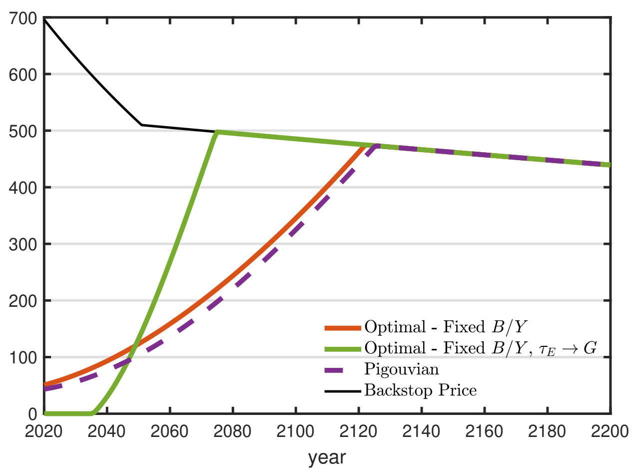
(c) Period Welfare Gains vs. SSP2



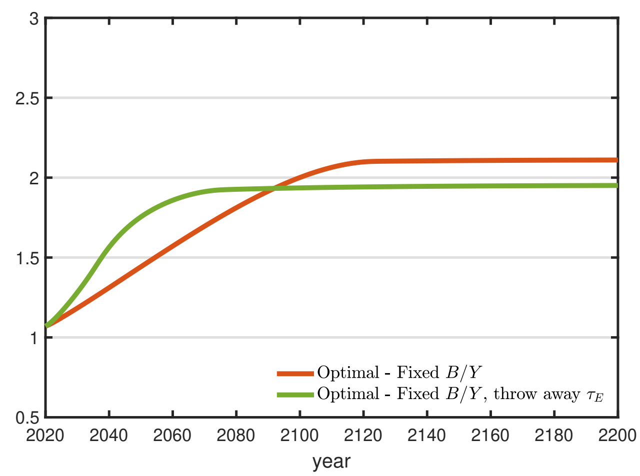
(d) Period Welfare Gains vs. SSP5

Figure G.3: Optimal Carbon Taxes vs. SSP2 and SSP5.

Note: Panel (a) shows the temperature path in the baseline scenario (blue) where the debt-to-GDP ratio and income taxes are set at their current level, and in the SSP2 (yellow) and SSP5 (red) scenarios where all instruments are set at the baseline levels except for the carbon tax and lump-sum transfers. The dashed lines correspond to the temperatures predicted in each of these scenarios by IPCC (2021), and the shaded blocks represent the corresponding "very likely range." Panel (b) shows the path of lump-sum transfers over GDP for the three scenarios. Panel (c) shows the period welfare gains of moving from SSP2 to the optimal carbon taxes for all households (purple), for the households in the lowest quintiles of both labor income and wealth (green), and in the highest quintiles (dark red). Panel (d) does the same for SSP5.



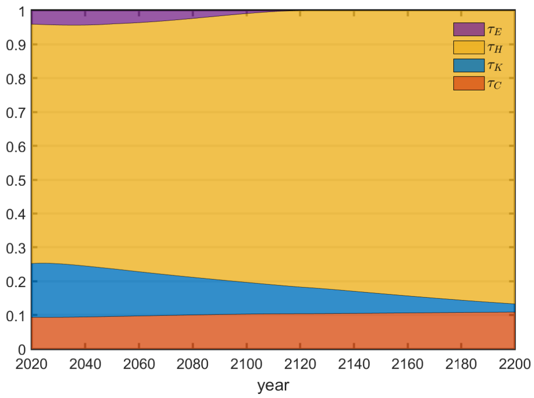
(a) Optimal Carbon Tax (in $/tCO$_2$)



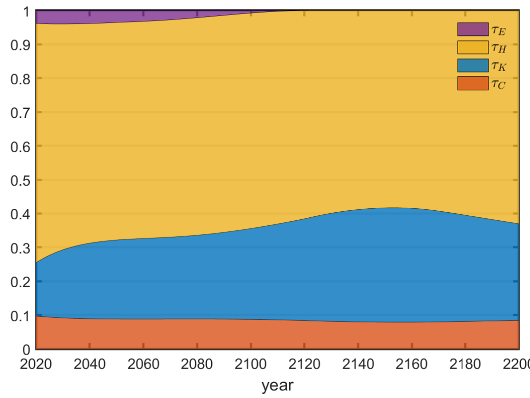
(b) Temperature Path (in °C)

Figure G.4: Optimal Carbon Tax and Temperature Path: Wasteful Spending vs. Baseline

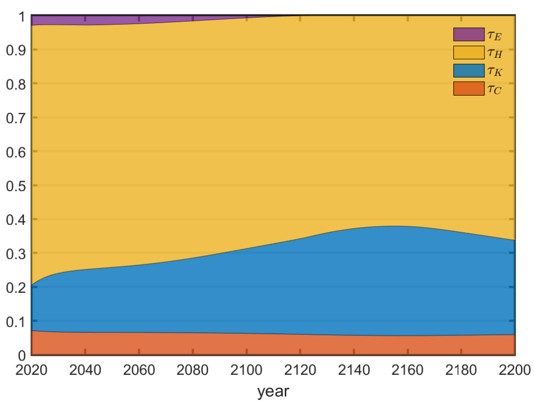
Note: The left figure plots the paths of optimal carbon taxes in the baseline scenario where the debt-to-GDP ratio and income taxes are set at their current level and where the tax is rebated lump-sum (red), and in a similar scenario where the tax finances wasteful government spending (green). The dashed purple line represents the SCC evaluated at the allocation of the baseline scenario, and the black line plots the backstop price. All values are expressed in $/tCO$_2$. The right figure plots the path of temperature for the two scenarios, with values expressed in °C.



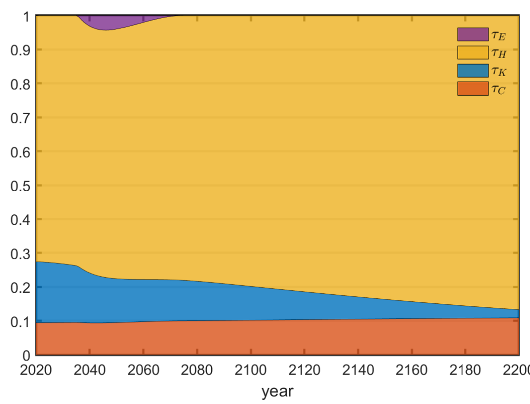
(a) Baseline: fixed B/Y



(b) Flexible B/Y



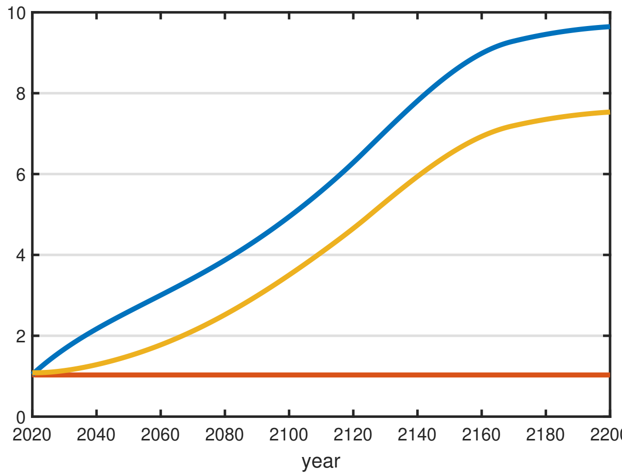
(c) Flexible B/Y and Opt Inc. Taxes



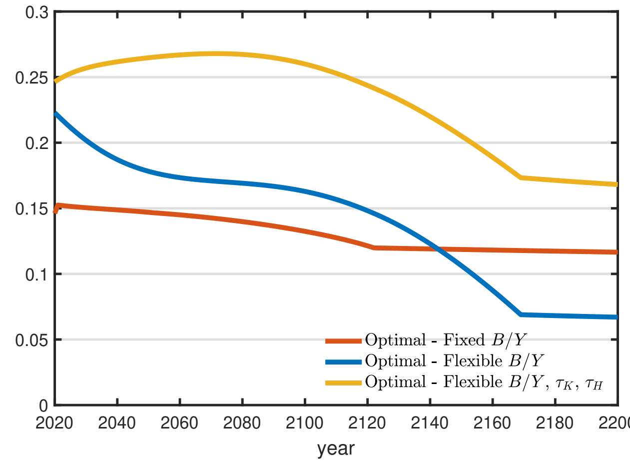
(d) Fixed B/Y and Wast. Spending

Figure G.5: Sources of tax revenue

Note: The figures plot the share of government revenue that comes from the carbon tax (purple), the labor tax (yellow), the capital tax (blue), and the consumption tax (orange) in four scenarios: in the baseline where the debt-to-GDP ratio and income taxes are set at their current level (top-left), in the second scenario where the government can choose the path of debt (top-right), in the third scenario where it can also choose income taxes (bottom-left) and in a scenario similar to the baseline but where the tax finances wasteful government spending (bottom-right). In the latter case, the figure displays the potential revenue raised by the carbon tax, even though it is actually thrown away.



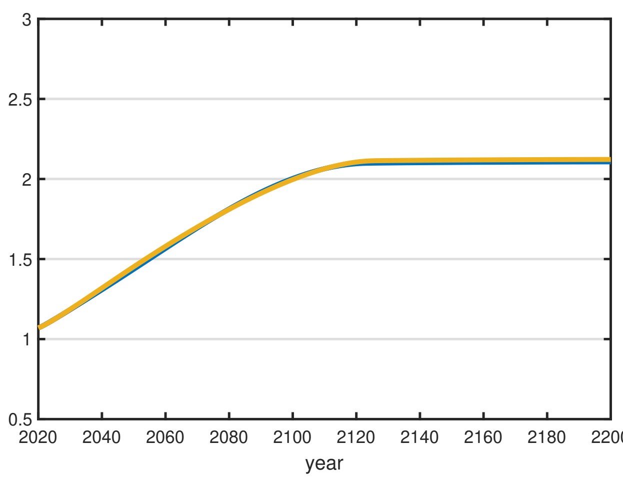
(a) Debt-to-GDP ratio (B/Y)



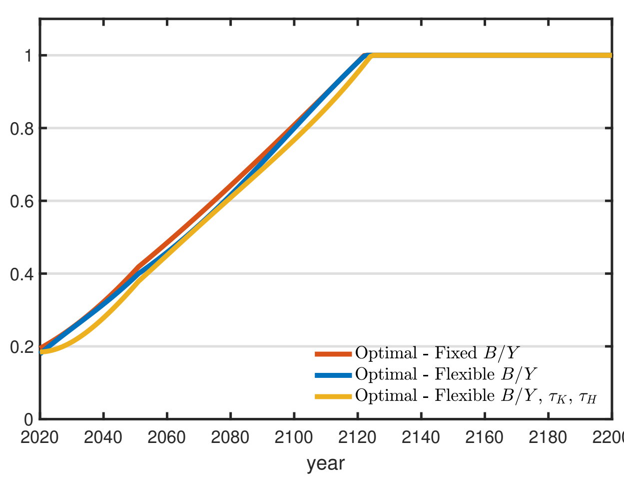
(b) Transfers-to-GDP ratio (T/Y)

Figure G.6: Ratios of Debt and Transfers to GDP

Note: The left figure plots the paths of debt-to-GDP ratio (B/Y) in the baseline scenario where the debt-to-GDP ratio and income taxes are set at their current level (red), in the second scenario where the government can choose the path of debt (blue), and in the third scenario where it can also choose income taxes (yellow). The right figure plots the path of transfers-to-GDP ratios (T/Y) for these three scenarios.



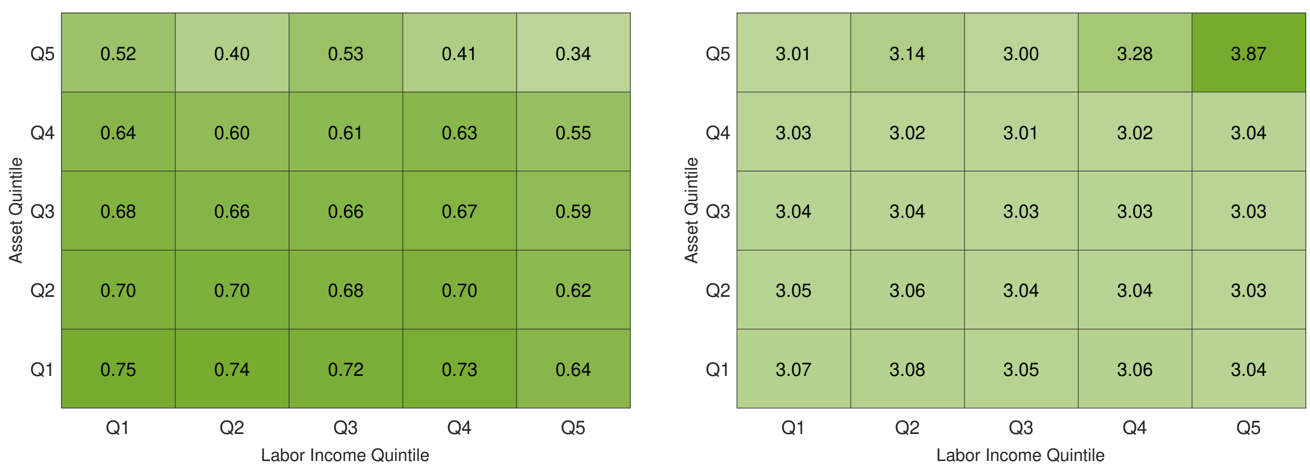
(a) Temperature (in °C)



(b) Abatement (in %)

Figure G.7: Paths of Temperature and Abatement.

Note: The figure plots the paths of temperature expressed in °C change compared to 1900 levels (left) and abatement expressed in % (right) in the baseline scenario where the debt-to-GDP ratio and income taxes are set at their current level (red), in the second scenario where the government can choose the path of debt (blue), and in the third scenario where it can also choose income taxes (yellow).



(a) Optimal vs. SSP2

(b) Optimal vs. SSP5

Figure G.8: Distribution of Welfare Gains by Assets and Labor Income with Stone-Geary Utility

Note: The figure plots the welfare gains expressed as the percentage increase in consumption required under policy SSP2 (left) and SSP5 (right) to make different groups of households indifferent between this policy and the optimal one. The groups correspond to quintiles of labor income, further divided into asset quintiles. The optimal policy is the baseline one in the extended economy with Stone-Geary preferences with fixed debt-to-GDP and income taxes. In the SSP scenarios, all instruments remain at their baseline levels except for the carbon tax and lump-sum transfers.